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Water sharing agreements sustainable to reduced flows

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ABSTRACT

By signing a water sharing agreement (WSA), countries agree to release an amount of river water in exchange for a negotiated compensation. We examine the vulnerability of such agreements to reduced water flows. Among all WSAs that are acceptable to riparian countries, we find out the one which is self-enforced under the most severe drought scenarios. The so-called upstream incremental WSA assigns to each country its marginal contribution to its followers in the river. Its mirror image, the downstream incremental WSA, is not sustainable to reduced flow at the source. Self-enforcement problems can be solved by setting water releases and compensations contingent to water flow. We apply our analysis to the Aral Sea Basin. We compute the upstream incremental compensations for the Bishkek agreement and asses its vulnerability with historical flows.

1. Introduction

Water scarcity is becoming one of the major challenges worldwide. Because of population and economic growth, demand for water has tremendously increased. At the same time, water becomes less available in many parts of the world because of global warming (climate change). The higher world temperatures are expected to increase the hydrological cycle activity, leading to a general change in precipitation patterns and increase in evapotranspiration (IPCC 2007, p. 7). Many semi-arid regions (e.g. Mediterranean, western United States, southern Africa and northeast Brazil) will suffer a decrease in water resources availability due to climate change (Bates et al., 2008). Moreover, other consequences of global warming such as the more frequent extreme events of precipitation and dry periods and the early melting of glaciers would lead to an increase in the variance in water supply.

Since at least Hardin (1968) and Ostrom (1990), it has been established that the sustainable exploitation of common-pool natural resources, such as water, requires cooperation among users. In practice, users such as farmers, industries, cities or countries, coordinate water extraction through various arrangements from irrigation communities (Ostrom, 1990), to water markets (Libecap, 2011) or international river treaties (Dinar, 2008). Those arrangements are designed by users. They specify water releases and, sometime, payments through monetary transfers.

Examples include international river sharing agreements in which countries commit to release water in exchange for compensations. For instance, by the Bishkek Treaty signed in 1998, compensation is paid for Kyrgyztan's compliance with release schedules that take into account its winter energy needs and Uzbekistan's and Kazakhstan's summer irrigation water demands. Future water releases in cubic meters per second for each month of the irrigation season are detailed in the treaty.

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In the USA, the inter-state river compacts signed by riparian states specify water releases that are fixed or proportional to water flows (Bennett et al., 2000). The 1996 Ganges treaty signed by India and Bangladesh specifies a combination of fixed and proportional releases contingent on water flow at Farakka barrage. It is divided equally for water flows below 70,000 cubic meters per second (cums), 35,000 cums are guaranteed to Bangladesh if water flows are above 70,000 and below 75,000 cums and 40,000 cums are allotted to India for water flows higher than 70,000 cums.

This paper addresses the vulnerability of existing water sharing arrangements to reduced flows. We model the problem of sharing water from a river with random water supplies. Riparian countries coordinate water extractions through water sharing agreements. Water releases can be fixed, proportional or contingent to water supply. Those agreements commit upstream countries to release volumes of water in exchange for compensations by downstream users. We analyze the design and self-enforcement of water sharing agreements by sovereign countries. Countries agree on water releases and transfers based on their expected welfare before water supplies are realized. In case of low water supply, an upstream country might be better off not releasing what it committed even if it has to renounce the compensation. We examine such defection strategies in case of droughts, where water supplies are below the long-term mean flow. We characterize the water sharing agreements that avoid countries' defection for the more severe drought scenarios.

Examples of countries defection during droughts have been observed. Dinar et al. (2010, Table 2) recorded complaints made during 1950–2005 regarding water sharing issues by states sharing international rivers. They found that a total of 112 complaints have been recorded regarding drought and floods between 1950 and 2005. One hundred and six of them regarding droughts and six regarding floods. In the Jordan River, while the Jordan–Israel water treaty of 1994 has mechanisms for dealing with shortages that cover a significant range of possible shortages, there is no stated mechanism for sharing shortages, mainly in prolonged droughts and extreme shortages, when they occur. This was the case in the 1998–2000 drought. Israel stated that it would not be possible to allow Jordan its water allocation according to the agreement, and it would have to reduce it.

Our framework is an extension of the river sharing problem introduced by Ambec and Sprumont (2002) to random water flows. We study a cooperative game in which countries negotiate water sharing agreements (WSAs). In a negotiation among sovereign countries, the agreement should be accepted in a voluntary manner. In particular, countries are free to reject any agreement at the basin-wide level if they are better off signing partial agreements with a smaller number of the basin riparians. To be accepted by all countries, the WSA should make any group of countries better-off in terms of their expectations compared with any other partial agreement (including no agreement at all). In other words, the WSA should be in the core of the cooperative game associated with the river sharing problem.

We first examine fixed water sharing agreements (FWSAs). We show that the cooperative game generated by the river sharing problem with random flows is convex. It implies that many river sharing agreements are in the core. One of them is the so-called downstream incremental agreement introduced by Ambec and Sprumont (2002). It assigns to any country its marginal contribution to the set of predecessors in the river. By doing so, it maximizes lexicographically the welfare of the most downstream countries in the river in the set of core FWSAs. It thus favors downstream countries against upstream countries. We consider the FWSA opposite to the downstream incremental in the core: the upstream incremental FWSA. It assigns to each country its marginal contribution to its followers in the river.

We then examine the vulnerability of core FWSAs to defection in the case of drought events. A FWSA agreement specifies some amount of water to be released in exchange for monetary transfers. With water flows lower than the mean, a country is obliged to consume less than its water allocation under the FWSA in order to fulfill its commitment. Yet, the payment it receives from the volume of water released is unchanged. With water being more valued by countries under reduced flows, a country might be better off by not releasing the volume of water it committed, although at a cost of not getting the monetary transfer from downstream countries. For a given level of reduced flow, a FWSA is *self-enforcing* if no country is better-off defecting, which means not releasing water. Among all core FWSAs, the upstream incremental FWSA is self-enforcing with the smallest water flows. It thus maximizes the range of water flows for which no country defects. By assigning the highest payment for water released, it avoids defection as much as possible. In contrast, the downstream incremental FWSA is the less sustainable core FWSA because it assigns the lowest payment for water released. It is indeed not self-enforcing for the first country in the river.

Next we consider proportional and contingent water sharing agreements. By signing a proportional water sharing agreement (PWSA), countries agree to release shares of water flows in exchange of compensations. As with a FWSA, compensations are fixed regardless of the realized water flows, although water releases are proportional to water flows. We show that our main result holds for proportional water sharing agreements: among all core PWSAs, the upstream PWSA is self-enforcing under lower reduced flows than any others core PWSA. In a contingent water sharing agreement (CWSA), water releases and compensations are defined contingently on the level of water supply. With such flexible agreements, self-enforcement is not any more an issue: the CWSA can be designed such that no country gains by defecting for any realized water flow.

The economic literature includes several works that focus on various aspects of international water sharing issues and their stability in a basin setting. Several studies analyze river sharing agreements but with deterministic water flows (Ambec and Sprumont, 2002; Ambec and Ehlers, 2008; Wang, 2011; Ansink and Weikard, 2012; van den Brink et al., 2012). Yet, others introduce the water supply variability into their analysis. Kilgour and Dinar (2001) review several sharing rules that are common in international water treaties and demonstrate how they may not meet the treaty parameters under increased water variability. Alternative sharing rules are suggested and their sustainability is demonstrated, using the case of the annual flow of the Ganges River at Farakka, the flash point between India and Bangladesh. Focussing on interstate river compacts in the United States, Bennett et al. (2000) compare the efficiency of fixed versus proportional allocation of water with variable water flow in inter-state water compacts. They do not address the issue of self-enforcement in case of drought, since the federal government has coercive power to enforce interstate compacts.

Ansink and Ruijs (2008) compare the performance of fixed and proportional agreements regarding their sustainability to reduced water flow. They rely on a two-country repeated game approach with self-enforcement constraints. They assume an exogenous division of the welfare from cooperation, which translates into a payment from the downstream country to the upstream country. The authors show that fixed agreements are less sustainable than proportional agreements when transfers divide equally the welfare from cooperation.[1] Our paper departs from Ansink and Ruijs (2008) in several ways. First, we do not restrict our analysis to bipartite agreements. We consider a river shared by $n \geq 2$ countries. By doing so, we allow for partial agreements in the river basin and coalition deviations during the negotiation.[2] Second, we consider self-enforcement problems in a one-shot game. Implicitly, we assume that a country can negotiate a new agreement for the next years after defection like in the case of the Bishkek Treaty in the Aral Sea. Our self-enforcement constraints are therefore more stringent. Third, and most importantly, we do not compare the performance of different types of agreements with exogenous division of the surplus from cooperation among countries.[3] We rather focus on one type or agreement (fixed, proportional or contingent) and endogenize the surplus division. We want to identify the surplus sharing rule (or equivalently the transfers among countries) that makes the river sharing agreement self-enforcing under the more severe droughts. Our paper is thus more on the *design* of water sharing agreements than on the comparison of different types of agreements. It aims to recommend transfers that are less vulnerable to defection in the case of drought events.

The paper proceeds as follow. We introduce the model in Section 2. We analyze the design and enforcement of fixed river sharing agreements in Section 3. In Section 4, we examine successfully contingent and proportional water sharing agreements. We then turn to a numerical application of the Aral Sea and conclude.

2. The river sharing problem

A set $N=\{1,\ldots,n\}$ of countries are located along a river and share its water. We identify countries by their locations along the river and number them from upstream to downstream: $i<j$ means that i is upstream to j. A coalition of countries is a non-empty subset of N. Given two coalitions S and T, we write $S<T$ if $i<j$ for all $i \in S$ and all $j \in T$. Given a coalition S, we denote by $\min S \equiv \min_i S$ and $\max S \equiv \max_i S$, respectively, the smallest and largest members of S, i.e. $S=\{\min S,\ldots,\max S\}$. Let $Pi=\{1,\ldots,i\}$ denote the set of predecessors of country i and $P^0 i = Pi \backslash \{i\}$ denote the set of strict predecessors of country i. Similarly, let $Fi=\{i,i+1,\ldots,n\}$ denote the set of followers of country i and let $F^0 i = Fi \backslash \{i\}$ denote the set of strict followers of i. For any n-dimension vector $\boldsymbol{y}=(y_i)_{i \in N}$, we denote by $\boldsymbol{y}_S=(y_i)_{i \in S}$ the vector of its components in S for any arbitrary $S \subset N$.

Each country $i \in N$ enjoys a benefit $b_i(x_i)$ from diverting x_i units of water from the river. As in Ambec and Sprumont (2002), we assume that the benefit function b_i is differentiable, increasing and strictly concave for all $x_i>0$. Furthermore, $b'_i(x_i)$ goes to infinity as x_i approaches 0. A country also values money linearly in the sense that the welfare realized by country i with x_i units of water and t_i units of money (or welfare or any numerary good) is $b_i(x_i)+t_i$.[4]

Each country $i \in N$ controls a flow of water $e_i \geq 0$ with $e_1>0$ at the river source. It includes water supplied by tributaries or stored in a reservoir controlled by i. The controlled water flows are random. The controlled flow e_i ranges in $[\underline{e}_i, \overline{e}_i]$ with $0 \leq \underline{e}_i \leq \overline{e}_i$ and $\underline{e}_1>0$. The vector of flows $\boldsymbol{e}$ is distributed according to a density f and cumulative F with $f(\boldsymbol{e})>0$ for every $\boldsymbol{e} \in \times_{i \in N}[\underline{e}_i, \overline{e}_i]$. A river problem with random water flows is defined by $(N, \boldsymbol{e}, \boldsymbol{b})$ where $\boldsymbol{e}$ is a random vector of water flows distributed according to f on $\times_{i \in N}[\underline{e}_i, \overline{e}_i]$.

In this setting, non-cooperative water extraction is generally inefficient. Under laisser-faire, each country i extracts water flowing down on its territory. Country 1 consumes e_1 leaving nothing to country 2 who itself extracts its controlled water flow and so on. Individual welfare is $b_i(e_i)$ ex post and $E[b_i(e_i)]$ ex ante for every $i \in N$. This outcome is usually inefficient: the welfare of two countries i and j with $i<j$ can be improved if country i releases some water to supply country j in exchange of some transfer. As long as $\boldsymbol{e}$ is not optimal, there exists $\epsilon>0$ such that $b_i(e_i-\epsilon)+b_j(e_j+\epsilon)>b_i(e_i)+b_j(e_j)$.[5] This can be done through a water sharing agreement (WSA).

By signing a *water sharing agreement* (WSA), riparian countries agree to release some amount of water in exchange of some payments. We consider three types of WSA: fixed, contingent and proportional. A *fixed water sharing agreement* (FWSA) is a vector of water releases $\boldsymbol{w}$ and payments $\boldsymbol{\tau}$ where w_i denotes the amount of water country i agrees to release

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[1] To be precise, they distinguish between two types of fixed agreements: fixed upstream and fixed downstream. In a fixed upstream agreement, the upstream country releases a fixed amount of water whereas in a fixed downstream agreement it diverts a fixed amount of water, thereby releasing the balance flow. They show that upstream fixed agreements are less sustainable than the proportional agreements while the reverse holds for the fixed downstream agreement. Here we consider only fixed upstream agreements, i.e. fixed water releases.

[2] Our paper highlights the importance of the spatial structure in a river sharing problem. As suggested by Dinar (2008), geography is an important aspect that explains many of the outcomes of treaty stability as affected by water supply variability. We address the geography aspect in the design of water sharing agreements.

[3] Note that, consistently with our results, Ansink and Ruijs (2008) found that water sharing agreements are more stable when upstream countries have all benefit from cooperation (we thank a referee for raising this point).

[4] In other words, the benefit of water consumption is expressed in money.

[5] The later condition of welfare improvement holds in many cases for instances if $b_i = b_j$ and $e_i > e_j$.

downstream while τ_i is the payment received by country i in exchange of w_i for $i=1,\dots,n$.[6] Water and financial transfers do not depend on realized water flows $\boldsymbol{e}$. With a *contingent water sharing agreement* (CWSA), countries agree on water releases $\boldsymbol{w}(\boldsymbol{e})$ and payments $\boldsymbol{\tau}(\boldsymbol{e})$ contingent on the realized water flows $\boldsymbol{e}$ for every potential flow $\boldsymbol{e} \in \times_{i \in N}[\underline{e}_i, \overline{e}_i]$. A *proportional water sharing agreement* (PWSA) assigns shares $\boldsymbol{\alpha}$ of water flows $\boldsymbol{e}$ against payments $\boldsymbol{\tau}$. It specifies shares α_{ji} of the water flow e_j for $j=1,\dots,i$ that is assigned to country i against a payment τ_i for $i=1,\dots,n$.

Along international rivers, riparian countries are sovereign regarding their decisions. First, they are free to sign WSAs or not. They might design a WSA only with a subset of riparian countries. Second, once the agreement is signed, countries can comply or not with the WSA. For a given realized flow of water, it might be in the interest of the country not to supply the amount of water it committed to. For instance, in periods of droughts, water being too valuable for a country, the loss of welfare might not be compensated by the financial transfer. Since no supranational authority can enforce the WSA with fees in case of defection, the WSA must be self-enforcing in the sense that no country should be better-off by not releasing water it committed to. Consistently with the literature on self-enforcing contracts (Thomas and Worral, 1988; Gauthier et al., 1997; Levine, 2003), we consider water sharing agreements such that no country defects for all realized water flows along the equilibrium path.

More precisely, we consider the following sequence of decisions undertaken by countries.

1. Countries agree on a water sharing agreement.
2. Water flows $\boldsymbol{e}$ are realized.
3. Each country decides to release or not water in exchange of the transfer.

The water sharing agreement is designed ex ante, i.e. before the water flows are realized, while compliance or defection is decided ex post. We focus on WSAs that are accepted by all riparian countries in stage 1 and self-enforced in stage 3. First, the WSA must satisfy coalitional participation constraints ex ante: any coalition of countries must be better-off with the WSA at the river-wide basin compared with any potential WSA among a subset of countries.[7] Second, the WSA must satisfy the self-enforcing constraints ex post: each country i is not worse-off by releasing w_i in exchange of τ_i in stage 3 rather than consuming all water it controls for some realized $\boldsymbol{e}$.

To analyze self-enforcing WSAs, we proceed as follow. We first find out the WSAs that satisfy the coalitional participation constraints. Next, among those WSAs, we characterize the ones that are self-enforcing for the lowest water flows. A WSA is *sustainable* to reduced flows $\boldsymbol{e}$ if it is self-enforced with the realized water flows $\boldsymbol{e}$. We identify the lowest water flow for which a WSA can remain sustainable. It allows us to characterize the range of water flows $\boldsymbol{e}$ such that a self-enforcing WSA does exist. We consider fixed water sharing agreements in Section 3 before moving to contingent and proportional water sharing agreements in Section 4.

3. Fixed water sharing agreements

By signing the FWSA $(\boldsymbol{w}, \boldsymbol{\tau})$, country i agrees to release w_i units of water to country $i+1$ in exchange of a payment τ_i for $i=1,\dots,n-1$. It thus consumes $x_i = e_i + w_{i-1} - w_i$ units of water when the realized water flow is e_i and obtains $t_i = -\tau_{i-1} + \tau_i$ units of money. Given e_i, the ex post utility or welfare of country i with the FWSA $(\boldsymbol{w}, \boldsymbol{\tau})$ is

$$b_i(x_i) + t_i = b_i(e_i + w_{i-1} - w_i) - \tau_{i-1} + \tau_i.$$

Countries are expected-utility maximizers. The ex ante welfare of country i with the FWSA $(\boldsymbol{w}, \boldsymbol{\tau})$ is defined by its expected welfare given the distribution of e_i

$$E[b_i(x_i) + t_i] = E[b_i(x_i)] + t_i = E[b_i(e_i + w_{i-1} - w_i)] - \tau_{i-1} + \tau_i. \tag{1}$$

The concavity of b_i makes country i dislike the variability of water flow. We now briefly describe the efficient FWSAs at the basin level before moving to the design and the self-enforcement of FWSAs.

3.1. *Efficient fixed water sharing agreements*

Since utility is transferable, the efficient water releases vector is defined as the one that maximizes total welfare ex ante subject to feasibility constraints. It is unique given our assumptions on the benefit functions. It is denoted $\boldsymbol{w}^*$. It defines a water consumption vector $\boldsymbol{x}^* = (\boldsymbol{x}^*)_{i \in N}$ where $x_i^* = e_i + w_{i-1}^* - w_i^*$ for any realization $e_i \in [\underline{e}_i, \overline{e}_i]$, for every $i \in N$. Formally, $\boldsymbol{w}^*$ solves the following maximization problem:

$$\max_{\boldsymbol{w}} \quad E\left[\sum_{i \in N} b_i(e_i + w_{i-1} - w_i)\right],$$
$$\text{s.t.} \quad w_i \geq 0 \quad \text{for every } i \in N,$$

[6] Of course, $w_n = 0$ since the most downstream country has no reason to release water and, therefore, receives no payment from downstream, i.e. $\tau_n = 0$. Symmetrically, $\tau_0 = 0$ and $w_0 = 0$ because the most upstream country 1 does not receive water from other countries.

[7] Note that it implies individual rationality for each country when applied to coalition of size one.

$$\underline{e}_i + w_{i-1} - w_i \ge 0 \quad \text{for every } i \in N.$$

The first set of feasibility constraints $w_i \ge 0$ for every $i \in N$ are on water releases: since water can only be transferred from upstream to downstream, water releases cannot be negative. The second set of feasibility constraints $\underline{e}_i + w_{i-1} - w_i \ge 0$ are on water consumption under the lowest water supply $\underline{e}_i$. These constraints guarantee that consumption $x_i = e_i + w_{i-1} - w_i$ is non-negative for any realized water flow $e_i \in [\underline{e}, \overline{e}_i]$ so that country i will always be able to release what it was committed to. This constraint should hold for every country $i \in N$. Denoting μ_i and λ_i the Lagrangian multipliers of the first and the second set of feasibility constraints respectively, the first-order conditions yield

$$E[b'_{i+1}(x^*_{i+1}) - b'_i(x^*_i)] = \lambda_i - \lambda_{i+1} - \mu_i,$$

for $i = 1, \dots, n-1$. The above conditions imply for any $j > i$

$$E[b'_j(x^*_j) - b'_i(x^*_i)] = \lambda_i - \lambda_j - \sum_{l=i}^{j} \mu_l. \tag{2}$$

The first-order conditions prescribe equalizing ex ante marginal benefits of water consumption whenever it is possible. If not, then one of the constraints is binding. It could be that the non-negative water release constraint is binding at say i, and, therefore, $\mu_i > 0$. Or that the non-negative water consumption constraint is binding, in which case $\lambda_i > 0$.

It is easy to show that, under infinite marginal benefit at zero water consumption, the non-negative water consumption constraints are not binding.[8] Therefore $\lambda_i = 0$ for every $i \in N$ so that the first-order condition (2) becomes

$$E[b'_i(x^*_i) - b'_j(x^*_j)] = \sum_{l=i}^{j} \mu_l. \tag{3}$$

Ex ante marginal benefits are equalized whenever the non-negative water release constraints are not binding between i and j. If it is at some location l with $i < l < j$ then $\mu_l > 0$ which implies $E[b'_i(x^*_i)] > E[b'_j(x^*_j)]$: country i enjoys a higher ex ante marginal benefit from water consumption than country j. Moreover, in this case, $w_l = 0$ and, therefore, no water transferred from i to j. Indeed, binding the constraint at l would imply not releasing water from country l. It is optimal to do that if it is expected that water is relatively more abundant downstream to l. Since marginal benefits reflect water scarcity in the sense that more water leads to a lower marginal benefit, ex ante marginal benefit is lower downstream.

Condition (3) is similar to conditions (3) and (4) in Ambec and Sprumont (2002) for deterministic water flows except that marginal benefits are ex ante (in expectation) rather than ex post (see also Kilgour and Dinar, 2001). Similarly, we can conclude that the efficient FWSA partitions the set of agents N into consecutive subsets $\{N_k\}_{k=1}^{K}$ with N_k upstream of N_{k+1} for $k = 1, \dots, K-1$. It defines the ex ante shadow value of water $\{\beta_k\}_{k=1}^{K}$ at each segment N_k of the river with $\beta_k > \beta_{k+1}$. Ex ante marginal benefits from water consumption are equalized among countries within N_k. They are equal to the ex ante shadow value of water β_k at N_k. Countries in N_k share the water flows they control $\sum_{i \in N_k} e_k$ and, therefore, do not transfer water downstream of N_k. Ex ante marginal benefit decreases moving from N_k to N_{k+1} as well as the shadow value of water.

3.2. Design of fixed water sharing agreements

When leaving the negotiation for a basin-wide FWSA, a coalition $S \subset N$ can agree on a FWSA on the water flow it controls as in Ambec and Sprumont (2002). The welfare that coalition S can secure is the highest welfare achieved by designing a partial FWSA $(\boldsymbol{w}_S, \boldsymbol{\tau}_S)$ among the members of S for sharing the water flow they control $e_S = (e_i)_{i \in S}$. For simplicity, we assume that, when computing its worth, the coalition expects all its members to comply with the partial FWSA in stage 3 of the game. Denoted $v(S)$, the worth of a coalition S can easily be defined for a connected coalition. A coalition S is connected if for all $i, j \in S$ and all $k \in N$, $i < k < j$ implies $k \in S$. For a connected coalition S

$$v(S) = \max_{\boldsymbol{w}_S} \quad E\left[\sum_{i \in S} b_i(e_i + w_{i-1} - w_i)\right],$$
$$\text{s.t.} \quad w_i \ge 0 \quad \text{for every } i \in S,$$
$$\underline{e}_i + w_{i-1} - w_i \ge 0 \quad \text{for every } i \in S, \tag{4}$$

where $w_{\min S - 1} = 0$. In particular, the stand-alone welfare of country i is simply $v(\{i\}) = E[b_i(e_i)]$. Let us denote by $\boldsymbol{w}^S_S$ the solution to (4). It is the efficient vector of water releases of the reduced river problem $(S, \boldsymbol{e}_S, \boldsymbol{b}_S)$.

For a disconnected coalition, we first need to decompose the coalition into its connected components (subsets). Let $\mathcal{P}(S) = \{S_l\}_{l=1}^{L}$ be the unique coarsest partition of S into its connected components. Since water cannot be transferred between two components S_l and S_{l+1} of $\mathcal{P}(S)$, the worth of coalition S is obtained by summing up the worth of its connected

[8] If it was binding for, say, country j, then water consumption in case of extreme drought $\underline{e}_j$ would be set to zero for i which implies an infinite marginal benefit in this case, formally $b'_j(\underline{x}^*_j) = +\infty$ where $\underline{x}^*_j = \underline{e}_j + w^*_{j-1} - w^*_j$. Since the density is positive in $\underline{e}_j$, it implies that j's marginal benefit is also infinite in expectation: $E[b'_j(x^*_j)] = +\infty$. On the other hand, since $\underline{e}_1 > 0$, at least some country i consumes water in all states of nature. For this country i, expected marginal benefit is finite: $E[b'_i(x^*_i)] < +\infty$. The last two conditions on expected marginal benefits for i and j contradict the first-order condition (2).

components

$$v(S) = \sum_{S_l \in \mathcal{P}(S)} v(S_l), \tag{5}$$

where $v(S_l)$ is given by (4). A FWSA $(\boldsymbol{w}, \boldsymbol{\tau})$ satisfies the participation constraints for coalition $S \subset N$ if

$$\sum_{i \in S} (E[b_i(e_i + w_{i-1} - w_i)] - \tau_{i-1} + \tau_i) \geq v(S). \tag{6}$$

We say that a FWSA is in the core of the cooperative game generated by the problem $(N, \boldsymbol{b}, \boldsymbol{e})$ if the participation condition (6) holds for every $S \subset N$. We call $v(S)$ the core lower bound for coalition S for every $S \subset N$.

Clearly, the core lower bound for the "grand coalition" N forces the FWSA to be efficient. Indeed, since $v(N) = \sum_{i \in N} E[b_i(x_i^*)] = \sum_{i \in N} E[b_i(e_i + w_{i-1}^* - w_i^*)]$, the core lower bounds determine fully water releases $\boldsymbol{w} = \boldsymbol{w}^*$. Monetary transfers $\boldsymbol{\tau}$ still need to be defined. To do so, it is convenient to work with welfare distributions instead of payments. Let us define $\boldsymbol{u} = (u_i)_{i \in N}$ as a distribution of the total ex ante welfare $v(N)$ with $\sum_{i \in N} u_i = v(N)$. There is a mapping between welfare distributions and transfers. A given transfer scheme $\boldsymbol{\tau}$ corresponds to a unique distribution of the welfare $\boldsymbol{u}$ where $u_1 = E[b_1(x_1^*)] + \tau_1$, $u_i = E[b_i(x_i^*)] - \tau_{i-1} + \tau_i$ for $i = 2, \ldots, n-1$ and $u_n = E[b_n(x_n^*)] - \tau_{n-1}$. Hence, from the welfare distribution $\boldsymbol{u}$ with $\sum_{i \in N} u_i = v(N)$, one can compute the monetary transfers defined as $\tau_i = \sum_{j \in Pi} (u_i - E[b_j(x_j^*)])$ for $i = 1, \ldots, n-1$.

We will say that a welfare distribution $\boldsymbol{u}$ satisfies the core lower bounds or the coalitional participation constraints if for every $S \subset N$

$$\sum_{i \in S} u_i \geq v(S).$$

A welfare distribution that satisfies the core lower bounds is called a core welfare distribution. We now establish a useful property of the characteristic function v, namely its convexity. The proof is adapted from Ambec and Sprumont (2000). It is available in Ambec et al. (2011).

Proposition 1. *The cooperative game v is convex in the sense that* $v(S) - v(S \backslash i) \geq v(T) - v(T \backslash i)$ *for every* $i \in T \subset S \subset N$.

The above proposition allows us to describe the full set of core welfare distributions. Shapley (1971) has shown that the core of a convex game is the convex hull of the so-called *marginal contribution vectors*. A marginal contribution vector assigns to each agent its marginal contribution to the coalition composed by its strict predecessors in a specific ordering of all agents. Let us define such an ordering by γ which is a bijection from N to N. The vector of marginal contributions of the ordering γ assigns $u_i = v(P\gamma(i)) - v(P^0\gamma(i))$ to agent i for $i = 1, \ldots, n$. All these marginal contribution vectors are in the core. Moreover, the core contains all linear combinations of marginal contribution vectors. One example is the *Shapley value* which assigns to every agent i its marginal contribution to all possible orderings weighted by uniform probabilities over the orderings. It is indeed the barycenter of the core for convex games. Another interesting element of the core is the so-called *downstream incremental* distribution proposed by Ambec and Sprumont (2002). Denoted $\boldsymbol{u}^d$, it considers the natural ordering along the river $\gamma(i) = i$. It assigns to any country i its marginal contribution to the coalition composed by its predecessors along the river: $u_i^d = v(Pi) - v(P^0 i)$ for $i = 1, \ldots, n$. It is the unique welfare distribution in the core that maximizes lexicographically the welfare of the most downstream users $n, n-1, \ldots, 1$. Given the above definition of u_i^d for every $i \in N$, the downstream incremental distribution determines the payments for water releases τ_i^d for every $i \in N$ as

$$\tau_i^d = v(Pi) - E\left[\sum_{j \in P_i} b_j(e_j + w_{j-1}^* - w_j^*)\right]. \tag{7}$$

Payments are based on losses for upstream countries. The compensation paid by country $i+1$ to country i is the expected loss from releasing w_i^* units of water at i for all upstream countries.

A welfare distribution opposite to the downstream incremental distribution in the core is the *upstream incremental* distribution. It considers the reverse ordering of agents $\gamma(i) = n - i$. Defined as $u_i^u = v(Fi) - v(F^0 i)$ for $i = 1, \ldots, n$, it assigns country i's marginal contribution to its successors along the river. The upstream incremental distribution is the core welfare distribution that maximizes lexicographically the welfare of the most upstream agents $1, 2, \ldots, n$. It has been analyzed in van den Brink et al. (2007) for deterministic water flows. The upstream incremental distribution determines the payments for water releases τ_i^u for every $i \in N$

$$\tau_i^u = E\left[\sum_{j \in F_i^0} b_j(e_j + w_{j-1}^* - w_j^*)\right] - v(F^0 i). \tag{8}$$

Payments are based on gains for downstream countries. The compensation paid by country $i+1$ to country i is the expected gain from releasing w_i^* units of water at i for all downstream countries.

3.3. *Self-enforcing fixed water sharing agreements*

We examine self-enforcement of FWSAs in stage 3. By signing an FWSA $(\boldsymbol{w}, \boldsymbol{\tau})$, countries agree to release water against money regardless of the realized water flows. For some realized water flow, some countries might be tempted not to release water. Indeed, even if signing a FWSA is welfare increasing ex ante, ex post some countries might be better off not complying with their commitments. Second, a country might be better off not buying the water it committed to the next upstream country. The former defection arises with lower water flows than expected whereas the later is tempting when water is more abundant than expected. We focus on self-enforcement in case of drought as defined below.

Definition 1. A FWSA $(\boldsymbol{w}, \boldsymbol{\tau})$ is self-enforced with flow e_i at i if

$$b_i(e_i + w_{i-1} - w_i) - \tau_{i-1} + \tau_i \geq b_i(e_i + w_{i-1}) - \tau_{i-1}.$$

The above self-enforcing constraint assures that country i is better-off by releasing w_i rather than consuming all water. A FWSA is self-enforced under realized water flows $\boldsymbol{e}$ if the self-enforcing constraints are satisfied for all riparian countries $i = 1, \dots, n$. We will then say that the FWSA is *sustainable* to water flows $\boldsymbol{e}$.

The self-enforcing constraint for country i and realized flow e_i is simplified to

$$\tau_i \geq b_i(e_i + w_{i-1}) - b_i(e_i + w_{i-1} - w_i). \tag{9}$$

The transfer paid for w_i should exceed the relative value of w_i for country i for any realized water flow. Since the right-hand side is decreasing with e_i, one need to consider only the lowest water flow $\underline{e}_i$ to asses the self-enforcement of a FWSA.[9]

Definition 2. Given the range of water flows $\times_{i \in N}[\underline{e}_i, \overline{e}_i]$, an FWSA $(\boldsymbol{w}, \boldsymbol{\tau})$ is self-enforcing if

$$\tau_i \geq b_i(\underline{e}_i + w_{i-1}) - b_i(\underline{e}_i + w_{i-1} - w_i) \quad \text{for every } i \in N.$$

We are now able to establish the main result of the paper. It characterizes the upstream incremental FWSA as the (unique) core FWSA that is sustainable to the most severe drought. The proof is in Appendix A.

Proposition 2. *The upstream incremental FWSA is self-enforced under lower water flows than any other core FWSA.*

Proposition 2 provides a characterization of the upstream incremental solution for river sharing problems with random flows. It also allows us to determine the minimal flow of water such that the upstream incremental FWSA is self-enforcing. It implies that, if a realized water flow is not self-enforced under the upstream incremental FWSA, no FWSA is self-enforcing. Combining the definition of $\boldsymbol{\tau}_i^u$ in (8) with the self-enforcing constraint (9) defines the minimal flow $\tilde{e}_i$ such that $(\boldsymbol{w}^*, \boldsymbol{\tau}^u)$ is self-enforcing

$$b_i(\tilde{e}_i) - b_i(\tilde{e}_i + w_{i-1}^* - w_i^*) = \sum_{j \in F^0 i} E[b_j(e_j + w_{j-1}^* - w_j^*)] - v(F^0 i) \tag{10}$$

For flow ranges $[\underline{e}_i, \overline{e}_i]$ with $\underline{e}_i \geq \tilde{e}_i$ for $i = 1, \dots, n$, the upstream incremental FWSA is self-enforcing. Other FWSAs might or might not be self-enforcing. Nevertheless, one can always design a FWSA that is self-enforcing by picking the upstream incremental FWSA. We summarize this results in the following Corollary.

Corollary 1. *A FWSA can be designed to be self-enforcing if and only if $\underline{e}_i \geq \tilde{e}_i$ for every $i \in N$.*

In the particular case where $\underline{e}_i = \tilde{e}_i$ for every $i \in N$ then, among all the core FWSA, only the upstream incremental FWSA is self-enforcing. The minimal flow $\tilde{e}_i$ for $i = 1, \dots, n$ indicates wether compliance in case of drought is a serious issue or not. If it is, the upstream incremental FWSA should be selected. If not, other FWSAs might be self-enforcing. Therefore, other considerations such as fairness concerns might be invoked to select a core distribution. For instance, Ambec and Sprumont (2002) propose a fairness criterion called the aspiration welfare upper bounds that selects the downstream incremental FWSA under deterministic flows. Under random water flows, the next proposition shows that the downstream incremental FWSA is not a good candidate among all core FWSAs to insure self-enforcement (see Appendix B for proof).[10]

Proposition 3. *All core FWSA are self-enforced under lower water flows than the downstream incremental FWSA. It is not self-enforced under reduced flows at the source.*

3.4. Satiated benefits

We now extend our model to allow for water satiation. As argued by Ambec and Ehlers (2008), overconsumption might be costly because of flooding or increased sanitation costs with higher water extraction costs. It is therefore likely that the benefit for water consumption is decreasing after satiation. Let us assume that the benefit function b_i of a country i is decreasing for water consumption above $\hat{x}_i$, i.e. the satiated level. More precisely, b_i is increasing up to $\hat{x}_i$ and then

[9] Note that the last constraint in the problem defining the FWSA in (4) insures that a country has always enough water to be able to comply with the FWSA. Defection is thus always a choice, not an obligation, for a country.

[10] Proposition 3 provides a characterization of the downstream incremental FWSA: among all core FWSA, it is the less sustainable one. It differs from the one provided by Ambec and Sprumont (2002) and van den Brink et al. (2011) as it makes the downstream incremental solution less appealing.

decreasing $b'_i(x)<0$ for every $x>\hat{x}_i$ for $i=1,\dots n$. The benefit function is still assumed to be strictly concave $b'_i(x)<0$ for every x with $b'_i(\hat{x}_i)=0$ for $i=1,\dots n$.

We examine how our results can generalize to the river sharing problem with satiation. Under deterministic water flow and satiation, Ambec and Ehlers (2008) have shown that the downstream incremental FWSA is in the core. Surprisingly, under the same assumptions, the upstream incremental FWSA might not be in the core as shown by the following example. Let us consider a river shared among three countries with identical benefit functions $b_1(a)=b_2(a)=b_3(a)=a(12-a)=b(a)$ for every $a\in[0,10]$ but unequal controlled water flows $\boldsymbol{e}=(e_1,e_2,e_3)=(4,6,2)$. The satiated consumption levels are $\hat{x}_i=6$ for a maximal benefit of $b_i(\hat{x}_i)=36$ for $i=1,2,3$. The optimal water allocation prescribes to share equally the total flow of water $e_1+e_2+e_3=12$ which requires that country 2 supplies country 3 with 2 units of water. It leads to a total welfare $v(1,2,3)=3b(4)=96$. The coalition $\{2,3\}$ can secure a welfare $v(2,3)=2b(4)=64$ by dividing equally $e_2+e_3=8$ (country 2 releases 2 units of water out of 6 units to country 3). The upstream incremental FWSA assigns $u_1^u=v(1,2,3)-v(2,3)=96-64=32$ and $u_3^u=v(3)=b(2)=20$ to countries 1 and 3 respectively. On the other hand, both countries can secure a welfare of $v(1,3)=2b(3)=54>52=u_1^u+u_3^u$ if country 1 releases one unit of water to supply country 3. Such water transfer would not be possible without satiation because country 2 would consume all water flowing down into its territory.

With satiated benefits, the upstream incremental FWSA is not the most sustainable FWSA because some countries would refuse to sign it. It sometime fails to satisfy the coalitional participation constraints. In the above example, it failed to satisfy the participation constraint of coalition $\{1,3\}$ because countries 1 and 3 do better together with their own FWSA than with the upstream incremental FWSA at the basin level. This is true under deterministic flows. Of course, one can easily find other examples in the more general case of random flows. Therefore, another FWSA should be recommended to meet the coalitional participation constraints together with the self-enforcing constraints. We posit such a FWSA by imposing an additional requirement: an upper bound on countries' welfare.

In addition to not being in the core, the upstream incremental FWSA is not a good agreement for another reason: it sometime assigns to a country more than its satiated benefit. In the above example, country 2 obtains 44 whereas it would enjoy at most $b_2(\hat{x}_2)=36$ by consuming as much water as it wants. Country 2 somehow takes advantage of its position on the river by extracting more welfare than it could enjoy if water was abundant. It does so at the expense of the other countries. As suggested by Moulin (1990), in such games in which resource scarcity prevents all agents to enjoy their desired welfare, it is fair that none of them obtains strictly more than her or his desired welfare. We thus impose the fairness principle that no country enjoy a welfare higher than its satiated benefit $b_i(\hat{x}_i)$. This principle defines upper bounds on countries' individual welfare.[11]

Definition 3. A FWSA $(\boldsymbol{w},\boldsymbol{\tau})$ satisfies the satiated benefit upper bounds if

$$E[b_i(e_i+w_{i-1}-w_i)]-\tau_{i-1}+\tau_i\le b_i(\hat{x}_i)\text{ for all } i\in N. \tag{11}$$

We define the *constrained upstream incremental FWSA* as the core FWSA satisfying the satiated benefit upper bounds that lexicographically maximizes the welfare of country 1, 2,…, n. Denoted with the superscript "cu", the constrained upstream incremental RSA $(\boldsymbol{w}^*,\boldsymbol{\tau}^{cu})$ assigns to country 1 a welfare $u_1^{cu}=\min\{v(F1)-v(F^01),b_1(\hat{x}_1)\}$. If $v(F1)-v(F^01)>b_1(\hat{x}_1)$ (i.e. country 1's marginal contribution to its followers is strictly higher than its satiated benefit) the remaining welfare $u_1^{cu}-b_1(\hat{x}_1)$ is assigned to the next country in the river. Let us denote it $r(1)=\min\{u_1^{cu}-b_1(\hat{x}_1),0\}$. Country 2 obtains $u_2^{cu}=\min\{v(F2)-v(F^02)+r(1),b_2(\hat{x}_2)\}$. And so on and so forth for the next downstream countries. More generally, the constrained upstream incremental distribution assigns $u_i^{cu}=\min\{v(Fi)-v(F^0i)+r(i-1),b_i(\hat{x}_i)\}$ to each country $i\in N$ where $r(i-1)=\min\{u_{i-1}^{cu}-b_{i-1}(\hat{x}_{i-1}),0\}$ for every $i\in N\backslash\{1\}$ and $r(0)=0$. The constrained upstream incremental transfer scheme $\boldsymbol{\tau}^{cu}$ is thus defined by $\tau_i^{cu}=\sum_{j\in Pi}(u_j^{cu}-E[b_j(x_j^*)])$ for $i=1,\dots,n$.

The constrained upstream incremental FWSA satisfies the satiated benefit upper bounds by construction. The next proposition establishes that it is a core FWSA (see Appendix C1 for proof). Importantly, for any country $i\in N$, the constrained upstream incremental FWSA assigns to the coalition of countries upstream of i its marginal contribution to the other countries up to their satiated benefit

$$\sum_{j\in Pi}u_j^{cu}=\min\left\{v(N)-v(F^0i),\ \sum_{j\in Pi}b_j(\hat{x}_j)\right\}. \tag{12}$$

for $i=1,\dots,n-1$. Property (12) guarantees that no other core FWSA that satisfies the satiated benefit upper bounds are self-enforced under lower water flows (see Appendix C for proof).

Proposition 4. *Among all core FWSAs that satisfy the satiated benefit upper bounds, the constrained upstream incremental FWSA is self-enforced under the lowest water flows.*

[11] With similar concave and single peak benefit functions but with equal access to water, Ambec (2008) shows that the Walrasian allocation with equal division of water might assign to some agents more than their satiated benefits. It therefore violated the below fairness upper bounds. Note that the Walrasian allocation can be achieved with bilateral trade among (price-takers) neighboring countries as shown by Wang (2011).

By assigning to a set of countries upstream a country i the surplus from cooperation $v(N)-v(F^0 i)$ up to the satiated benefits $\sum_{j\in Pi} b_j(\hat{x}_j)$ for $i=1,\ldots,n-1$, the constrained upstream incremental FWSA maximizes the payments made to the upstream countries. It thus maximizes the loss in case of defection. Any other core FWSA that satisfies the satiated upper bounds would prescribe a lower payment for the same amount of water released for at least one country. The loss in case of defection would thus be lower. During some droughts events, the country would be better of defecting while it is not under the constrained upstream incremental FWSA. We now turn to contingent and proportional water sharing agreements.

4. Contingent and proportional water sharing agreements

4.1. Contingent water sharing agreements

We now allow to write water sharing agreements contingent on water flow $\boldsymbol{e}$. A contingent water sharing agreement (CWSA) $\{\boldsymbol{w}(\boldsymbol{e}),\boldsymbol{\tau}(\boldsymbol{e})\}$ is a set of vectors of water transfers $\boldsymbol{w}(\boldsymbol{e})$ and payments $\boldsymbol{\tau}(\boldsymbol{e})$ for every possible flow $\boldsymbol{e}\in\times_{i\in N}[\underline{e}_i,\overline{e}_i]$.

With CWSAs, it is convenient to derive the worth of a coalition from its ex post welfare, i.e. after the realization of water flows $\boldsymbol{e}$. Denoted $v(S,\boldsymbol{e})$, the ex post worth of a connected coalition S is the highest welfare that S can achieve with water flows $\boldsymbol{e}$ for every $S\subset N$ and $\boldsymbol{e}\in\times_{i\in N}[\underline{e}_i,\overline{e}_i]$

$$v(S,\boldsymbol{e})=\max_{\boldsymbol{w}_S(\boldsymbol{e})}\ \sum_{i\in S} b_i(e_i+w_{i-1}(\boldsymbol{e})-w_i(\boldsymbol{e})),$$
$$\text{s.t.}\quad w_i(\boldsymbol{e})\geq 0 \quad \text{for every } i\in S,$$
$$e_i+w_{i-1}(\boldsymbol{e})-w_i(\boldsymbol{e})\geq 0 \quad \text{for every } i\in S. \tag{13}$$

As before, the worth of any coalition $S\subset N$ is obtained by summing up the worth of its connected components: $v(S,\boldsymbol{e})=\sum_{S_l\in\mathcal{P}(S)} v(S_l,\boldsymbol{e})$ for any $\boldsymbol{e}\in\times_{i\in N}[\underline{e}_i,\overline{e}_i]$ where $v(S_l,\boldsymbol{e})$ is given by (13) and $\mathcal{P}(S)=\{S_l\}_{l=1}^{L}$ is the partition of S into its connected components. The ex ante worth of coalition S is $v(S)=E[v(S,\boldsymbol{e})]$. Coalitional participation constraints are defined ex ante as before. A CWSA $\{\boldsymbol{w}(\boldsymbol{e}),\boldsymbol{\tau}(\boldsymbol{e})\}$ satisfies the coalitional participation constraint for coalition S if

$$\sum_{i\in S}(E[b_i(e_i+w_{i-1}(\boldsymbol{e})-w_i(\boldsymbol{e}))-\tau_{i-1}(\boldsymbol{e})+\tau_i(\boldsymbol{e})])\geq E[v(S,\boldsymbol{e})]. \tag{14}$$

Note that a major difference between the coalition participation constraints for CWSAs in (14) and the ones for FWSAs in (6) is that, since payments are contingent on water flows, the expectation operator is applied not only to the benefit from water consumption but also to payments. Therefore the coalitional participation constraints restrict expected payments $E[(\tau_i(\boldsymbol{e}))]$ but not payments per see $\tau_i(\boldsymbol{e})$ for $i=1,\ldots,n-1$, thereby leaving some freedom in fixing payments. We will see later that they can be fixed to satisfy the (more stringent) ex post coalitional participation constraints. Before examining payments, let us focus on water releases.

The participation constraint of grand coalition N implies that the contingent water releases $\boldsymbol{w}(\boldsymbol{e})$ must be efficient ex post.[12] Formally, the efficient vector of water releases $\boldsymbol{w}^*(\boldsymbol{e})$ for realized water flows $\boldsymbol{e}$ solves the problem defined by $v(N,\boldsymbol{e})$ in (13) for every $\boldsymbol{e}\in\times_{i\in N}[\underline{e}_i,\overline{e}_i]$. It corresponds to the efficient water allocation in a deterministic setting with water flows $\boldsymbol{e}$ as described by Ambec and Sprumont (2002). The first-order conditions imply the equalization of marginal benefit $b'_i(x_i^*(\boldsymbol{e}))=b'_j(x_j^*(\boldsymbol{e}))$ whenever the non-negative water release constraints are not binding between i and j.[13] If it is binding at l with $i<l<j$, then the marginal benefit is strictly higher upstream than downstream: $b'_i(x_i^*(\boldsymbol{e}))>b'_j(x_j^*(\boldsymbol{e}))$.

We now examine the contingent payments. The self-enforcing constraints are defined as before but with contingent water releases and payments.

Definition 4. A CWSA $\{\boldsymbol{w}^*(\boldsymbol{e}),\boldsymbol{\tau}(\boldsymbol{e})\}$ is self-enforced with flow e_i if

$$b_i(e_i+w_{i-1}^*(\boldsymbol{e})-w_i^*(\boldsymbol{e}))+\tau_i(\boldsymbol{e})-\tau_{i-1}(\boldsymbol{e})\geq b_i(e_i+w_{i-1}^*(\boldsymbol{e}))-\tau_{i-1}(\boldsymbol{e})$$

for every $\boldsymbol{e}$ such that the realized flow controlled by country i is e_i.

As before, the self-enforcing constraints define lower bounds on payments that are now contingent on water flows

$$\tau_i(\boldsymbol{e})\geq b_i(e_i+w_{i-1}^*(\boldsymbol{e}))-b_i(e_i+w_{i-1}^*(\boldsymbol{e})-w_i^*(\boldsymbol{e})). \tag{15}$$

As for FWSA in Definition 2, a CWSA $\{\boldsymbol{w}^*(\boldsymbol{e}),\boldsymbol{\tau}(\boldsymbol{e})\}$ is self-enforcing is it is self-enforced for every water flow $\boldsymbol{e}\in\times_{i\in N}[\underline{e}_i,\overline{e}_i]$.

With water sharing agreements contingent on water flows, payments can be designed to satisfy the coalition participation constraints not only ex ante (before the realization of $\boldsymbol{e}$) but also ex post (after the realization of $\boldsymbol{e}$). Formally, contingent transfers $\boldsymbol{\tau}(\boldsymbol{e})$ satisfy the ex post participation constraints for a coalition $S\subset N$ if

$$\sum_{i\in S}(b_i(e_i+w_{i-1}^*(\boldsymbol{e})-w_i^*(\boldsymbol{e}))-\tau_{i-1}(\boldsymbol{e})+\tau_i(\boldsymbol{e}))\geq v(S,\boldsymbol{e})\quad \text{for every } \boldsymbol{e}\in\times_{i\in N}[\underline{e}_i,\overline{e}_i] \tag{16}$$

[12] If it is not the case for say water flows $\boldsymbol{e}'$, the welfare of riparian countries can be strictly increased by agreeing on efficient water releases $\boldsymbol{w}^*(\boldsymbol{e}')$ if water flows turn out to be $\boldsymbol{e}'$ with other contingent water transfers being unchanged.

[13] Remember that $x_i^*(\boldsymbol{e})=e_i+w_{i-1}^*(\boldsymbol{e})-w_i^*(\boldsymbol{e})$ denote country i's consumption with the efficient water transfers given the realized water flows $\boldsymbol{e}$.

The CWSA assigns to any group of countries $S \subset N$ at least the welfare that this group could achieve by signing its own CWSA among them for every potential water flow $\boldsymbol{e}$. Of course, CWSAs that satisfy the ex post coalitional participation constraints also satisfy the (ex ante) participation constraints as defined by (16). Moreover, since countries are risk-neutral regarding monetary transfers, there is no welfare loss in designing contingent payments that satisfy the participation constraints not only ex ante but also ex post.

It turns out that the CWSAs that satisfy the ex post participation constraints are always self-enforcing. It is easy to show that the ex post participation constraint for coalition $Pi = \{1, \ldots, i\}$ of countries upstream of country i implies that the CWSA is self-enforced for country i for every water flows $\boldsymbol{e} \in \times_{i \in N}[\underline{e}_i, \overline{e}_i]$. The property holds for every coalition Pi for $i = 1, \ldots, n-1$. Consider an arbitrary agent $i \in N$ and a given vector of water flow $\boldsymbol{e} \in \times_{i \in N}[\underline{e}_i, \overline{e}_i]$. The ex post welfare of coalition Pi with the CWSA $\{\boldsymbol{w}^*(\boldsymbol{e}), \boldsymbol{\tau}(\boldsymbol{e})\}$ under the realized water flow $\boldsymbol{e}$ is

$$\sum_{j \in Pi} b_j(e_j + w^*_{j-1}(\boldsymbol{e}) - w^*_j(\boldsymbol{e})) + \tau_i(\boldsymbol{e}).$$

The ex post coalitional participation constraint for Pi can therefore be written as

$$\tau_i(\boldsymbol{e}) \geq v(Pi, \boldsymbol{e}) - \sum_{j \in Pi} b_j(e_j + w^*_{j-1}(\boldsymbol{e}) - w^*_j(\boldsymbol{e})). \tag{17}$$

Given water supply $\boldsymbol{e}$, coalition Pi can agree on the efficient water releases up to country i in their own CWSA, that is releasing $w^*_j(\boldsymbol{e})$ from country j to $j+1$ for $j = 1, \ldots, i-1$. Hence

$$v(Pi, \boldsymbol{e}) \geq \sum_{j=1}^{i-1} b_j(e_j + w^*_{j-1}(\boldsymbol{e}) - w^*_j(\boldsymbol{e})) + b_i(e_i + w^*_{i-1}(\boldsymbol{e})). \tag{18}$$

Combining (17) with (18) leads to

$$\tau_i(\boldsymbol{e}) \geq b_i(e_i + w^*_{i-1}(\boldsymbol{e})) - b_i(e_i + w^*_{i-1}(\boldsymbol{e}) - w^*_i(\boldsymbol{e})),$$

which is the self-enforcing constraint (15) for country i. We thus have proved that a CWSA that satisfies the ex post participation constraints (16) is self-enforcing. Hence self-enforcement is not an issue with a contingent agreement: a CWSA can always be designed to be self-enforcing for any range of water flows by selecting payments that satisfy the ex post participation constraints.

Proposition 5. *Contingent water sharing agreements can be designed to be self-enforcing.*

When contingent to water flows, a water sharing agreement can be made acceptable for coalitions of countries not only ex ante but also ex post. When applied to successive coalitions starting from the source, the ex post coalitional participation constraints insure that coalition Pi is better-off releasing $w^*_i(\boldsymbol{e})$ to country $i+1$ in exchange of compensation $\tau_i(\boldsymbol{e})$ than with its own CWSA, for every $i \in N$. It yields lower bounds on transfer as defined in (17), which guarantees that country i is also better off releasing $w^*_i(\boldsymbol{e})$ in exchange of $\tau_i(\boldsymbol{e})$. In a nutshell, the welfare that country i enjoys if it defects is lower than what it would have enjoyed over a CWSA with its upstream partners. We now examine proportional water sharing agreements.

4.2. Proportional water sharing agreements

A proportional water sharing agreement (PWSA) defines water flow shares $\boldsymbol{\alpha} = \{\alpha_{ji}\}_{(j,i) \in P_i \times N}$ and payments $\boldsymbol{\tau} = (\tau_i)_{i \in N}$. The share α_{ji} corresponds to the proportion of water flow e_j originated from country j's territory that is assigned to country i for every $i \in F_j$ for all $j \in N$. Payments do not depend on water flows as with the FWSAs. An arbitrary country i pays τ_{i-1} for a right on shares α_{ji} of the flows e_j for $j = 1 \ldots, i-1$. It is allowed to consume a share α_{ii} of the water flow e_i originated from its territory. It supplies a share $1 - \alpha_{ii} = \sum_{l \in F^0 i} \alpha_{il}$ of e_i to downstream countries against a payment τ_i. Country i's ex ante welfare with the PWSA $(\boldsymbol{\alpha}, \boldsymbol{\tau})$ is

$$E\left[b_i\left(\sum_{j \in Pi} \alpha_{ji} e_j\right)\right] - \tau_{i-1} + \tau_i,$$

for $i = 1, \ldots n$ with the conventions $\tau_0 = 0$ and $\tau_n = 0$ so that country 1 and n obtain $E[b_1(\alpha_{11} e_1)] + \tau_1$ and $E[b_n(\sum_{j \in N} \alpha_{jn} e_n)] - \tau_{n-1}$ respectively.

The worth of a connected coalition S is defined as follows:

$$v(S) = \max_{\{\alpha_{ji}\}_{(j,i) \in (Pi \cap S) \times S}} E\left[\sum_{i \in S} b_i\left(\sum_{j \in Pi \cap S} \alpha_{ji} e_j\right)\right],$$
$$\text{s.t.} \; 0 \leq \alpha_{ji} \leq 1 \quad \text{for every } (j,i) \in (Pi \cap S) \times S,$$
$$\sum_{i \in Fj \cap S} \alpha_{ji} = 1 \quad \text{for every } j \in S.$$

The last constraint of the above program guarantee that the water flow e_j is shared among country j and its downstream partners within coalition S. As before, the worth of an arbitrary coalition $S \subset N$ is simply the sum of its connected components $v(S) = \sum_{S_l \in \mathcal{P}(S)} v(S_l)$ where $\mathcal{P}(S) = \{S_l\}_{l=1}^{L}$ is the partition of S into its connected components. A PWSA $(\boldsymbol{\alpha}, \boldsymbol{\tau})$ satisfies the coalitional participation constraint for coalition S if

$$\sum_{i\in S}\left(E\left[b_i\left(\sum_{j\in Pi}\alpha_{ji}e_j\right)\right]-\tau_{i-1}+\tau_i\right)\geq v(S).$$

A core PWSA is a PWSA that satisfies the coalitional participation constraints for all coalitions $S\subset N$. The participation constraint of the grand coalition implies that the water sharing rule $\boldsymbol{\alpha}$ of a core PWSA is efficient in the sense that it solves the problem defined by $v(N)$. Denoting $\boldsymbol{\alpha}^*$ the efficient shares of water flows, $\underline{\lambda}_{ji}$ and $\overline{\lambda}_{ji}$ the Lagrangian multipliers of the lower and upper bound constraints on α_{ji} for every $j\in Pi$ for $i=1,\ldots,n$, and γ_j the Lagrangian multiplier for the last constraints for every $j=1,\ldots,n$, the first-order conditions yield

$$E[e_j b'_i(x_i^*)]=\underline{\lambda}_{ji}-\overline{\lambda}_{ji}+\gamma_j$$

for every $j\in Pi$ for $i=1,\ldots,n$ where $x_i^*=\sum_{j\in Pi}\alpha_{ji}^*e_j$ denotes i's water consumption. When the constraints $0\leq\alpha_{ji}\leq 1$ are not binding, then $\underline{\lambda}_{ji}=\overline{\lambda}_{ji}=0$. Hence, for any two countries h and l downstream of flow e_j, the first-order conditions yield

$$E[e_j b'_h(x_h^*)]=E[e_j b'_l(x_l^*)]. \tag{19}$$

Since the marginal benefit of water defines its economic value in a country (at a location along the river), the first-order condition (19) equalizes the expected value of the water flow to be shared among countries. With water value of $b'_l(x_l^*)$ in country l and $b'_h(x_h^*)$ in country h, it guarantees that the expected value of the e_j units of water is equal in the two countries. If it is higher in country l than in country h, a higher share of e_j should be assigned to country l. Reversely, if the expected value of water is higher in country h, a higher share of e_j should be released to country l. When the equalization of water flow values among countries is not feasible, at least one constraint is binding. Suppose that the constraint is binding for, say, water flow e_k. Then e_k should be assigned exclusively to one country, say, country r so that $\alpha_{kr}=1$ and therefore $\alpha_{ki}=0$ for every $i\in Fj\backslash\{r\}$.

The self-enforcement constraints for PWSAs are defined the same manner than for the FWSAs in Definition 1. A PWSA $(\boldsymbol{\alpha},\boldsymbol{\tau})$ is self-enforced with flow e_i at i if

$$b_i\left(\sum_{j\in Pi}\alpha_{ji}e_j\right)-\tau_{i-1}+\tau_i\geq b_i\left(e_i+\sum_{j\in P^0 i}\alpha_{ji}e_j\right)-\tau_{i-1}.$$

As for FWSA in Definition 2, a PWSA $(\boldsymbol{\alpha},\boldsymbol{\tau})$ is self-enforcing if it is self-enforced for every water flow $\boldsymbol{e}\in\times_{i\in N}[\underline{e}_i,\overline{e}_i]$.

The upstream incremental PWSA $(\boldsymbol{\alpha}^*,\boldsymbol{\tau}^u)$ is defined with the transfer scheme $\boldsymbol{\tau}^u$ that assigns to every country its marginal contribution to its followers in the river:

$$u_i^u=E\left[b_i\left(\sum_{j\in Pi}\alpha_{ji}^*e_j\right)-\tau_{i-1}^u+\tau_i^u\right]=v(Fi)-v(F^0 i),$$

so that

$$\tau_i^u=E\left[\sum_{j\in F_i^0}b_j\left(\sum_{l\in Pj}\alpha_{lj}^*e_l\right)\right]-v(F^0 i)$$

for $i=1,\ldots,n$.

We now establish our result for proportional water sharing agreements. The proof is in Appendix D.

Proposition 6. *The upstream incremental PWSA is self-enforced under lower water flows than any other core PWSA.*

Although proportional and fixed agreements share differently water, they both define payments regardless of water flow. As a consequence, self-enforcement is an issue: with reduced water supply, countries might gain by not releasing the amount of water they agree to. In order to avoid countries' defection, the welfare from cooperative water sharing should be assigned lexicographically to the most upstream countries. Although the two types of agreement lead to different welfare, the upstream incremental welfare distribution is build in similar way by assigning to each country its marginal contribution to the downstream countries. It defines payments $\boldsymbol{\tau}^u$ based on value function $v(S)$ which differs if water releases are fixed or proportional to water flow.

5. Application to the Aral Sea Basin

We illustrate our approach with a simple example of three players, calibrated to the Aral Sea Basin. More precisely, we focus on the Bishkek international agreement signed in 1998 by Kyrgyzstan (KG), Uzbekistan (UZ) and Kazakhstan (KZ) on the Syr Darya river. The Syr Darya is one of the main streams that create the Aral Sea Basin in Central Asia. A description of the various features of the Syr Darya River, within the Aral Sea Basin are provided in Dinar et al. (2007). Dukhovny and de Schutter (2011) estimate the average annual river runoff between 1951 and 1975 to be 37.2 km^3. Of that volume, the runoff formed within KG, UZ and KZ is 74.2%, 16.6%, and 6.5%, respectively. Tajikistan contributes a minuscule amount of 2.7%, and for practical purposes it is not considered a riparian to this river. KG is the upstream riparian, using the water for electricity generation. UZ and KZ are both downstream riparians that use the water for irrigation of field crops (mainly wheat and cotton). The heart of the conflict between the three riparians stems from the reciprocal need-period of water for production of electricity (winter) and irrigation (summer). These conflicts are exacerbated by two factors related to climate change, namely variation in water availability across years, and extreme temperature low values in winter experienced by the upstream riparian KG. After several conflict incidents that followed the 1991 collapse of the Soviet Union, the riparian states reached several agreements, including the 1998 Bishkek Water Agreement. Without entering the agreement features and usefulness, the barter details (Dinar et al., 2007) suggest that KG receives from KZ the equivalent of 1.1 billion kWh of electric power in the form of coal (valued at 22 million dollars) and 400 million kWh + 500 million m^3 of natural gas (valued at 48.5 million dollars) from UZ. The total compensation transferred to KG is thus valued at $22+48.5=70.5$ million dollars. In return, KG releases 3.25 billion m^3 of water from the Toktogul Reservoir in monthly flows during the irrigation season and 2.2 billion kWh of summer electricity (from its hydropower facility on the Toktogul reservoir and downstream cascade) to KZ and UZ. Water release in summer was renegotiated to 1.3 billion m^3 in 2000 and 2.5 billion m^3 in 2001.[14] The 2000 agreement specifies that the summer water release should be allocated equally between KZ and UZ.

Using an integrated hydrologic–agronomic–economic model of the Syr Darya basin (Cai et al., 2003), we estimated a quadratic water benefit function for each of the three countries. Releasing D_1 billion cubic meters from the Toktogul Reservoir allows KG to produce hydropower with an estimated benefit of $B_1(D_1)=10.9D_1-0.032D_1^2$ in millions of dollars. From the D_1 billion m^3 released by KG, let us denote UZ and KZ's water consumption in billion m^3 by D_2 and D_3 respectively with $D_2+D_3=D_1$. The agricultural benefit from KG's water releases is $B_2(D_2)=12.749+538D_2-22D_2^2$ for UZ and $B_3(D_3)=3.148+540D_3-23D_3^2$ for KZ. The intercepts 12.749 and 3.148 represent the value of crop produced with the water inflows controlled by UZ and KZ, respectively. Under the above benefit functions, we estimate the upstream and downstream incremental transfers paid to KG for the 1998, 2000 and 2001 agreements. Consistent with theory, under the downstream incremental transfer t^d, the most upstream country is compensated exactly for its loss of welfare. That means that KG is paid for the loss of hydropower in winter due to water release in summer. The transfer t^d is thus defined as the expected loss of welfare for KG due to summer water releases. If KG has to release 3.25 billion m^3 in summer in compliance with the 1998 agreement, then the downstream incremental transfer is the difference between the expected value of hydropower production with and without 3.25 billions m^3.[15] Symmetrically, the upstream incremental transfer is the increased welfare due to summer water releases in UZ and KZ. Since the intercept represents the benefit without (summer) water releases, it is simply the difference between the benefit with $3.25/2=1.625$ billion m^3 and the intercept for each country.[16] We sum up the two differences to obtain the transfer received by KG under the upstream incremental distribution. The estimated transfers are presented in Table 1.

Our Aral Sea Basin example illustrates the magnitude of the difference between the two solutions. It also suggests that the range of acceptable transfers defined as $[t^u,t^d]$ is quite significant. The transfer of 70.5 million dollars negotiated in the 1998 agreement turns out to be included in this range. In Table 2, we compute the loss of welfare for all water inflows under the agreements signed in 1998, 2000 and 2001. That is the difference between $B_1(q)$ and $B_1(q-R)$ for any realized water inflow q under the committed release R with $R=3250$ for 1998, $R=1300$ for 2000 and $R=2500$ for 2001.

Each line refers to a level of water inflow observed in the past 100 years (first column). The second column presents the probability to obtain at least this level of water inflow based on historical data (CAWater-Info, 2011). The third to fifth columns yield the loss of welfare for KG from releasing water following the agreements signed in 1998, 2000 and 2001. It has been calculated with the benefit function $B_1(D_1)$ described above. As expected, the loss of benefit is increasing with a decline in water inflow. For a given inflow q, KG is better-off defecting if the loss of benefit from releasing water is higher than the transfer it receives. Consider the two payments t^d and t^u computed in Table 1. None of the agreements would be sustainable with the downstream incremental payment t^d when inflow is lower than 11 billions m^3 (approximately) which occurs 40% of the time. However, all agreements are sustainable with the upstream incremental payment t^u for any

Table 1
Water and monetary transfers.

Year	Delivery (billion m^3)	t^d (million \$)	t^u (million \$)
1998	3.25	33.3	1633
2000	1.3	13.2	682
2001	2.5	25.5	1277

[14] Sources: www.ce.utexas.edu/prof/mckinney/papers/aral/central_asia_regional_water.htm and www.cawater-info.net/bk/water_law/part3_e.htm.

[15] More precisely, we compute the expected benefits $E[B_1(D_1)]$ with water releases D_1 corresponding to the water inflows described in Table 2 (with the probabilities computed in the first column) and the expected benefit with the same water releases minus 3.25 billion m^3.

[16] Consistently with the 2000 agreement, water released by KG, D_1, is shared equally between UZ and KZ: $D_2=D_3=D_1/2$. It is also approximatively the optimal split of D_1 given UZ's and KZ's benefit functions.

Table 2
Loss of welfare due to water release depending on water inflow under the three agreements.

Water Inflow q (million m^3)	Probability that inflows are at least q	Loss 1998 agreement (million $)	Loss 2000 agreement (million $)	Loss 2001 agreement (million $)
6525	0.990	34.4	13.7	26.4
7478	0.980	34.2	13.6	26.3
7750	0.970	34.2	13.6	26.2
8290	0.945	34.0	13.5	26.1
8810	0.895	33.9	13.5	26.0
9232	0.830	33.8	13.5	26.0
9714	0.780	33.7	13.4	25.9
10,267	0.720	33.6	13.4	25.8
10,763	0.605	33.5	13.3	25.7
11,286	0.495	33.4	13.3	25.6
11,746	0.430	33.3	13.2	25.6
12,130	0.390	33.2	13.2	25.5
12,755	0.330	33.1	13.2	25.4
13,207	0.260	33.0	13.1	25.3
13,686	0.210	32.9	13.1	25.3
14,329	0.165	32.8	13.0	25.2
14,702	0.110	32.7	13.0	25.1
15,152	0.065	32.6	13.0	25.0
15,763	0.050	32.5	12.9	24.9
16,250	0.041	32.4	12.9	24.9
16,590	0.035	32.3	12.8	24.8
17,250	0.030	32.2	12.8	24.7
17,750	0.027	32.1	12.7	24.6
18,250	0.023	32.0	12.7	24.5
18,754	0.020	31.9	12.7	24.4
19,250	0.017	31.8	12.6	24.4
19,750	0.015	31.7	12.6	24.3
20,725	0.010	31.5	12.5	24.1

historical inflow. Hence, it is always the self-interest of KG to release what it committed to in the Bishkek treaty if it is paid its marginal contribution to the benefit of UZ and KG.[17]

6. Conclusion

By signing international river sharing treaties voluntarily, countries agree to release some fixed amount of water in exchange for some compensation. They have a self-interest in complying with the releases when water inflow is high enough. Even if an agreement specifies water supply to downstream countries, a country is better off by releasing what it had committed to, since the payment it receives from downstream countries offsets its welfare loss from releasing water. This is not always the case under water drought conditions within its territory. To release the same amount of water, the country is obliged to consume less water. It might be tempted to defect if the payment it receives does not compensate its welfare loss from releasing the water.

In this paper, we analyze the design of water sharing agreements under variable water flow and their robustness to the above defection strategy by countries. When water releases and payments can be set contingent to water flows, water sharing agreements can be designed such that no country defects. Defection is an issue for fixed and proportional water sharing agreements because payments are fixed regardless water flows. We first fully characterize the set of water sharing agreements that are acceptable by all groups of riparian countries. They all prescribe the same water releases: those which maximize the expected welfare of water extraction along the river. In contrast, many monetary transfers can be part of an acceptable water agreement including the ones defined by the Shapley value, the Walrasian allocation and the downstream incremental welfare distribution.

Among the set of acceptable monetary transfers, we identify the one which is the most robust to defection in case of drought for both fixed and proportional water sharing agreements. It is the upstream incremental transfer scheme which requires that each country receives the marginal contribution of its water releases to all the countries located downstream. It maximizes lexicographically the welfare of the most upstream countries in the set of acceptable transfers. Opposite in this set is the downstream incremental transfer scheme which maximizes lexicographically the welfare of the most downstream countries. The downstream incremental transfer scheme turns out to be less robust to defection than any other acceptable

[17] Note that the 70.5 million dollars compensation for KG stipulated in the 1998 agreement seems also to prevent KG from defecting for any historical inflow.

transfer scheme. Our computation from a simple representation of the Aral Sea Basin provides evidence that the two types of solutions can differ substantially. It thus suggests that picking the right agreement can greatly reduce the vulnerability of fixed water sharing agreements to global warming.

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Appendix A. Proof of Proposition 2

Suppose that a core FWSA $(\boldsymbol{w}^*, \boldsymbol{\tau}')$ with $\boldsymbol{\tau}' \neq \boldsymbol{\tau}^u$ is self-enforced with reduced flow e_i at i while $(\boldsymbol{w}^*, \boldsymbol{\tau}^u)$ is not. Then, by (9), we have

$$\tau_i' \geq b_i(e_i + w_{i-1}^*) - b_i(e_i + w_{i-1}^* - w_i^*) > \tau_i^u.$$

By the definition of τ_i^u in (8), the above inequality implies

$$\tau'_i > \sum_{j \in F^0 i} E[b_j(e_j + w_{j-1}^* - w_j^*)] - v(F^0 i),$$

or, equivalently

$$v(F^0 i) > \sum_{j \in F^0 i} E[b_j(e_j + w_{j-1}^* - w_j^*)] - \tau_i'. \tag{20}$$

Now by (1) the ex ante welfare of country j with $(\boldsymbol{w}^*, \boldsymbol{\tau}')$ is defined by

$$u'_j = E[b_j(e_j + w_{j-1}^* - w_j^*)] - \tau'_{j-1} + \tau'_j.$$

The total ex ante welfare of coalition $F^0 i = \{i+1, \ldots, n\}$ is then

$$\sum_{j \in F^0 i} u'_j = \sum_{j \in F^0 i} E[b_j(e_j + w_{j-1}^* - w_j^*)] - \tau'_i.$$

Combined with (20), it leads to $v(F^0 i) > \sum_{j \in F^0 i} u'_j$ which contradicts that $(\boldsymbol{w}^*, \boldsymbol{\tau}')$ is a core FWSA.

Appendix B. Proof of Proposition 3

The proof of the first part of Proposition 3 is similar than the proof of Proposition 2. Suppose that a core FWSA $(\boldsymbol{w}^*, \boldsymbol{\tau}')$ with $\boldsymbol{\tau}' \neq \boldsymbol{\tau}^d$ is not self-enforced under reduced flow e_i at i while $(\boldsymbol{w}^*, \boldsymbol{\tau}^d)$ is. Then, by (9), we have

$$\tau_i^d \geq b_i(e_i + w_{i-1}^*) - b_i(e_i + w_{i-1}^* - w_i^*) > \tau'_i.$$

By the definition of τ_i^d in (7), the above inequality implies

$$v(Pi) - \sum_{j \in Pi} E[b_j(e_j + w_{j-1}^* - w_j^*)] > \tau'_i,$$

or, equivalently

$$v(Pi) > \sum_{j \in Pi} E[b_j(e_j + w_{j-1}^* - w_j^*)] + \tau_i'. \tag{21}$$

The ex ante welfare of country j with $(\boldsymbol{w}^*, \boldsymbol{\tau}')$ is defined by

$$u'_j = E[b_j(e_j + w_{j-1}^* - w_j^*)] - \tau'_{j-1} + \tau'_j.$$

The total ex ante welfare of coalition $Pi = \{1, \ldots, i\}$ is then

$$\sum_{j \in Pi} u'_j = \sum_{j \in Pi} E[b_j((e_j + w_{j-1}^* - w_j^*)] + \tau'_i.$$

Combined with (21), it yields $v(Pi) > \sum_{j \in Pi} u'_j$ which contradicts that $(\boldsymbol{w}^*, \boldsymbol{\tau}')$ is a core FWSA.

For the second part of Proposition 3, first remark that, since $b_1(e_1) - b_1(e_1 - w_1^*)$ is decreasing with $e_1 \in [\underline{e}_1, \overline{e}_1]$ for every $w_1^* \in (0, e_i)$, $b_1(\underline{e}_1) - b_1(\underline{e}_1 - w_1^*) \geq b_1(e_1) - b_1(e_1 - w_1^*)$ for every $e_1 \in [\underline{e}_1, \overline{e}_1]$ with a strict inequality for $e_1 > \underline{e}_1$. The last inequalities imply

$$b_1(\underline{e}_1) - b_1(\underline{e}_1 - w_1^*) > E[b_1(e_1) - b_1(e_1 - w_1^*)]. \tag{22}$$

Since, by (7), $\tau_1^d = v(1) - E[b_1(\underline{e}_1 - w_1^*)]$ and $v(1) = E[b_1(e_1)]$, the inequality (22) can be re-arranged as $b_1(\underline{e}_1) - b_1(\underline{e}_1 - w_1^*) > \tau_1^d$ which shows that the downstream incremental FWSA is not sustainable to reduced flow at the source for any minimal flow $\underline{e}_1 < \overline{e}_1$, that is as long as e_1 is random.

Appendix C. Proof of Proposition 4

C.1. The constrained upstream incremental FWSA is in the core

Consider an arbitrary coalition $S \subset N$. If S is connected, the constrained upstream incremental WSA yields to coalition S a welfare[18]

$$\sum_{i \in S} u_i^{cu} = \min\left\{ v(FS) - v(F^0 S) + r(\min S - 1), \sum_{i \in S} b_i(\hat{x}_i) \right\}. \tag{23}$$

Since $FS = S \cup F^0 S$ for every S connected, by superadditivity of v, $v(FS) \geq v(S) + v(F^0 S)$. Moreover, $v(S) \leq \sum_{i \in S} b_i(\hat{x}_i)$. The last two inequalities combined with (23) and $r(j) \geq 0$ for any $j \in N$ establish $\sum_{i \in S} u_i^{cu} \geq v(S)$ for any connected coalition S.

Suppose now that S is not connected. Consider the last country in S that consumes water $l(S) = \max_i \{ i \in S : x_i^S > 0\}$. If $l(S)$ does not exist, $v(S) = 0 \leq \sum_{i \in S} u_i^{cu}$. Let $\overline{S} = Pl(S) \backslash P^0 \min S$ be the coalition composed by all countries from $\min S$ to $l(S)$. Since $\overline{S}$ is connected, the welfare of $\overline{S}$ is

$$\sum_{i \in \overline{S}} u_i^{cu} = \min\left\{ v(F\overline{S}) - v(F^0 \overline{S}) + r(\min \overline{S} - 1), \sum_{i \in \overline{S}} b_i(\hat{x}_i) \right\}$$

The last equation with $\min \overline{S} = \min S$ and $\sum_{i \in \overline{S}} u_i^{cu} = \sum_{i \in S} u_i^{cu} + \sum_{i \in \overline{S} \backslash S} u_i^{cu}$ imply

$$\sum_{i \in S} u_i^{cu} = \min\left\{ v(F\overline{S}) - v(F^0 \overline{S}) + r(\min S - 1), \sum_{i \in \overline{S}} b_i(\hat{x}_i) \right\} - \sum_{i \in \overline{S} \backslash S} u_i^{cu}. \tag{24}$$

Suppose first that $v(F\overline{S}) - v(F^0 \overline{S}) + r(\min S - 1) \geq \sum_{i \in \overline{S}} b_i(\hat{x}_i)$. Then $\sum_{i \in S} u_i^{cu} = \sum_{i \in \overline{S}} b_i(\hat{x}_i)$. Since $u_i^{cu} \leq b_i(\hat{x}_i)$ for every $i \in \overline{S} \backslash S$, (24) implies

$$\sum_{i \in S} u_i^{cu} = \sum_{i \in \overline{S}} b_i(\hat{x}_i) - \sum_{i \in \overline{S} \backslash S} u_i^{cu} \geq \sum_{i \in \overline{S}} b_i(\hat{x}_i) - \sum_{i \in \overline{S} \backslash S} b_i(\hat{x}_i) = \sum_{i \in S} b_i(\hat{x}_i) \geq v(S).$$

Suppose now that $v(F\overline{S}) - v(F^0 \overline{S}) + r(\min S - 1) < \sum_{i \in \overline{S}} b_i(\hat{x}_i)$. Then, since $F\overline{S} = \overline{S} \cup F^0 \overline{S}$, by superadditivity of v, $v(F\overline{S}) \geq v(\overline{S}) + v(F^0 \overline{S})$ which, combined with (24) and $r(\min S - 1) \geq 0$, implies

$$\sum_{i \in S} u_i^{cu} \geq v(\overline{S}) - \sum_{i \in \overline{S} \backslash S} z_i^{cu}. \tag{25}$$

Since countries in-between connected coalitions in S up to $l(S)$ divert $\hat{x}_i$ for every $i \in \overline{S} \backslash S$, the water allocation $((x_i^S)_{i \in S \cap \overline{S}}, (\hat{x}_i)_{i \in \overline{S} \backslash S})$ can be implemented in $\overline{S}$ and, therefore, $v(\overline{S}) \geq v(S \cap \overline{S}) + \sum_{i \in \overline{S} \backslash S} b_i(\hat{x}_i)$. Furthermore, since $x_i^S = 0$ downstream $l(S)$ in S for every $i \in S \backslash Pl(S)$, and, therefore $b_i(x_i^S) = 0$ for every $i \in S \backslash Pl(S)$, $v(S) = v(S \cap Pl(S)) = v(S \cap \overline{S})$. Thus we have

$$v(\overline{S}) \geq v(S) + \sum_{i \in \overline{S} \backslash S} b_i(\hat{x}_i). \tag{26}$$

Since $u_i^{cu} \leq b_i(\hat{x}_i)$ for every $i \in \overline{S} \backslash S$ by definition, (25) and (26) imply $\sum_{i \in S} u_i^{cu} \geq v(S) + \sum_{i \in \overline{S} \backslash S} (b_i(\hat{x}_i) - u_i^{cu})$. Since by definition $u_i^{cu} \leq b_i(\hat{x}_i)$, it implies $\sum_{i \in S} u_i^{cu} \geq v(S)$ the desired conclusion.

C.2. The constrained upstream incremental FWSA is self-enforced under lowest water flows than any other core FWSA that satisfies the satiated benefit upper bounds

Suppose that a core FWSA $(\boldsymbol{w}^*, \boldsymbol{\tau}')$ with $\boldsymbol{\tau}' \neq \boldsymbol{\tau}^{cu}$ is self-enforced with reduced flow e_i at $\boldsymbol{i}$ while $(\boldsymbol{w}^*, \boldsymbol{\tau}^u)$ is not. Then, by (9), we have

$$\tau_i{}' \geq b_i(e_i + w_{i-1}^*) - b_i(e_i + w_{i-1}^* - w_i^*) > \tau_i^{cu}.$$

Adding $\sum_{j \in Pi} E[b_j(x_j^*)]$ both sides of the inequalities, it implies

$$\sum_{j \in Pi} E[b_j(x_j^*)] + \tau'_i > \sum_{j \in Pi} E[b_j(x_j^*)] + \tau_j^{cu}. \tag{27}$$

[18] Recall that $\min S$ denotes the most upstream country in S.

On the other hand, the participation constraint (6) for coalition $F^0 i = \{i+1, \ldots, n\}$ under the FWSA $(\boldsymbol{w}^*, \boldsymbol{\tau}')$ yields

$$\sum_{j \in F^0 i} E[b_j(x_j^*)] - \tau'_i \geq v(F^0 i).$$

Since $N = Pi \cup F^0 i$, by definition of $v(N)$

$$v(N) = \sum_{j \in Pi} E[b_j(x_j^*)] + \sum_{j \in F^0 i} E[b_j(x_j^*)].$$

The last two relationships imply

$$\sum_{j \in Pi} E[b_j(x_j^*)] + \tau'_i \leq v(N) - v(F^0 i). \tag{28}$$

Combining (27) and (28) yields

$$v(N) - v(F^0 i) \geq \sum_{j \in Pi} E[b_j(x_j^*)] + \tau_i^{cu}.$$

Since $\sum_{j \in Pi} E[b_j(x_j^*)] + \tau_i^{cu} = \sum_{j \in Pi} u_j^{cu}$, the above condition combined with (12) implies that the satiated benefit upper bounds are binding for countries $j = 1, \ldots, i$. Therefore

$$\sum_{j \in Pi} E[b_j(x_j^*)] + \tau_i^{cu} = \sum_{j \in Pi} b_j(\hat{x}_j).$$

By (27), it yields

$$\sum_{j \in Pi} E[b_j(x_j^*)] + \tau'_i > \sum_{j \in Pi} b_j(\hat{x}_j),$$

that is the FWSA $(\boldsymbol{w}^*, \boldsymbol{\tau}')$ fails to satisfy the satiated benefit upper bound for at least one country $j \in Pi$, a contradiction.

Appendix D. Proof of Proposition 6

First we show that the upstream incremental PWSA is a core PWSA. Consider an arbitrary coalition S and its partition into connected components $\mathcal{P}(S) = \{S_l\}_{l=1}^{L}$. The upstream incremental PWSA $(\boldsymbol{\alpha}^*, \boldsymbol{\tau}^u)$ assigns to the members of coalition S a welfare of

$$\sum_{i \in S} u_i^u = \sum_{S_l \in \mathcal{P}(S)} \sum_{i \in S_l} u_i^u = \sum_{S_l \in \mathcal{P}(S)} \Big(v(FS_l) - v(F^0 S_l) \Big), \tag{29}$$

where FS_l denotes the set of follower of S_l including S_l while $F^0 S_l$ is the set of strict followers of S_l. Since the value function v is supper-additive, $FS_l = F^0 S_l \cup S_l$ implies

$$v(FS_l) \geq v(F^0 S_l) + v(S_l), \tag{30}$$

for every $S_l \in \mathcal{P}(S)$. Combining (29) and (30) with $v(S) = \sum_{S_l \in \mathcal{P}(S)} v(S_l)$ shows that the coalitional participation constraint hold for coalition S.

The proof that $(\boldsymbol{\alpha}^*, \boldsymbol{\tau}^u)$ is self-enforcing under lower reduced flow than any other core PWSA proceeds similarly than in Proposition 2. It is therefore omitted.

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