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RELATIVISTIC TREATMENT OF QUARKONIUM IN QCD ☆

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We have formulated a relativistic two-body equation for quark−antiquark bound states using the expansion in intermediate states. We have used the asymptotically free QCD gluon exchange with either a scalar or equal vector and scalar linear potential. Using these we have fitted the upsilon, charmonium, D, F, $s\bar{s}$, $\bar{s}$−u,d and u,d quark mesons for the rho and above. We have also calculated relativistically their widths, singlet−triplet and P-state splittings.

We have formulated a relativistic two-body equation for quark−antiquark systems using the multi-quantum intermediate state expansion of the field equations used by Greenberg [1], Gross [2], and Johnson [3]. We will summarize the formulation below. A more detailed analysis will be published elsewhere [4]. We have applied this to quark−antiquark systems and have compared the results with the Υ, ψ, B, F, and D families as well as lower mass mesons which are not affected by chiral symmetry breaking or large mixings. We have used an asymptotically free QCD vector exchange and a linear part which is either totally a scalar exchange or an equal combination of vector and scalar exchanges. The scalar results are presented in this letter; see ref. [4] for the other case.

We will present our fits to the spectra and leptonic decay widths for the mesonic systems excluding the lightest pion, kaon, and etas. The parameters used are the linear slope, the QCD potential scale Λ_R, the quark masses, and a cutoff on the slowly diverging integral equations which represents the effects of

higher mass intermediate states, and is reset for each $\bar{q}q$ system.

The derivation starts with the field theory equations for the quark field and for the vector and scalar potential fields. We use the asymptotic freedom modified vector gluon exchange plus effective linear exchange as our best knowledge of the solution to the potential field equations. We then proceed with the equation for the quark field in the potentials produced by the antiquark:

$$(i\not{\partial} - m_1)\,\psi(x) = g_v\not{A}(x)\,\psi(x) + g_s S(x)\,\psi(x)\,, \qquad (1)$$

where m_1 is the quark mass. The equation may be solved in terms of all of its matrix elements between the bound state and the quark−gluon basis. To do this we insert a complete set of states on the right-hand side between a potential field and the quark field. The valence quark approximation which we use is obtained by limiting the intermediate states to the single antiquark states, which gives an integral equation for the matrix element of the quark field between the bound state and the on-shell antiquark state.

The resulting integral equation has the following properties: (a) Upon performing a partial wave analysis it becomes a one-variable integral equation.

(b) Since one leg is on-shell it is current conserving and even the one-gluon exchange is gauge invariant. (c) In the nonrelativistic limit it reduces to the Darwin–Breit hamiltonian, which includes the v^2/c^2 effects. (d) It has the Dirac equation as a limit as the on-shell antiquark becomes extremely heavy relative to the off-shell quark. (e) It includes retardation effects which cannot be reproduced by spatial potentials. For another application to deeply bound composite models of fermions, see ref. [5].

We define the matrix element of the quark field between the bound state and the on-shell antiquark state as

$$\Psi_\alpha(p, \lambda) = (2\pi)^3 (2\omega_B \omega/m_2)^{1/2} \langle \bar{p}, \lambda | \psi_\alpha(0) | \bar{B} \rangle, \quad (2)$$

where $\omega_B = (\bar{B}^2 + M^2)^{1/2}$, $\omega = (\bar{p}^2 + m_2^2)^{1/2}$, m_2 is the antiquark mass and M is the unknown bound state mass. We transform this to a 4×4 component wave function $\Phi(p)$,

$$\Phi_{\alpha\beta}(p) = \sum_\lambda \Psi_\alpha(p, \lambda) \, \bar{v}_\beta(p, \lambda), \quad (3)$$

with subsidiary condition that $\Phi(\not{p} + m_2) = 0$. The equation for Φ is then

$$(\not{B} - \not{p} - m_1) \, \Phi(p)$$

$$= \int \frac{d^3 p'}{(2\pi)^3 \, 2\omega'} \, [V_v((p' - p)^2) \, \gamma_\mu \Phi \gamma^\mu (\not{p} - m_2)$$

$$+ V_s((p' - p)^2) \, \Phi(\not{p} - m_2)] \, . \quad (4)$$

We then define large and small Dirac components for the quark field, labeled by G, F, and for the antiquark field, labeled by u, d, as parts of Φ

$$\Phi = \begin{bmatrix} \tilde{G}_u & \tilde{G}_d \\ \tilde{F}_u & \tilde{F}_d \end{bmatrix}, \quad (5)$$

where $\tilde{G}_d$ is the large component in the non relativistic limit. The subsidiary condition on Φ leaves only $\tilde{G}_d$ and $\tilde{F}_d$ as independent:

$$\tilde{G}_u = -\tilde{G}_d \, \frac{\bar{\sigma} \cdot \bar{p}}{\omega + m_2}, \quad \tilde{F}_u = -\tilde{F}_d \, \frac{\bar{\sigma} \cdot \bar{p}}{\omega + m_2}. \quad (6)$$

In order to perform an angular momentum decomposition we go to a direct product representation with $G_d = \tilde{G}_d \sigma_y$ and $F_d = \tilde{F}_d \sigma_y$. We then expand in an angular momentum basis

$$G_d(\bar{p}) = g_-(p) | J, M; L = J - 1, S = 1 \rangle$$

$$+ g_+(p) | J, M; L = J + 1, S = 1 \rangle,$$

$$F_d(\bar{p}) = f_0(p) | J, M; L = J, S = 0 \rangle$$

$$+ f_1(p) | J, M; L = J, S = 1 \rangle. \quad (7)$$

Expanding the potential in Legendre polynomials of $\cos \theta = \hat{p} \cdot \hat{p}'$ and performing the angular integrals gives the "radial" momentum integral equation containing the bound state mass as an eigenvalue

$$[M + T'(p)] \begin{pmatrix} g_- \\ g_+ \\ f_0 \\ f_1 \end{pmatrix} (p) = \int dp' \, K_J(p, p') \begin{pmatrix} g_- \\ g_+ \\ f_0 \\ f_1 \end{pmatrix} (p'), \quad (8)$$

where $T'(p)$ is a matrix of kinetic factors and $K_J(p, p')$ is the kernel matrix containing the angular momentum projections of the potentials. (See ref. [4] for details.)

The integral equation diverges slowly at large momentum no worse than logarithmically. Before that becomes important, however, the neglected multi-quanta states become important in competing for probability or amplitude with the valence quark states. This is put in as a cutoff which can vary to describe the differing physical cutoffs in each channel and is of the form:

$$S(\Lambda_c, p) = \Lambda_c^2 / (\Lambda_c^2 + p^2). \quad (9)$$

Other forms of the cutoff were also tried but do not affect the success of the fits.

For the potential we start with the one-gluon exchange modified by the renormalization group asymptotic freedom corrections at short distances in a form [6] [+1] that avoids pseudopoles at $\bar{q}^2 = \Lambda_R^2$ and behaves as $[\bar{q}^2 \ln(1 + \bar{q}^2/\Lambda_R^2)]^{-1}$. As a gluon exchange this must occur in the vector potential, but it also contains a q^{-4} limit at small $\bar{q}^2$ corresponding to a linear potential at large r. In a relativistic equation, however, this purely vector linear potential would lead to a Klein paradox. To avoid this we subtract off the linear part and use for our gluon exchange the relativistic form:

[+1] For spectroscopy with a more exact asymptotic freedom form see ref. [7].

$$V_c(q^2) = -4\pi \tfrac{16}{27}\pi \{[-q^2 \ln(1-q^2/\Lambda_R^2)]^{-1}$$

$$- \Lambda_R^2/(q^2)^2\} .\tag{10}$$

We then add in the linear potentials separately in the scalar part for the fits of this paper (see also ref. [4] for a combination of vector and scalar linear parts).

We first present our fit to the charmonium system. We expect that for the centers of the levels before splitting, the parameters used in previous nonrelativistic fits [6] will be most useful. This is indeed the case, and we take $\Lambda_R = 0.4$ GeV and $\kappa_s = 0.15$ GeV^2. The P state splittings, the leptonic decay widths and the $\psi-\eta_c$ splittings are then fitted by choosing the cutoff Λ_c appropriately. The quark mass is then fixed to obtain the correct value for the $\psi(3097)$. The fits are thus presented in table 1 in terms of the excitation energy above the ψ. The location of the levels are in general of the same accuracy as other fits [8]. We

take due caution in the accuracy expected since studies of the mixing to D–$\overline{\text{D}}$ channels have shown shifts from the bound state equation masses of the order of 100 MeV [9].

Besides the spectral levels, in table 1 we include the leptonic decay width of the ψ and the ratios of the leptonic decay widths of the higher S states; the splitting of the $^3P_2-^3P_0$ levels; the $\psi-\eta_c$ and $\psi'-\eta_c'$ splittings and the ratio $R_p = (^3P_2-^3P_1)/(^3P_1-^3P_0)$. In further words of caution, the QCD corrections to the leptonic widths are not firmly established [10] and the ψ width was only constrained to be no more than a factor of two greater than the observed width, in agreement with many other fits. The ratios of widths are considered to be less ambiguous and are consistent with the data. The ratio R_p is unique, independent of matrix elements, if the potential is a single pure power law [11]. Here, however, two powers are involved in the potential in even the simplest approximation and

Table 1
Charmonium and upsilonium results. $\Lambda_R = 0.4$ GeV, $\kappa_s = 0.15$ GeV^2.

Data	Charmonium ($m_c = 1.57$ GeV, $\Lambda_c = 4.0$ GeV)		Upsilonium ($m_b = 4.90$ GeV, $\Lambda_c = 7.0$ GeV)	
	expt.	calc.	expt.	calc.
4^3S_1	1318	1314	1116	1118
2^3D_1	1063	1016		953
3^3S_1	933	991	890	873
2^3P_{COG}		827	796	788
1^3D_1	671	650		673
2^3S_1	589	586	561	563
1^3P_{COG}	426	395	444	431
1^3S_1	0	0	0	0
$2^3P_2-2^3P_0$		67	38	40
$1^3P_2-1^3P_0$	141	136	41	62
R_{2P}		0.86	0.85 ± 0.1	0.82
R_{1P}	0.48	0.74	0.93 ± 0.1	0.77
$2^3S_1-2^1S_0$	94	38		22
$1^3S_1-1^1S_0$	113	102		60
$\Gamma(1^3S_1)$ (keV)	4.8	9.0	1.24	1.6
$\Gamma(2S)/\Gamma(1S)$	0.40	0.51	0.44	0.41
$\Gamma(3S)/\Gamma(1S)$	0.16	0.34	0.32	0.28
$\Gamma(4S)/\Gamma(1S)$	0.09	0.27	0.22	0.11

R_p depends on the details of the wave functions and is of less fundamental importance. A level shift from multimeson channel mixing of even 25 MeV can change this from 1.0 to 0.5, and such mixing calculations are called for.

For the upsilonium spectra and widths, we use the same value of κ_s and of Λ_R found in charmonium and vary Λ_c to fit the splittings and widths, and then set m_b to set the upsilon (9460) correctly. As before we measure the excitation energy from the upsilon, and give the results in table 1. The widths are closer to the experimental ones than in the charmonium case, indicating the expected decrease in the QCD corrections at larger Q^2. One tentative discrepancy is that the newest results for the P states [12] are showing near equality for the 2^3P_2–2^3P_0 splitting with the 1^3P_2–1^3P_0 splitting, being 38 MeV and 41 MeV, respectively. The calculations yield a ratio of about 0.65 between these splittings.

Since the relativistic equation which we use reduces to the Dirac equation for a heavy antiquark and a light quark, we should be able to describe such mesons as the D, D*, F, F*, and B [13]. We do fit the D and D* states but it requires using two parameters, the light (u or d) quark mass and the cutoff for that system. Until more states are found in that system, the approach cannot really be tested there. The same is true of the F and F* which are fit by setting the strange quark mass and the cutoff for the system. For the newly found B meson, since the b and u, d quark masses are now fixed, we only need to fit the cutoff and then predict the B* mass. The D and D* determine the u and d quark masses to be 250 MeV and the

F and F* determine the strange quark mass to be 410 MeV. The mass difference of the B* and B is predicted to be about 50 MeV, as in other calculations.

The (u, d) or light quarks, and the s quark mesons require a description by a relativistic equation. In this work we must neglect the pion, kaon, and η mesons, i.e. the 0^- nonet, since we do not include chiral symmetry breaking or dynamical mixing calculations. We also leave out the δ $0^+(983)$ and the S^* $0^+(975)$–ϵ $0^+(1300)$ since these show strong couplings to mesonic channels [14] and are not well described by the valence quark approximation. Thus in table 2 we present the $L = 1$ light and strange quark mesons and the 1^{--} $L = 0$ mesons.

In this work [4] we have used a relativistic bound state expansion and equation to simultaneously study the heavy quark, heavy-light and light quark mesons. Our relativistic treatment included spin–orbit, spin–spin, and retardation effects automatically without recourse to a v/c expansion [+2]. Multiquark state effects were included approximately by using a high momentum cutoff. The approach shows successful spectra and leptonic decay rate calculations but does not explain all discrepancies of the physical spectra from the nonrelativistic calculations. A more detailed presentation including results with equal vector and scalar linear potentials will be published elsewhere [4]. Finally we note that the strange or light (u or d) quark mass values increase as the energy scale decreases. This is qualitatively consistent with the mass

[+2] For a recent review of results of v^2/c^2 expansions see ref. [15].

Table 2
Strange and light quark mesons.

Meson		J^{PC}			
		1^{--}	2^{++}	1^{++}	1^{+-}
$s\bar{s}$	experiment	1020	1520	1418	–
	$m_s = 0.52$, $\Lambda_c = 3.0$ GeV	1056	1524	1390	1421
$\bar{s}u$, $\bar{s}d$	experiment	892	1434	1414	1270
	$m_s = 0.55$, $m_{u,d} = 0.35$, $\Lambda_c = 3.5$ GeV	916	1390	1215	1257
u, d ($\bar{u}$, $\bar{d}$)	experiment, $I = 1$	770	1318	1275	1233
	$I = 0$	783	1273	1283	1190
	$m_{u,d} = 0.35$, $\Lambda_c = 2.0$ GeV	832	1264	1167	1186

evolution predicted by QCD in the much higher mass scale where the renormalization group equations are valid.

References

[1] O.W. Greenberg, Phys. Rev. 139B (1965) 1038;
O.W. Greenberg and R.J. Genolio, Phys. Rev. 150 (1966) 1070;
O.W. Greenberg, Orbis Scientiae 1982, University of Maryland preprint 82-195.
[2] F. Gross, Phys. Rev. 186 (1969) 1448.
[3] K. Johnson, Phys. Rev. D4 (1972) 1101.
[4] M. Bander, D. Silverman, B. Klima and U. Maor, Relativistic treatment of quark–antiquark spectra in QCD, University of California (Irvine) report No. 83-22 (1983).
[5] M. Bander, T.-W. Chiu G. Shaw and D. Silverman, Phys. Rev. Lett. 47 (1981) 549.
[6] J.L. Richardson, Phys. Lett. 82B (1979) 272;
R.D. Carlitz and D.B. Creamer, Ann. Phys. (NY) 118 (1979) 429;
R. Levine and Y. Tomozawa, Phys. Rev. D21 (1979) 840.

[7] W. Buchmüller, G. Grunberg and S.-H. Tye, Phys. Rev. Lett. 45 (1980) 103.
[8] W. Buchmüller and S.-H. Tye, Phys. Rev. D24 (1981) 132;
S. Ono, Phys. Rev. D23 (1981) 1119;
A. Martin, Phys. Lett. 100B (1981) 511;
G. Bahnot and S. Rudaz, Phys. Lett. 78B (1978) 119;
H. Kraseman and S. Ono, Nucl. Phys. B154 (1979) 283;
S. Gupta, S. Radford and W. Repko, Phys. Rev. D26 (1982) 3305;
C. Quigg and J.L. Rosner, Phys. Rev. D23 (1981) 2625.
[9] E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane and T.M. Yan, Phys. Rev. D21 (1980) 203.
[10] S.J. Brodsky, G.P. Lepage and P.B. Mackenzie, Phys. Rev. D28 (1983) 228.
[11] T.D. Lee, Particle physics and introduction to field theory (Harwood Academic Publishers, New York, 1981).
[12] K. Han et al., Phys. Rev. Lett. 49 (1982) 1612;
G. Eigen et al., Phys. Rev. Lett. 49 (1982) 1616;
J. Lee-Franzini, CUSB at CESR, talk Moriond Conference (1983).
[13] S. Behrends et al., Phys. Rev. Lett. 50 (1983) 881.
[14] Particle Data Group, Review of Particle Properties, Phys. Lett. 111B; and LBL-100 revised, minireviews on delta (980) and epsilon (1300).
[15] J.L. Rosner, Progress in the description of heavy quarkonium, Preprint EFI 83/17;
P. Moxhay and J.L. Rosner, Relativistic corrections in quarkonium, EFI 83-15 (1983).