Title

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ABSTRACT

A Fano factor of 0.30±0.03 has been measured for germanium with a lithium-drifted semiconductor detector used for measuring gamma-ray energies ranging from 100 to 2800 keV. A value of 2.98±0.01 eV for the average energy per hole-electron pair at 77°K was measured for gamma-ray peaks in the energy region from 100 to 1400 keV.

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1. THE FANO FACTOR FOR GERMANIUM

1.1 INTRODUCTION

The ability of semiconductor detector systems to measure a wide energy range while simultaneously providing very good energy resolution has led to their widespread use in modern nuclear spectroscopy. In this paper the ultimate resolution, as related to the Fano factor, for lithium-drifted germanium semiconductor detectors is discussed. The resolution of the system is determined (apart from experimental sources that adversely affect resolution, such as high counting rate and bad experimental geometry) by two fundamentally different characteristics:

(a) Noise from the associated electronic equipment, mainly from the input stage of the preamplifier;

(b) Statistical fluctuation in the number of electron-hole pairs created in the crystal by the incident particle.

The noise is minimized by well-matched preamplifier–linear-amplifier systems[1,2]. Although the noise performance of such systems has been investigated by many authors the statistical fluctuation in the number of charges created in the crystal has hitherto received little attention. This statistical fluctuation, being a fundamental property of the detector material, sets an ultimate resolution limit.

For a rough estimate of the statistical fluctuation Δn in the number of hole-electron pairs created in the detector by an incident particle with energy E, the following equation can be used:

$$\Delta n = (E/\epsilon)^{1/2} , \qquad (1)$$

where ϵ is the average energy per hole-electron pair. Experimentally, however, the statistical fluctuation is less than that calculated from eq. (1). The magnitude of this discrepancy is given by the Fano factor F, defined by

$$(\Delta n)^2_{exp} = F\,(\Delta n)^2 , \qquad (2)$$

where $(\Delta n)_{exp}$ is the experimental statistical fluctuation.

In 1947 Fano[3]) treated the statistical fluctuation in the number of ions produced by radiation in gases and predicted a factor of approximately 1/3 to 1/2. Van Roosbroeck[4]) did a detailed theoretical investigation, using analytical and Monte Carlo methods for the calculation of yield and Fano factor for hole-electron pairs generated in semiconductors; we use some of his results in this report.

From eq. (2) it can be seen that a Fano factor of 1 would correspond to the Poisson distribution that applies when the events in the ionization process are independent. But the energy-loss mechanisms in the semiconductor are not independent, and therefore a Fano factor between 0 and 1 would be expected. Very little experimental information is available, and the values reported, ranging from 0.3 to 0.5 for germanium[5,6]), have usually been obtained as incidental information.

1.2 THEORY

Energetic particles incident on a semiconductor produce phonons and electron-hole pairs through a branching process, the mechanism of which is shown in fig. 1. An electron or a hole created by the energetic particle can lose energy in two fundamentally different ways:

(a) Creation of a hole-electron pair. The available energy (primary electron energy minus band-gap energy loss) is randomly shared by the degraded primary electron and the pair. The electron and the hole are assumed to share equally the energy given to the pair.

(b) Phonon losses to the crystal lattice. These phonons arise from electrons having energies both above and below the band-gap energy.

For a mathematical description of the creation of hole-electron pairs in semiconductors, the following parameters are introduced. Theoretically, the relative yield Y is defined by the ionization threshold energy E_{th} divided by the average energy per hole-electron pair ϵ

$$Y = \frac{E_{th}}{\epsilon} . \qquad (3)$$

The statistical behavior of the energy-loss process is determined mainly at the end of the branching process, where most of the secondaries having an energy of just a few ionization-threshold units are generated. The yield and Fano factor, and therefore the average energy per hole-electron pair, are essentially independent of the type of incident particle.

The dependence of yield and Fano factor on the number N of generated phonons per ionizing collision has been calculated by Van Roosbroeck[4]), from whose report fig. 2 has been reproduced. For the application of this theory to the germanium detector discussed in our paper, an estimate of the relative yield is necessary. This estimate requires knowledge of the threshold energy and the average energy per hole-electron pair. The average energy per hole-electron pair has been determined to be 2.98±0.01 eV. (See Section II). Although the threshold energy for germanium at 77°K is not accurately known, Van Roosbroeck[4]) has demonstrated that $E_{th} \approx E_g$ (E_g is the energy gap) is a good approximation. Since $E_g = 0.74$ eV, the relative yield can then be calculated by means of eq. (3)

$$Y = \frac{0.74}{2.98} = 0.25.$$

From fig. 2 a corresponding theoretical Fano factor of 0.32 is determined.

1.3 EXPERIMENTAL METHOD

A lithium-drifted germanium detector with a 0.5-cm depletion region and a rectangular 1- by 1.5-cm area was used in these experiments. This detector was operated at 77°K and 600 volts bias, with a leakage current of less than 10^{-9} A. The charge-sensitive preamplifier, with an EC-1000 input stage, and the linear amplifier system have been reported previously[1, 2]). The spectra were recorded in a 400-channel pulse-height analyzer.

Simultaneous observations of gamma-ray and pulser peaks were made for gamma-ray energies ranging from 100 to 2800 keV. Sources having one or two strongly dominant gamma transitions of well-known energies were used. The pulser was a transistorized type with a short-term stability better than 0.01% (Lawrence Radiation Laboratory Drawing 11X3281 P-1). The full widths at half maximum (FWHM) of the gamma-ray and pulser peaks were determined by means of the energy calibration provided by the pulser, which was calibrated against known gamma-ray energies. In those cases where sources were available, a better energy calibration was obtained by calibrating the system directly from two known gamma-ray lines in the energy region being studied. In addition to the energy calibration, the pulser peaks provided a means of resolution control throughout the experiment. Typical spectra are shown in figs. 3 and 4.

The linearity of the amplifier system, carefully investigated, was determined to be better than 3 parts in 1000, and the linearity of the pulse-height analyzer was better than 1 part in 200. Investigations of the time stability of the system indicated no drifts in peak positions over a period of two hours, which was the longest counting time used. To avoid a loss of resolution due to baseline shift, the total counting rate in the system was kept below 2000 counts/sec. Since the pulser peak widths did not vary, these factors can be neglected. The accuracy of the resolution determination is given by three error contributions:

(a) Inaccuracy in the pulser dial reading causing an energy determination error, in all recordings less than 1%;

(b) Error in the energy-scale determination arising from an error in the estimation of the peak positions, in all recordings less than 0.5%;

(c) The main error, arising from our inability to determine very accurately the number of channels for FWHM. The accuracy of this determination is essentially dependent on the total number of counts in the peak and this error is typically about 5%.

1.4 RESULTS AND DISCUSSION

In the measurements, an addition in quadrature of the electronic noise and the statistical fluctuation in the detector will correspond to the observed FWHM of the peak.

$$(\mathrm{FWHM})^2_{\mathrm{observed}} = (\mathrm{FWHM})^2_{\mathrm{electronic}} + 2.35^2\, F\varepsilon, \qquad (4)$$

where the last term represents the statistical fluctuation obtained from eq. (2), converted to energy units by multiplication by ε, and to FWHM of a Gaussian distribution by the factor 2.35.

The observed resolutions are listed in table 1, and these data are plotted in fig. 5 where lines calculated from eq. (4) are drawn. The linearity of the plot indicates that very little charge trapping occurs in this detector. The observed broadening on the low-energy side of the 2754-keV Na^{24} peak can be accounted for by low-energy secondaries escaping from the detector, although some trapping may have been present. On the basis of the lines drawn in fig. 5, we estimate a Fano factor of

$$F = 0.30 \pm 0.03,$$

which is in excellent agreement with the value predicted by the theory of Van Roosbroeck[4]. It is noteworthy that this factor remained constant throughout the energy region investigated. Using fig. 2 and the experimental value of the Fano factor, one can determine that the number of phonons per ionizing collision is $N = 52$, and that the value of the threshold energy is

$$E_{th} = 0.80 \text{ eV},$$

a value only 6 percent higher than the energy gap and thus consistent with the assumption $E_{th} \approx E_g$.

Using the relation $\epsilon = E_{th} + N E_R$, where E_R = Raman energy for phonons in germanium, we obtain a value of

$$E_R = 42 \times 10^{-3} \text{ eV},$$

which corresponds to a wave number of 339 cm^{-1}, as compared with 370 cm^{-1} measured by infrared spectroscopy methods[7]. These comparisons indicate that our measured Fano factor leads to consistent results.

2. MEASUREMENT OF THE AVERAGE ENERGY PER HOLE-ELECTRON PAIR FOR GAMMA RAYS IN GERMANIUM AT 77°K

The average energy per hole-electron pair ϵ has been measured by many authors whose values range from 2.8 to 3.0 eV. At present, the most accurate measurement known to the authors is [8] $\epsilon = 2.94 \pm 0.15$ eV. To determine ϵ more accurately, we measured gamma-ray peaks with the same experimental equipment described in Section I, except that an accurately calibrated mercury-relay pulse generator was used instead of the transistorized pulser. Each gamma-ray peak was recorded during the same time interval as a set of mercury pulser peaks whose voltages were determined by a precision digital voltmeter. This method will be described in detail in a later report [9]. Measurement of the pulse voltage V_m corresponding to the peak position of the gamma ray and the value of the pulser test capacitor C of the charge-sensitive preamplifier enables one to calculate ϵ from $\epsilon = e E_\gamma / V C$, where e is the electron charge and E_γ is the gamma-ray energy. To obtain the V used in the ϵ calculation, we lowered the measured value of the voltage V_m by about 0.25% because the charge-sensitive preamplifier does not have infinite gain. From these measurements, which are summarized in table 2 and are also shown in fig. 6, an average value of

$$\epsilon = 2.98 \pm 0.01 \text{ eV}$$

is obtained for gamma rays in germanium. (The test capacitor had a value of $C = 0.767 \pm 0.001$ pF.) Note that the accuracy of this measurement is strongly supported by the good linearity of fig. 6.

Knowledge of the Fano factor and ϵ enables one to determine the theoretical limitation of the energy resolution. Figures 7 and 8 show these limiting values as a function of the energy loss in the detector for different electronic (pulser) resolutions.

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Table 1. Measured FWHM values and the corresponding Fano factor of the gamma-ray peaks

Source	E_γ(keV)	$FWHM_{observed}$(keV)	F[a]
Co^{57}	122.05 ± 0.05	3.02 ± 0.12	0.35 ± 0.35
Cs^{137}	661.60 ± 0.08	3.41 ± 0.14	0.30 ± 0.08
Mn^{54}	834.9 ± 1.1	3.52 ± 0.14	0.29 ± 0.07
Co^{60}	1173.22 ± 0.04	3.79 ± 0.17	0.31 ± 0.06
Na^{22}	1274.8 ± 0.8	3.78 ± 0.19	0.29 ± 0.06
Co^{60}	1332.48 ± 0.05	3.82 ± 0.15	0.29 ± 0.06
Y^{88}	1836.2 ± 1.0	4.24 ± 0.21	0.32 ± 0.06
Na^{24}	2753.92 ± 0.12	4.62 ± 0.31[b]	0.28 ± 0.06

[a]The electronic noise contribution was assigned a value of 2.90 ± 0.05 keV, averaged from about 20 pulser peak recordings.

[b]This value has been calculated from the high-energy edge of the recorded gamma-ray peak.

Table 2. Summary of the results for the average energy per hole-electron pair

Source	E_γ (keV)	V (mV)	ϵ (eV)
Co^{57}	122.05 ± 0.05	8.56 ± 0.03	2.978 ± 0.015
Co^{57}	136.40 ± 0.06	9.55 ± 0.06	2.983 ± 0.014
Sn^{113}	393.0 ± 0.3	27.48 ± 0.04	2.987 ± 0.011
Cs^{137}	661.60 ± 0.08	46.36 ± 0.05	2.981 ± 0.007
Mn^{54}	839.4 ± 1.1	58.91 ± 0.05	2.976 ± 0.010
Co^{60}	1173.23 ± 0.04	82.16 ± 0.07	2.968 ± 0.015
Co^{60}	1332.48 ± 0.05	93.32 ± 0.08	2.982 ± 0.007

FIGURE CAPTIONS

Fig. 1. Diagrammatic representation of the energy-loss process in a solid-state detector: E_R = Raman phonon energy for the lattice; E_g = band-gap of the material; p = assumes a random value from 0 to 1.

Fig. 2. Dependence of yield and Fano factor on number of phonons per ionizing collision. (Reproduced from the paper of Van Roosbroeck[4]). The theoretical and experimental Fano factors are shown.

Fig. 3. Example of spectra recorded, showing the 834.9-keV gamma-ray peak of Mn^{54}.

Fig. 4. Example of spectra recorded, showing the 2753.92-keV gamma-ray peak of Na^{24}. The quoted FWHM value has been calculated from the high-energy side of the peak because low-energy tailing was observed.

Fig. 5. Plot of the resolution of the germanium detector versus gamma-ray energy. Lines representing Fano factors calculated on the basis of the value of ϵ reported in Section II of this paper are shown.

Fig. 6. Gamma-ray energies and corresponding pulser voltages, listed in table II, illustrating the excellent linearity.

Fig. 7. Total energy resolution as a function of the energy loss in the detector (up to 3 MeV) for different electronic (pulser) resolutions.

Fig. 8. Total energy resolution as a function of the energy loss in the detector (up to 30 MeV) for different electronic (pulser) resolutions.

FOOTNOTES AND REFERENCES

*This work was done under the auspices of the U. S. Atomic Energy Commission.

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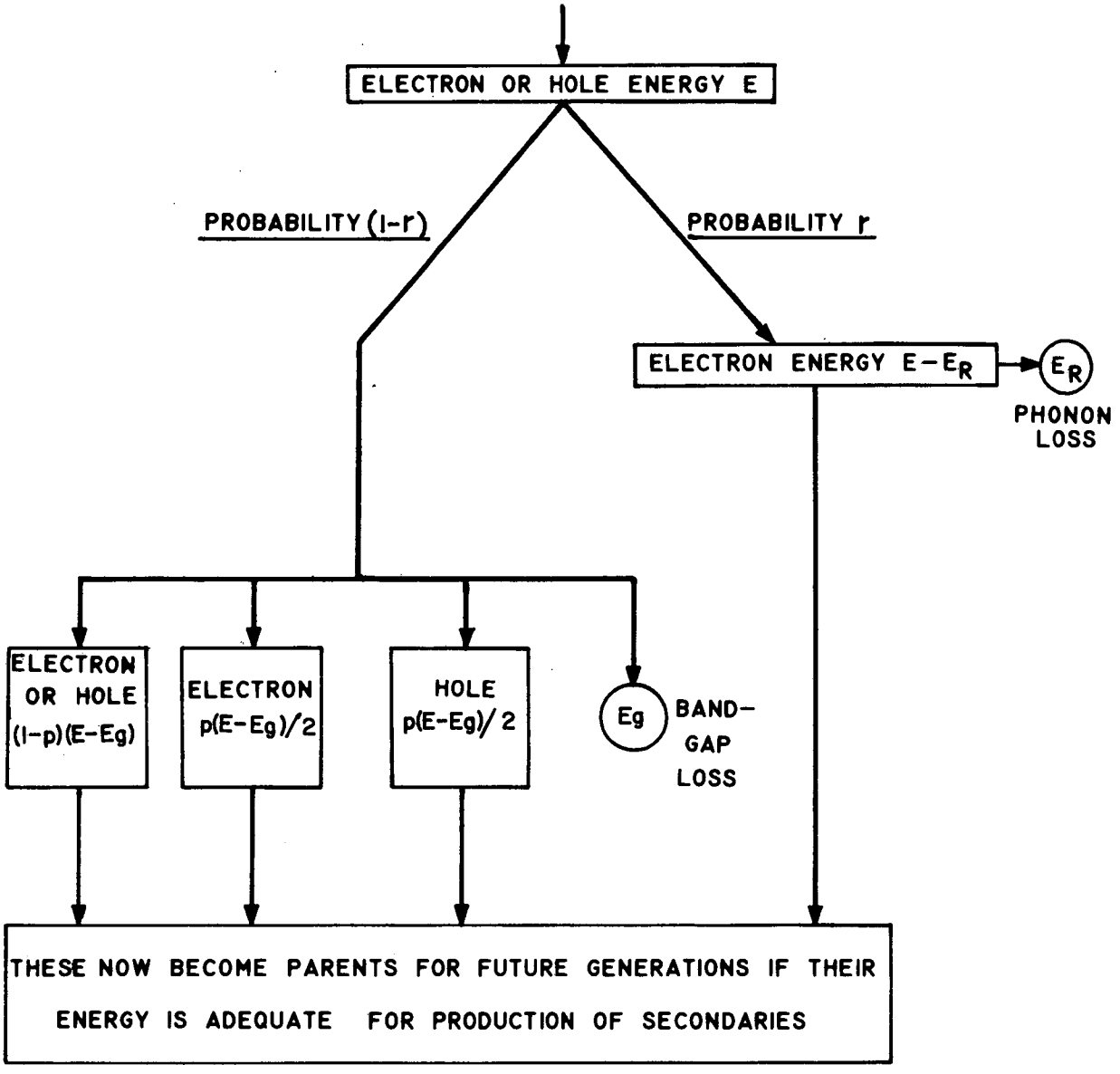

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Fig. 1

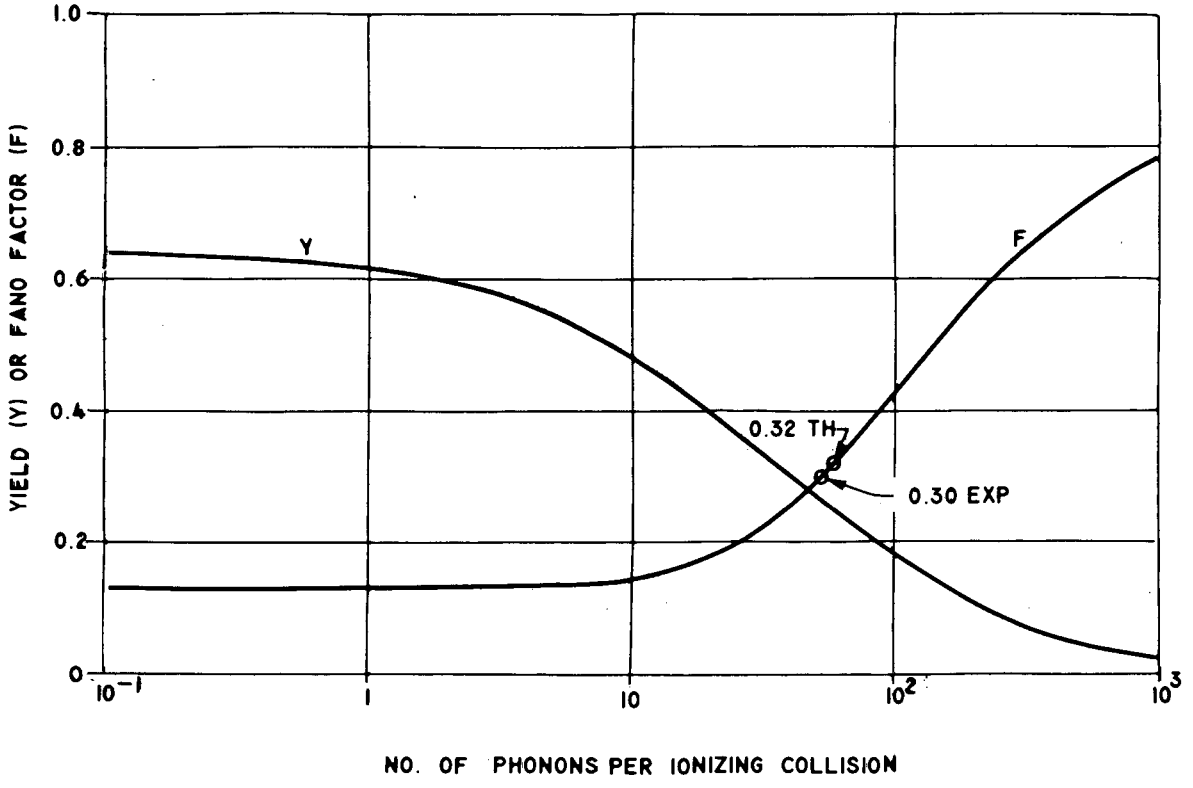

1.0
0.8
0.6
0.4
0.2
0
YIELD (Y) OR FANO FACTOR (F)
Y
F
0.32 TH
0.30 EXP
10^{-1}
1
10
10^2
10^3
NO. OF PHONONS PER IONIZING COLLISION
MUB 7362

Fig. 2

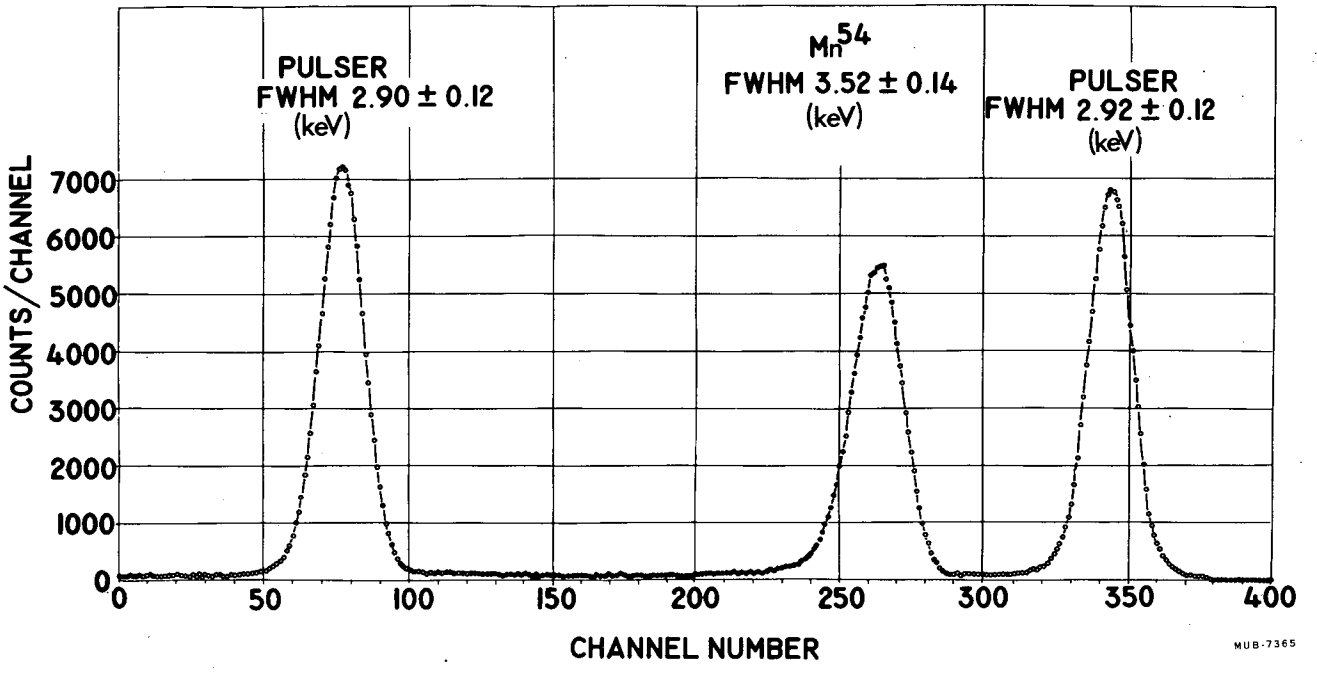

PULSER
FWHM 2.90 ± 0.12
(keV)
Mn54
FWHM 3.52 ± 0.14
(keV)
PULSER
FWHM 2.92 ± 0.12
(keV)
COUNTS/CHANNEL
7000
6000
5000
4000
3000
2000
1000
0
0
50
100
150
200
250
300
350
400
CHANNEL NUMBER
MUB-7365

Fig. 3

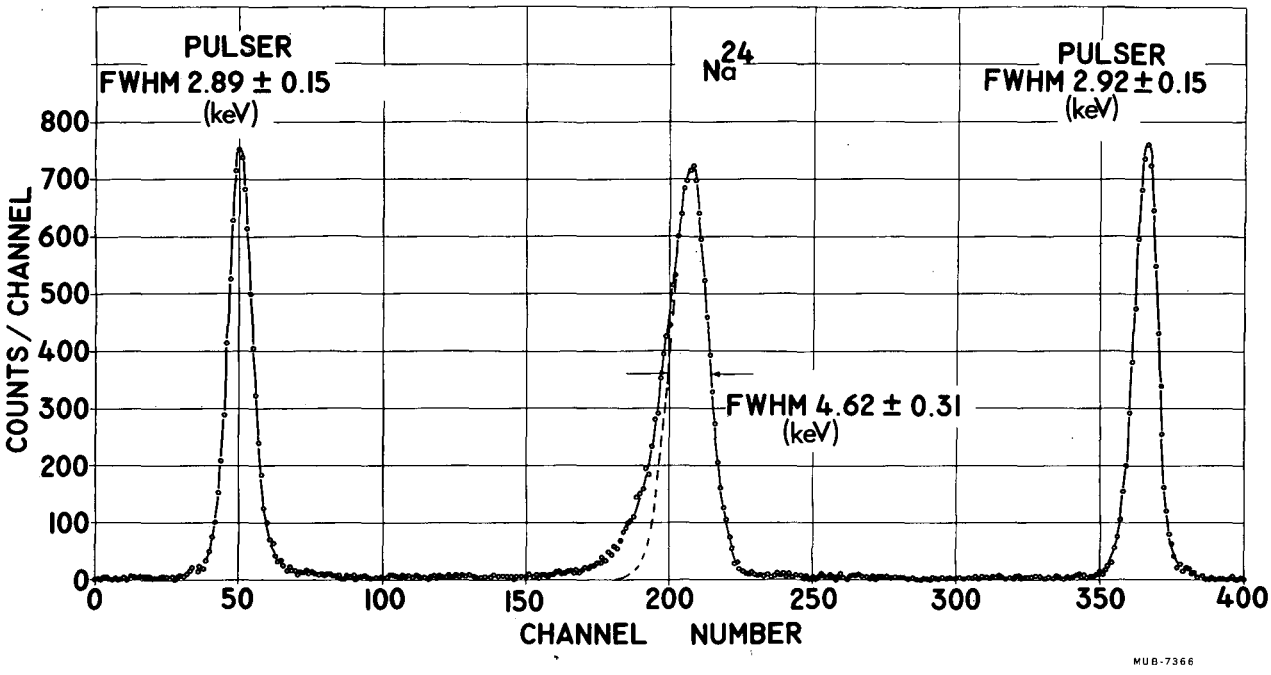

PULSER
FWHM 2.89 ± 0.15
(keV)
Na^{24}
PULSER
FWHM 2.92 ± 0.15
(keV)
FWHM 4.62 ± 0.31
(keV)
COUNTS / CHANNEL
CHANNEL NUMBER
0
100
200
300
400
500
600
700
800
0
50
100
150
200
250
300
350
400
MUB-7366

Fig. 4

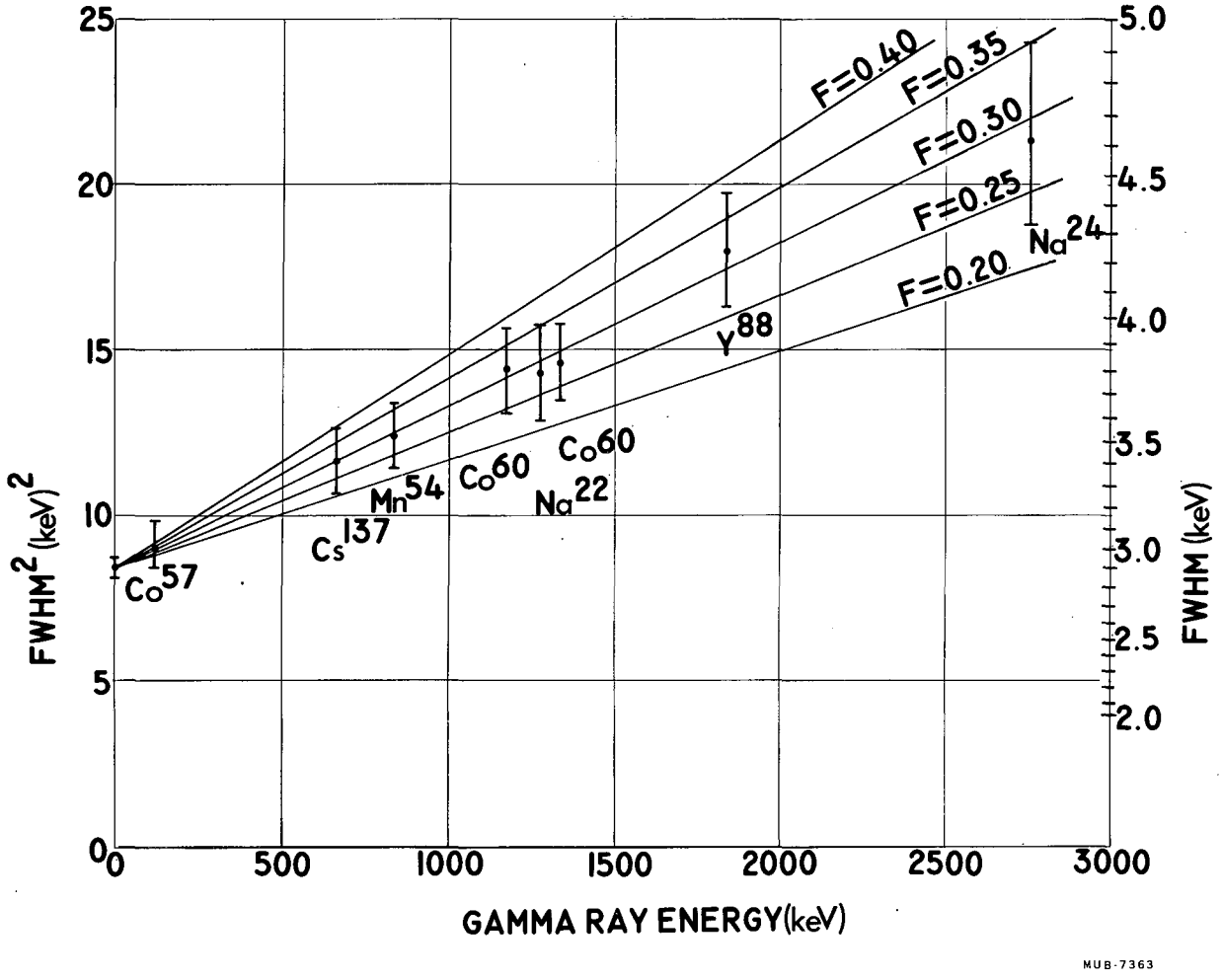
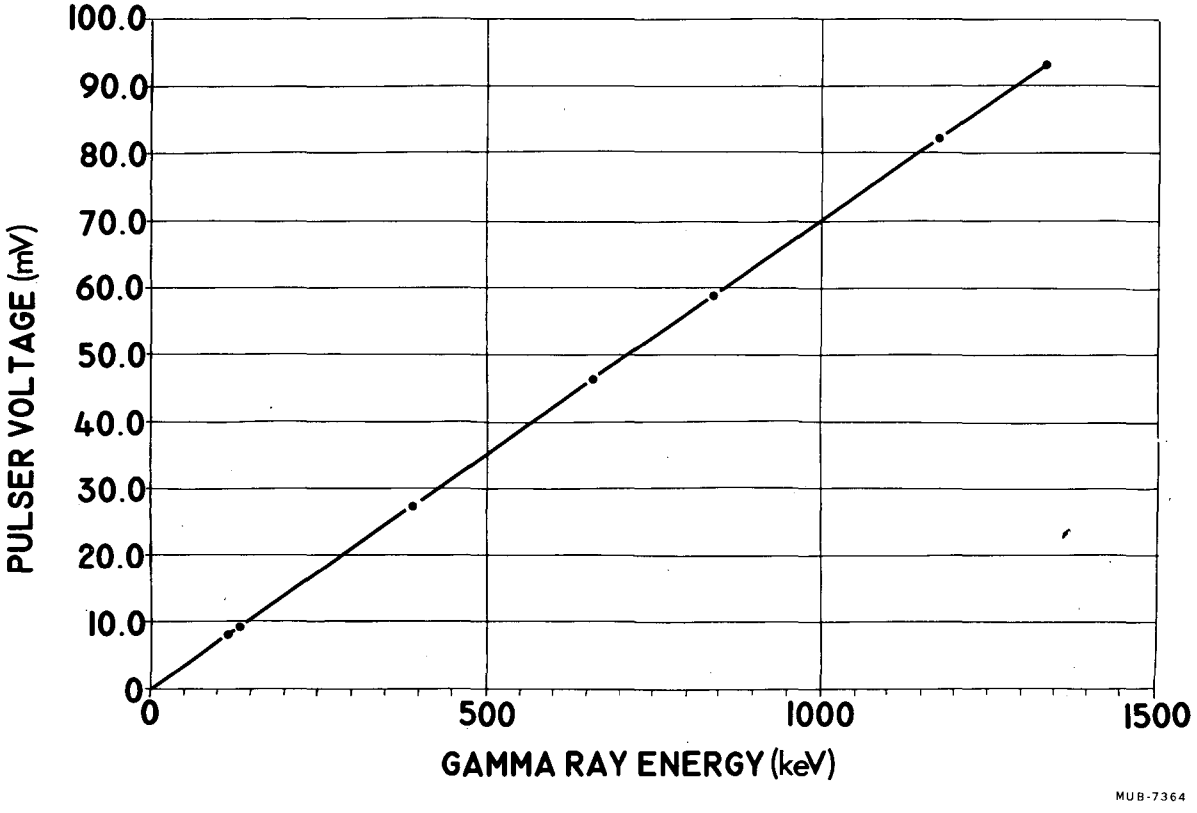

FWHM^2 (keV)^2
FWHM (keV)
GAMMA RAY ENERGY(keV)
F=0.40
F=0.35
F=0.30
F=0.25
F=0.20
Co^{57}
Cs^{137}
Mn^{54}
Co^{60}
Co^{60}
Na^{22}
Y^{88}
Na^{24}
MUB-7363

Fig. 5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90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
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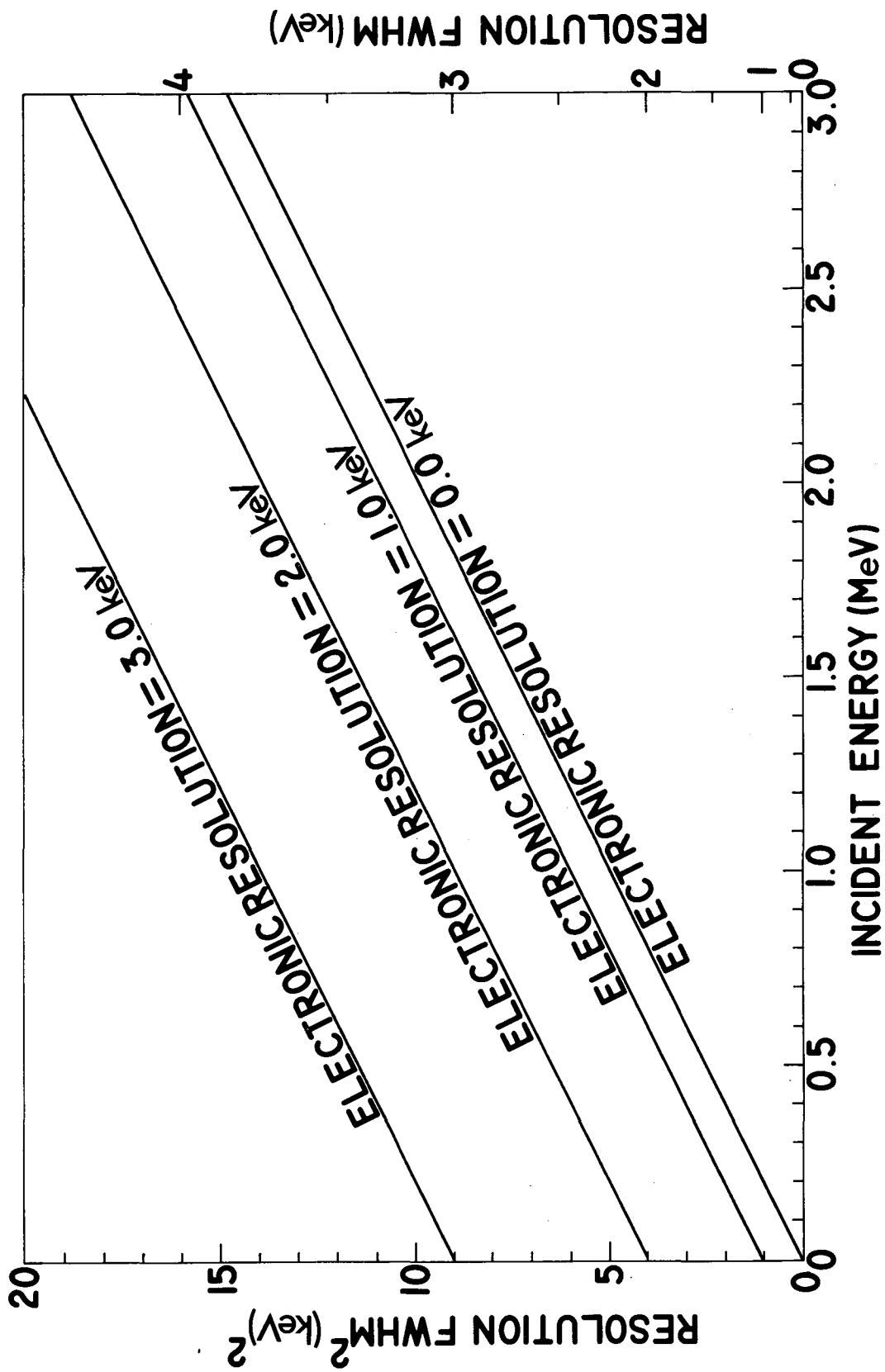
0
500
1000
1500
GAMMA RAY ENERGY (keV)
MUB-7364

Fig. 6



MUB-8265

Fig. 7

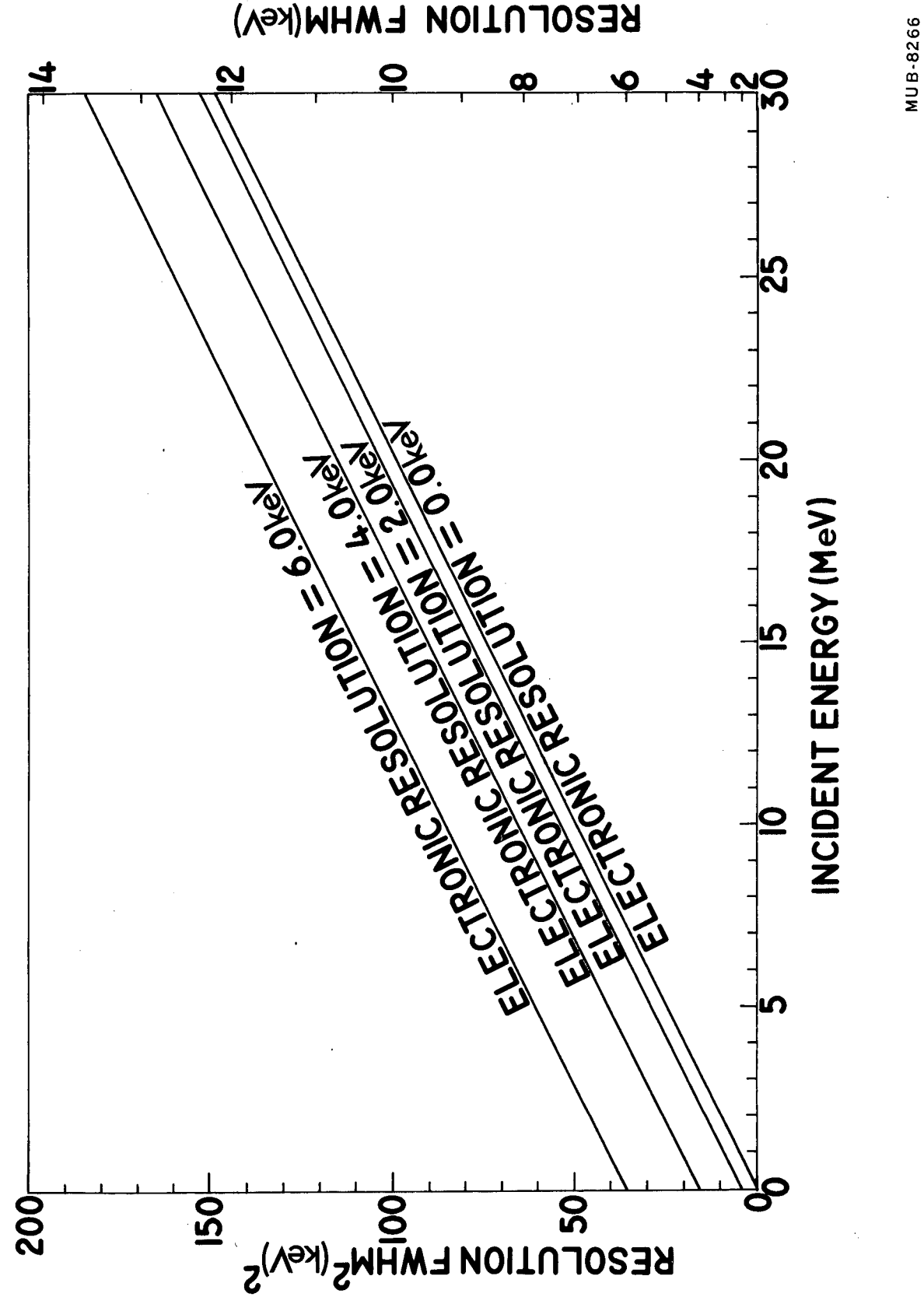


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Fig. 8