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Identification of supply models of retailer and manufacturer oligopoly pricing[☆]

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Abstract

We present sufficient conditions for data on an industry's product prices, quantities, and input prices to identify retailers' and manufacturers' vertical supply model. Identification requires nonlinear demand for homogeneous products and multi-product firms with non-constant markups for differentiated products.

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1. Introduction

A number of recent studies have introduced retailers into structural econometric models of sequential vertical-pricing games (Mortimer, 2004; Sudhir, 2001; Villas-Boas and Zhao, 2005; Villas-Boas, 2005), relaxing the conventional assumption that manufacturers set prices and that retailers act as neutral pass-through intermediaries (Bresnahan, 1982; Nevo, 1998). In traditional structural econometric models, the model of firms' pricing behavior is identified via the estimation

[☆] The views in this note are those of the authors and do not necessarily represent the position of the Federal Reserve Bank of New York or of the Federal Reserve System.

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of a conduct parameter that measures manufacturers' deviations from price-taking behavior for homogeneous-products industries and from Bertrand-pricing behavior for differentiated-products industries.

This note provides conditions under which a vertical model of multiple retailers' and manufacturers' oligopoly-pricing behavior is identified. Modeling firms' behavior along a vertical channel has important implications for the analysis of manufacturer mergers (Manuszak, 2001), of firm pass-through of foreign-trade policies (Feenstra, 1989; Hellerstein, 2005), and of price dynamics in the economy as a whole (Chevalier et al., 2003).

Suppose a researcher observes a time series of retail price-quantity pairs which she believes to be market-equilibrium outcomes of demand and supply conditions. The general identification problem is to infer the distributions of consumers' and firms' decision rules, which are not observable, from the decisions themselves, which are observable, as price-quantity pairs. But without additional information, various combinations of demand and supply models may appear to produce the same observable decisions, or price-quantity pairs, over time (Working, 1926; Bresnahan, 1982).

The econometric problem is, thus, a standard simultaneous-equation model in which a demand and a supply pricing equation, both derived from behavioral assumptions, must be estimated. The demand equation relates quantity purchased to price, product characteristics, and unobserved demand determinants. The supply equation relates prices to a markup and to observed and unobserved cost determinants. The researcher typically does not have access to the prices retailers pay to manufacturers. In many industries the researcher can get data on retailers' and manufacturers' input prices. This paper's main goal is to establish when data on an industry's retail prices, quantities, and input prices over time are sufficient to identify the vertical model of manufacturers' and retailers' oligopoly-pricing behavior given that demand and supply relations are not known a priori.

This note's framework, with its assumption of symmetric retailers, differs in several important ways from the theoretical literature on vertical arrangements that focuses on differences across retailers. Rather than dealing with these more general forms of vertical arrangements, this note's contribution is to establish conditions under which one may free up the assumption of a single ownership matrix and single conduct parameter, those of the manufacturer, in empirical work to incorporate the behavior of retailers with their own ownership matrices and conduct parameters.

The next section sets out the identification problem for a homogeneous-products model and shows that the vertical supply model is identified given nonlinear demand. The Third section shows that for a differentiated-products model the vertical supply model is identified under very general conditions, even with linear demand, except in special cases where markups are constant or an industry has exclusive dealerships with single-product firms.

2. Homogeneous products

Let the inverse demand for a particular product be given by

$$p = h(Q, Y, \alpha) + \varepsilon, \tag{1}$$

where p is the retail price, Q is quantity, Y contains exogenous variables that affect demand, α contains demand parameters to be estimated, and ε is the random error term.

On the supply side, assume the standard linear-pricing model that leads to double-marginalization in which manufacturers set wholesale prices p^w and retailers follow setting retail prices p . Retailers have constant marginal costs: $c^r = \beta_r + \gamma_r W$ where W represents exogenous variables that affect cost and β_r and γ_r are parameters to be estimated. Manufacturers have constant marginal costs: $c^w = \beta_w + \gamma_w W$, where β_w and γ_w are manufacturer cost parameters to be estimated.

If retailers behave as price-takers, one can write $p = p^w + c^r$. Otherwise, they set their perceived marginal revenue equal to marginal cost. Define a parameter λ_r to be estimated that is interpreted as measuring retail firms' deviations from price-taking behavior. Note that λ_r does not vary by retailer. As a scalar parameter for the market, it requires symmetric retailers. Conventional models of supply that ignore vertical structure implicitly assume that $\lambda_r = 0$.¹ Marginal revenue is given by $p + h'(Q,.)Q$, and retailers' perceived marginal revenue is given by $p + \lambda_r h'(Q,.)Q$. Retailers' supply relation is:

$$p = p^w - \lambda_r h'(Q)Q + \beta_r + \gamma_r W + \eta_r, \quad (2)$$

where η_r is the retail supply random term with unobserved components of retail costs.

Given that retailers behave according to Eq. (2), the inverse derived demand faced by manufacturers is: $p^w = h(Q) + \lambda_r h'(Q)Q - \beta_r - \gamma_r W$. Define a parameter λ_w to be estimated that is interpreted as measuring manufacturer firms' deviations from price-taking behavior. Note that λ_w does not vary by manufacturer. As a scalar parameter for the market, it requires symmetric manufacturers. Manufacturers' marginal revenue is: $p^w + h'(Q)Q + \lambda_r h''(Q)Q^2 + \lambda_r h'(Q)Q$, their perceived marginal revenue is: $p^w + \lambda_w [h'(Q)Q + \lambda_r h''(Q)Q^2 + \lambda_r h'(Q)Q]$, and their supply relation is:

$$p^w = -\lambda_w \lambda_r [h''(Q)Q^2 + h'(Q)Q] - \lambda_w h'(Q)Q + \beta_w + \gamma_w W + \eta_w, \quad (3)$$

where η_w is the manufacturer supply random term with unobserved components of manufacturer costs.

If the researcher has wholesale-price data as well as retail and manufacturer cost data, he can estimate Eqs. (1) (2), and (3) simultaneously, treating price and quantity as endogenous variables. In most cases, however, neither wholesale-price data nor information on what part of marginal cost is attributable to retailers and what to manufacturers are available. Then the pricing equation to be estimated is obtained by substituting Eq. (3) into Eq. (2), which gives:

$$p = -(\lambda_r + \lambda_w)h'(Q)Q - \lambda_r \lambda_w [h''(Q)Q^2 + h'(Q)Q] + \beta_{r+w} + \gamma_{r+w}W + \eta. \quad (4)$$

Under what conditions are the parameters λ_r and λ_w , the demand parameters α , and the cost parameters β and γ identified? First, given constant marginal costs, the exogenous cost variables W must differ from the exogenous demand variables Y and the dimensions of W must be such that the demand parameters α are identified. We require additive separability in costs across products for both the retailer and the manufacturer. We also require that there be no interaction between retailer and manufacturer cost variables. Correlation between retailer and manufacturer cost variables, for example, makes identification of λ_r and λ_w problematic. Second, the parameters λ_r and λ_w are identified if demand is non-linear. In the special case of linear demand the parameters λ_r and λ_w cannot be identified separately. A linear demand function, e.g., $Q = \alpha_0 + \alpha_1 p + \alpha_2 Y$ yields $h = \frac{-\alpha_0}{\alpha_1} + \frac{1}{\alpha_1} Q - \frac{\alpha_2}{\alpha_1} Y$ and $h'(Q) = \frac{1}{\alpha_1}$ and

¹ With linear demand and constant marginal costs, for homogeneous products: $\lambda_r = 0$ given price-taking behavior and $\lambda_r \neq 0$ given deviation from price-taking behavior; for differentiated products: $\lambda_r = 0$ given Bertrand-pricing behavior, $\lambda_r = 1$ given Cournot-pricing behavior, and $\lambda_r = \frac{1}{H}$ given perfect collusion across firms where H is a Herfindahl index.

finally $h''(Q)=0$, and Eq. (4) becomes $p = \frac{-[\lambda_r + \lambda_w + \lambda_r \lambda_w] \frac{1}{\alpha_1} Q + \beta_{r+w} + \gamma_{r+w} W + \eta}{[\lambda_r + \lambda_w + \lambda_r \lambda_w] \frac{1}{\alpha_1}}$ and, though we can treat α_1 as known since demand can be estimated, we cannot identify λ_r and λ_w separately. Note also that only β_{r+w} and γ_{r+w} can be identified and not the retailers' β_r and γ_r or the manufacturers' β_w and γ_w separately. Note that in the special case with exclusion restrictions on W , one can identify γ_r and γ_w separately with linear demand. This case requires that input 1 only affect manufacturer costs and input 2 only retailer costs. Finally, in the special case of nonlinear demand with constant demand elasticities, identification fails as markups are constant so one cannot distinguish markup adjustments from marginal-cost movements. In this case, only when markups are constant and equal to zero can conduct be identified correctly as price-taking behavior.²

3. Differentiated products

Assume N differentiated products and let q_n denote demand for product n :

$$q_n = q(p_1, \dots, p_N, Y, \alpha) + \varepsilon_n, \tag{5}$$

where p_1, \dots, p_N are retail prices, Y contains exogenous variables that affect demand, α contains demand parameters to be estimated, and ε_n is the random error term.

On the supply side, assume the standard linear-pricing model that leads to double marginalization where M manufacturers set wholesale prices p^w and R retailers follow setting retail prices p . Let retailers' marginal costs be constant: $c^r = \beta_r + \gamma_r W$ as well as manufacturers' marginal costs: $c^w = \beta_w + \gamma_w W$.

Each retailer maximizes his profit function:

$$\pi_r = \sum_{j \in S_r} [p_j - p_j^w - c_j^r] q_j(p) \quad \text{for } r = 1, \dots, R. \tag{6}$$

where S_r is the set of products sold by retailer r . The first-order conditions, assuming a pure-strategy Nash equilibrium in retail prices, are:

$$q_j + \sum_{m \in S_r} T_r(m, j) [p_m - p_m^w - c_m^r] \frac{\partial q_m}{\partial p_j} = 0 \quad \text{for } j = 1, \dots, N. \tag{7}$$

Switching to matrix notation, define $[A * B]$ as the element-by-element multiplication of two matrices of the same dimensions A and B . Define a matrix T_r with general element $T_r(i, j) = 1$ if the retailer sells products i and j and equal to zero otherwise. Let Δ_r be a matrix with general element $\Delta_r(i, j) = \frac{\partial q_j}{\partial p_i}$. Solving Eq. (7) for price-cost margins gives, in vector notation:

$$\underbrace{p - p^w - c^r}_{m^r} = - [T_r * \Delta_r]^{-1} q(p), \tag{8}$$

which is a system of N implicit functions that expresses the N retail prices as functions of the wholesale prices. If retailers behave as Nash-Bertrand players then Eq. (8) describes their supply relation. Define an N -by-1 vector of parameters, \mathcal{A}_r , that measures the deviation from the underlying retail-pricing model for

² In the extraordinary case where markup adjustments exactly offset marginal-cost changes, prices will be constant, and identification will fail.

each product as well as a matrix M_r with diagonal elements given by the vector m^r . The supply relation becomes

$$p = p^w + M_r A_r + \beta_r + \gamma_r W + \eta_r, \tag{9}$$

where η_r is the retail supply random error term with unobserved components of retail costs.

Manufacturers choose wholesale prices p^w to maximize their profits knowing that retailers behave according to Eq. (9). Solving for manufacturers’ first-order conditions, assuming again a pure-strategy Nash equilibrium in wholesale prices and using matrix notation, yields:

$$\underbrace{(p^w - c^w)}_{m^w} = - [T_w * \Delta_w]^{-1} q(p), \tag{10}$$

where T_w is a matrix with general element $T_w(i,j)=1$ if the manufacturer sells products i and j and equal to zero otherwise, Δ_w is a matrix with general element $\Delta_w(i,j) = \frac{\partial q_i}{\partial p_i^w}$, and $*$ represents the element-by-element multiplication of both matrices.

To obtain Δ_w , note that $\Delta_w = \Delta_p' A_r$, where Δ_p is a matrix of derivatives of all retail prices with respect to all wholesale prices. To get the expression for Δ_p , we totally differentiate for a given j in Eq. (7) with respect to all retail prices ($dp_k, k=1, \dots, N$) and with respect to the wholesale price p_j^w , with variation dp_j^w :

$$\sum_{k=1}^N \left[\frac{\partial q_j}{\partial p_k} + \sum_{i=1}^N (T_r(i,j) \frac{\partial^2 q_i}{\partial p_j \partial p_k} (p_i - p_i^w - c_i^r)) + T_r(k,j) \frac{\partial q_k}{\partial p_j} \right] dp_k - \underbrace{T_r(f,j) \frac{\partial q_f}{\partial p_j}}_{S(j,f)} dp_j^w = 0. \tag{11}$$

Putting all $j=1, \dots, N$ products together, let G be a matrix with general element $g(j,k)$ and let S_f be an N -dimensional vector with general element $S(j,f)$. Then $G dp - S_f dp_j^w = 0$. Solving for the derivatives of all retail prices with respect to the wholesale price p_j^w , the f -th column of Δ_p is obtained: $\frac{dp}{dp_j^w} = G^{-1} S_f$. Stacking all N columns together, $\Delta_p = G^{-1} S$, which has the derivatives of all retail prices with respect to all wholesale prices. The general element of Δ_p is $(i,j) = \frac{\partial p_i}{\partial p_j^w}$.

If manufacturers behave as Nash-Bertrand players then Eq. (10) describes their supply relation. If one associates an N -by-1 vector of parameters A_w , one for each product, that measures deviations from the underlying model of manufacturer-pricing behavior, then the supply relation for the manufacturers becomes:

$$p^w = M_w(A_r, M_r) A_w + \beta_w + \gamma_w W + \eta_w, \tag{12}$$

where η_w is a supply random term with unobserved components of manufacturer costs and $M_w(A_r, M_r)$ is a matrix with the m^w defined by Eq. (10) on its diagonal elements, and is in general a function of the retail-pricing “behavior” represented by A_r and of the retail margins M_r . The supply equation to be estimated is obtained by substituting (12) into Eq. (9) which yields:

$$p = M_r A_r + M_w(A_r, M_r) A_w + \beta_{r+w} + \gamma_{r+w} W + \eta. \tag{13}$$

Assume that W in (13) and Y in Eq. (5) are exogenous variables that differ from one another and that the dimension of W is such that the parameters of demand α are identified. For each product j , Eq. (13) is:

$$p_j = \underbrace{\lambda_j^w [- [T_w^* \Delta_p' \Delta_r]^{-1}]_{j-\text{line}} q(p)}_{m_j^w(T_w, T_r)} + \underbrace{\lambda_j^r [- [T_r^* \Delta_r]^{-1}]_{j-\text{line}} q(p)}_{m_j^r(T_r)} + \beta_{r+w} + \gamma_{r+w} W + \eta. \tag{14}$$

We require additive separability in costs across products for both the retailer and the manufacturer. We also require there to be no interaction between retailer and manufacturer cost variables. Correlation between retailer and manufacturer cost variables, for example, makes identification of λ_r and λ_w problematic. If for a product j we have $m_j^w(T_w, T_r) = K m_j^r(T_r)$ where K is a non-zero constant (where manufacturer margins are proportional to retail margins) then the retail and manufacturer models are not identified: for a certain j we could estimate $\lambda_r + K \lambda_w$ but not λ_r and λ_w separately. Furthermore, if $m_j^w(T_w, T_r) = a + K m_j^r(T_r)$, then we can estimate $\lambda_r + K \lambda_w$ and $a \lambda_w + \beta_{r+w}$ but not β_{r+w} , λ_r , and λ_w separately. Note further that if $K=0$ and so $m_j^w(T_w, T_r) = a$ (a constant wholesale mark-up independent of the retail mark-up and independent of the quantity sold), then λ_r is identified but only $a \lambda_w + \beta_{r+w}$ can be estimated and not β_{r+w} and λ_w separately. Unlike in the homogeneous-products model, identification in the differentiated-products model is possible with linear demand given non-constant markups. Next, we show formally that with linear demand the parameters in A_r and A_w can be identified separately.

Consider without loss of generality a simple model of two manufacturers selling two products to two retailers, where the goal is to examine Eq. (14). The two manufacturers a and b produce one good each, which they sell to the two retailers 1 and 2. Without loss of generality assume that manufacturer b 's product is retailer 2's private label. The model has three retail-level products: product 1 produced by manufacturer a and sold to retailer 1; product 2 produced by manufacturer a and sold to retailer 2; and product 3 produced by manufacturer b and sold to retailer 2, where the retailer's product matrix is:

$$T_r = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

and the manufacturers' product matrix is:

$$T_w = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Let $d_{i,j} + \frac{\partial q_i}{\partial p_j}$. Solving Eq. (8) for each product yields:

$$\begin{bmatrix} m_1^r \\ m_2^r \\ m_3^r \end{bmatrix} = \frac{1}{d_{11}(d_{22}d_{33} - d_{23}d_{32})} \begin{bmatrix} d_{22}d_{33} - d_{23}d_{32} & 0 & 0 \\ 0 & d_{11}d_{33} & -d_{11}d_{32} \\ 0 & -d_{11}d_{23} & d_{11}d_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}. \tag{15}$$

To obtain manufacturers’ price-cost margins, one must compute Δ_w , a matrix with a complicated expression even for this simple model. The matrix $\Delta_w = \Delta_p' \Delta_r$, where Δ_p is a matrix of derivatives of each retail price with respect to each wholesale price given by:

$$\begin{aligned} \Delta_p &= G^{-1}S \\ &= \begin{bmatrix} 2d_{11} + h_{111}m_1^r & d_{12} + h_{112}m_1^r & d_{13} + h_{113}m_1^r \\ d_{21} + h_{221}m_2^r + h_{321}m_3^r & 2d_{22} + h_{222}m_2^r + h_{322}m_3^r & d_{23} + h_{223}m_2^r + h_{323}m_3^r + d_{32} \\ d_{31} + h_{231}m_2^r + h_{331}m_3^r & d_{32} + h_{232}m_2^r + h_{332}m_3^r + d_{23} & 2d_{33} + h_{233}m_2^r + h_{333}m_3^r \end{bmatrix}^{-1} \\ &\quad \times \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{32} \\ 0 & d_{23} & d_{33} \end{bmatrix} \end{aligned} \tag{16}$$

and where $h_{ijk} = \frac{\partial q_i^2}{\partial p_j \partial p_k}$. Solving for the manufacturers’ mark-ups in Eq. (10) for each product gives us:

$$\begin{bmatrix} m_1^w \\ m_2^w \\ m_3^w \end{bmatrix} = \frac{1}{\frac{\det(T_w * \Delta_w)}{\det G}} A \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

where the expressions for the matrix A are given in the Appendix. Finally we must substitute the relevant lines of m_j^w from the matrix M and m_j^r from Eq. (15) into Eq. (14) and look at cases where m_j^w is proportional to or an affine transformation of m_j^r , which leaves us unable to identify manufacturer and retailer pricing behavior separately. Next, we show that even for the linear case, if m_j^w and m_j^r are non-constant markups, then we can separately identify λ_r and λ_w .

Let demand for each product be given by

$$q_n = \alpha_{n0} + \alpha_{n1}p_1 + \alpha_{n2}p_2 + \alpha_{n3}p_3 + Y\alpha_{n4} + \varepsilon_n, \text{ for } n = 1, 2, 3. \tag{17}$$

Consider without loss of generality product 1. The retail margin from Eq. (15) for the linear case is: $m_1^r = \frac{q_1}{\alpha_{11}}$, and the manufacturer margin is: $m_1^w = K \frac{q_1}{\alpha_{11}} - K_2q_2$, where

$$K = T_w(1, 1) \frac{\det G}{\det(T_w * \Delta_w)} \alpha_{11} [2\alpha_{11}^3 - \alpha_{12}\alpha_{11}\alpha_{21} + \alpha_{13}\alpha_{11}\alpha_{31}], \tag{18}$$

$$K_2 = T_w(1, 2) \frac{K}{\alpha_{11}} \underbrace{[2\alpha_{22}^2\alpha_{21} + \alpha_{31}\alpha_{33}\alpha_{11} - \alpha_{32}\alpha_{33}\alpha_{21} - \alpha_{23}\alpha_{33}\alpha_{21} - \alpha_{21}\alpha_{22}\alpha_{11} - \alpha_{12}\alpha_{11}\alpha_{21}]}_{K_3}, \tag{19}$$

and $T_w(1,1)=T_w(1,2)=1$. Note, first, that for this linear case, if $K_2=0$ then the manufacturer margin is proportional to the retail margin, and identification fails. This happens if $T_w(1,2)=0$ (which implies by definition that $T_w(2,1)=0$) if the matrix T_w is diagonal in its upper-left 2-by-2 minor. This corresponds to

a market structure of single-product manufacturers working with single-product retailers that each act as an exclusive dealer of one product in the upper-left minor. For the case where the demand parameters satisfy $K_3=0$, identification also fails in this simple model. Second, if $m_j^w(T_w, T_r) = a + Km_j^r(T_r)$, the manufacturer margin is an affine transformation of the retail margin, then we can estimate $\lambda_r + K\lambda_w$ and $a\lambda_w + \beta_{r+w}$ but not β_{r+w}, λ_r , and λ_w separately. Third, if $K=0$ and so $m_j^w(T_w, T_r) = a$ (a constant wholesale mark-up independent of the retail mark-up and independent of the quantity sold), then λ_r is identified but only $a\lambda_w + \beta_{r+w}$ can be estimated and not β_{r+w} and λ_w separately. Thus, for the linear case, if m_j^w and m_j^r are non-constant markups, then we can separately identify λ_r and λ_w . In the special case of nonlinear demand with constant demand elasticities, identification fails as markups are constant so one cannot distinguish markup adjustments from marginal-cost movements. Only when markups are constant and equal to zero can conduct be identified correctly as Bertrand-pricing behavior. Finally, in the extraordinary case where markup adjustments exactly offset marginal-cost changes, prices will be constant, and identification will fail.

4. Conclusion

This note outlines conditions under which we can identify a model of multiple retailers' and manufacturers' oligopoly-pricing behavior. We show that such models are identified in homogeneous-products models given nonlinear demand and in differentiated-products models in general for multi-product retailers and manufacturers, even with linear demand. Identification may fail in special cases where an industry has exclusive dealerships and single-product firms or where firms have constant markups.

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Appendix A

$$[T_w^*A_w]^{-1} = \frac{A}{\frac{\det(T_w^*A_w)}{\det G}}$$

$$= \frac{\begin{bmatrix} \mu d_{11}^2 + \varepsilon + & -((\eta + \theta)d_{11} + (1 + \kappa)d_{21} + \varepsilon) & 0 \\ -(\mu d_{11}d_{12} + \nu d_{11}d_{22} + \zeta d_{11}d_{32}) & (\eta + \theta)d_{12} + (1 + \kappa)d_{22} + (\rho + \varpi)d_{32} & 0 \\ 0 & 0 & (\pi + \theta)d_{13} + (\xi + \kappa)d_{23} + (\sigma + \varpi)d_{33} \end{bmatrix}}{\frac{\det(T_w^*A_w)}{\det G}}$$

where $d_{ij} = \frac{\partial q_i}{\partial p_j}$; $h_{ijk} = \frac{\partial^2 q_i}{\partial p_j \partial p_k}$; $\varepsilon = \nu d_{11}d_{21}$; $\zeta = \varsigma d_{11}d_{31}$; $\eta = (-d_{21} - h_{221}m_2^r - h_{321}m_3^r)d_{22}$; $\theta = (d_{31} + h_{231}m_2^r + h_{331}m_3^r)d_{33}$; $\iota = (2d_{22} + h_{222}m_2^r + h_{322}m_3^r)d_{22}$; $\kappa = (-d_{32} - h_{232}m_2^r - h_{332}m_3^r - d_{23})d_{33}$; $\mu = 2d_{11} + h_{111}m_1^r$; $\nu = -d_{12} - h_{112}m_1^r$; $\xi = (2d_{22} + h_{222}m_2^r + h_{322}m_3^r)d_{32}$; $\pi = (-d_{21} - h_{221}m_2^r - h_{321}m_3^r)d_{32}$; $\varpi = (2d_{33} + h_{233}m_2^r + h_{333}m_3^r)d_{33}$; $\rho = (-d_{23} - h_{223}m_2^r - h_{323}m_3^r - d_{32})d_{22}$; $\sigma = (-d_{23} - h_{223}m_2^r - h_{323}m_3^r - d_{32})d_{32}$; $\varsigma = d_{13} + h_{113}m_1^r$.

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