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Abstract

A new class of electron interferometers is proposed that make use of a laser modulated electron beam. The observable interference arises in second-order processes, since the first-order interference oscillates rapidly and averages to zero. It is shown that the detected signal varies sinusoidally with the separation of two sequential modulators, thus generating a fringe pattern with peak-to-peak spacing that may be smaller or larger than the laser wavelength. It is suggested that such devices could be used to detect and study the modulation process, and because they operate with massive charged particles, to perform a wide variety of new experiments and high precision measurements.

Introduction

Considerable interest and controversy has arisen from a 1969 experiment[1] which seems to indicate that an electron beam can be appreciably modulated by a laser beam using a solid material as a coupler. In that experiment, the electrons passed through a thin crystalline film irradiated on its edge with a focussed laser beam, and then impinged on a non-fluorescent screen. Light of the same color as the laser was reportedly emitted from the regions on the screen where the electrons impinged, and this was taken as evidence of electron modulation. The controversy has developed from the lack of a suitable model to account for the emitted intensity, and the failure of others[2] to reproduce the experiment may indicate that the original observations are not easily explained. Nevertheless, there appear to be reasonable models for modulation processes, and recent calculations indicate that appreciable (∿ percents) modulation can be achieved with present techniques.

Quantum-mechanical calculations of the modulation process are straightforward.[3] The incident electron wave is scattered into a coherent superposition of spherical waves of various frequencies; interference between these waves and the incident waves produces modifications of electron density, current, etc. that vary in space and time. The detection of such modifications (hence the modulation) presents a formidable experimental problem, however. The electrons may be pictured as oscillating rapidly among the various energy eigenstates, and linear detectors which cannot follow these oscillations must take some sort of average over many cycles. Hence all the first order interference effects will average to zero, making the modulation unobservable. Recently, Favro and co-workers[4] have shown that if electrons in a coherent superposition state strike an appropriately resonant target, the excitation cross-section will be larger than the incoherent value, and this could provide a means of detecting and studying the modulation process. However, such mechanisms are cooperative electron effects, and depend upon the square of the beam current.

We wish to point out that if the electron beam is modulated twice, either by passing through two crystals or by using two laser frequencies, new interference effects occur which could be observable. In these processes, the second modulation acts like a coherent detector or demodulator for the beam from the first modulator. Such a device would make possible observation and study of the quantum-mechanical modulation processes and new high precision measurements and experiments.

Theory

Consider the experimental arrangement diagrammed in Fig. 1, in which an electron beam passes through two thin solid films (a,b) illuminated by laser light of various frequencies, and is detected at some distant point $\vec{r}$. We assume the electrons can be described by a wavefunction $\psi(\vec{r},t)$ which evolves in time according to the hamiltonian $H_0(\vec{r}) + V(\vec{r},t)$, where $H_0(\vec{r})$ represents the free-particle hamiltonian. The electron + laser + crystal interactions are described by the operators

$$V(\vec{r},t) = \sum_n V_n(\vec{r} - \vec{a})e^{-i(\omega_n t - \phi_n)} + \sum_m V_m(\vec{r} - \vec{b})e^{-i(\omega_m t - \phi_m)} , \quad (1)$$

where the two terms represent the two modulators. The leading terms in $V(\vec{r},t)$ include the static periodic crystal potential, the linear electron-laser interaction, the linear optically induced polarization of the crystal, and the distortion of the laser field by the crystal.

We now make several simplifying assumptions: 1) The interaction $V(\vec{r},t)$ is weak so we can use perturbation theory; 2) The incident electron wave is plane, monoenergetic, and nonrelativistic; 3) The electron current is small so cooperative electron effects are negligible; 4) The modulator separations and observation distances are large compared with the dimensions of the modulators. With these assumptions it is a relatively simple matter to solve for $\psi(\vec{r},t)$. We have done this by using the free particle propagator to compute the perturbation series expansion of the Green's function, a technique that is well-known in time-dependent scattering theory.[5] The result can be written

$$\psi = \psi_0 + \psi_1^{(a)} + \psi_1^{(b)} + \psi_2^{(ba)} + \psi_2^{(ab)} + \psi_2^{(aa)} + \psi_2^{(bb)} + \ldots , \qquad (2)$$

where ψ_0 is the incident plane wave, $\psi_1^{(a)}$, $\psi_1^{(b)}$ are the first-order scattered spherical waves, $\psi_2^{(ba)}$ is the second-order spherical wave scattered first by a, then by b, $\psi_2^{(aa)}$ is the spherical wave scattered by a in second order, etc.

Thus, the modulated wave is a linear superposition of the incident wave and various spherical waves with certain amplitudes and phases. The electron probability density is $\rho_e = \psi^*\psi$, so we examine the various quadratic and bilinear products of Eq. (2). Consider the product[6]

$$\psi_0^*\psi_1^{(a)} \cong \frac{1}{R} \sum_n f_n(\vec{K}_n;\vec{K}_0)\, e^{i[(\vec{K}_n-\vec{K}_0)\cdot(\vec{R} + \frac{\vec{L}}{2}) - (\Omega_n-\Omega_0)t + \phi_n]} , \qquad (3)$$

which represents the interference of the first-order wave $\psi_1^{(a)}$ with the incident wave ψ_0. In the forward direction[7] $(\vec{K}_n \| \vec{K}_0)$ for n = 0, the exponent vanishes and the term is a constant, while for $n \neq 0$, it oscillates sinusoidally at the laser frequency $\omega_n \sim 10^{15}$ Hz. Any detector that cannot follow these oscillations will take some sort of average over many cycles, and that will be zero. Thus, $\psi_0^* \psi_1^{(a)}$ and the similar term $\psi_0^* \psi_1^{(b)}$ give no detectable contribution to ρ_e.

Now consider the product

$$\psi_1^{*(b)}\psi_1^{(a)} \cong \frac{1}{R^2}\sum_{nm} f_m(\vec{K}_m;\vec{K}_0)\, f_n(\vec{K}_n;\vec{K}_0)$$

$$\times\, e^{i[(\vec{K}_n-\vec{K}_m)\cdot\vec{R} - (\Omega_n-\Omega_m)t + (\frac{K_n + K_m}{2}\hat{R} - \vec{K}_0)\cdot\vec{L} + (\phi_n-\phi_m)]} \quad . \quad (4)$$

This product, unlike the previous ones, introduces the possibility of cancelling the time dependence nontrivially, when $\Omega_n - \Omega_m = \omega_n - \omega_m = 0$, with $n, m \neq 0$. This leaves a phase factor linear in the modulator separation $\vec{L}$ (and other parameters), so the term will vary sinusoidally with L. Other products like $\psi_0^* \psi_2^{(ab)}$ and $\psi_0^* \psi_0^{(ba)}$ show similar behavior. For instance, $\psi_0^* \psi_2^{(ab)}$ has the phase $(\vec{K}_{nm} - \vec{K}_0)\cdot(\vec{R} + \frac{\vec{L}}{2}) - (\Omega_{nm} - \Omega_0)t + K_m L + \vec{K}_0\cdot\vec{L} + (\phi_n + \phi_m)$ which is constant in time for $\Omega_{nm} = \Omega_0$. Homogeneous products like $\psi_1^{*(a)}\ \psi_1^{(a)}$, $\psi_1^{*(b)}\ \psi_1^{(b)}$, $\psi_0^*\ \psi_2^{(aa)}$, and $\psi_0^*\ \psi_2^{(bb)}$ can also have time-independent components, but these naturally do not involve $\vec{L}$ in a nontrivial way.

Now consider the case when two frequencies Ω_n, Ω_m are nearly (but not exactly) equal, so the time dependence is nearly (but non exactly) cancelled. In the forward direction, with $\vec{R}$, $\vec{L}$, $\vec{K}_0$, $\vec{K}_n$, $\vec{K}_m$, $\vec{K}_{nm}$, $\vec{K}_{mn}$, etc. all parallel, the phase in Eq. (4) becomes $\Phi_{nm} = (K_n-K_m)R - (\Omega_n-\Omega_m)t + (\frac{K_n + K_m}{2} - K_0)L + (\phi_n-\phi_m)$ $\equiv \Delta\omega_{nm}(R/v - t) + \omega_{nm} L/v + \Delta\phi_{nm}$ where we defined $\Delta\omega_{nm} \equiv \omega_n - \omega_m$, $\omega_{nm} \equiv (\omega_n + \omega_m)/2$, $\Delta\phi_{nm} = \phi_n - \phi_m$, and $v = \hbar K_0/m_e$ = electron velocity. Thus, this component of the electron density will have the form

$$\rho_e^{(b)(a)} \cong \sum_{nm} B_{nm} \cos\left[\Delta\omega_{nm}(\frac{R}{v} - t) + \frac{\omega_{nm} L}{v} + \Delta\phi_{nm} + \delta_{nm}\right] \quad , \quad (5)$$

plus rapidly oscillating terms. The phase $\delta_{nm} = \delta_n - \delta_m$ results from using $f_n = |f_n|e^{-i\delta_n}$. Consideration of other terms in ρ_e show that similar behavior is found in certain cross terms, as discussed above.

In a simple case where only two frequencies are present, only a single difference frequency $\Delta\omega \sim 0$ is generated. Thus, we conclude that the electron density will have the form

$$\rho_e \cong A + B \cos\left[\Delta\omega\left(\frac{R}{v} - t\right) + \frac{\omega L}{v} + \Delta\phi + \delta\right] \quad , \tag{6}$$

plus rapidly oscillating terms.

Applications

1. The modulation process. Observation of the fringe pattern described by $\rho_e \cong A + B \cos \Phi$ (Eq. (6)) would confirm the existence of the quantum-mechanical modulation. The fringes presumably could be observed by varying any of the parameters entering the phase Φ: v, L, $\Delta\phi$, etc. By using $\Delta\omega \neq 0$, the fringes can be made to oscillate slowly in time at the controllable frequency $\Delta\omega$, and in space with period $2\pi v/\Delta\omega$. Thus, a lock-in detector tuned to $\Delta\omega$ could be used as a very sensitive fringe detector. Note that if several frequencies $\Delta\omega_{nm}$ are present, lock-in techniques can be used to pick out the individual frequencies $\Delta\omega_{nm}$, thus determining the B_{nm} independently. The relative laser phase $\Delta\phi_{nm}$ can be conveniently adjusted by an optical delay line to give the fringe phase an arbitrary zero. The fringes could also be conveniently observed by varying the velocity v or the observation point R.

2. Length measurements. From Eq. (6), with $\Delta\omega = 0$ held constant, the fringe phase Φ changes by 2π when the modulator spacing L is changed by

$$\Delta L = \beta\lambda \quad , \tag{7}$$

where $\beta = v/c$ and λ is the laser wavelength. Thus, the electron fringe separations are smaller than the laser wavelength by $\beta < 1$, and in theory could be arbitrarily small. This "fringe compression" effect is similar to light traveling in a medium with index of refraction $n > 1$, and could be the basis of high precision measurements. Such a device would have sensitivity between optical and x-ray interferometers, and may lead to coupled electron/ optical and electron/ x-ray interferometers in the same way that coupled x-ray/optical interferometers have been discussed recently for length standard comparisons.[8] On the other hand, it may be more convenient to mechanically couple the laser and the crystals so that $\Delta\phi$ is an arbitrary function of L. Thus

$$\Delta L = \beta\lambda\left(1 - \frac{\beta\lambda}{2\pi}\frac{\partial\Delta\phi}{\partial L}\right)^{-1} \quad . \tag{8}$$

which can be larger than the laserwidth. Thus, "fringe expansion" is also possible, and can be quite dramatic. In fact the fringe wavelengths could be made infinite (a "zero beat" condition) or negative, meaning that the fringes move oppositely to the change ΔL. In this operation, the interferometer acts like a Moiré fringe magnifier. The "zero beat" technique could be used to eliminate overlapping fringes independently, thus unravelling a complex modulation spectrum. Alternatively, the Fourier transform of the signal versus distance gives the spectrum directly.

3. Velocity measurements. Since Φ depends on the particle velocity v, any interaction that changes v by δv will introduce a shift in the fringes of $\delta\Phi/\Phi = -\delta v/v$. We can write this in terms of the equivalent time delay $\delta t = -(L/v)(\delta v/v)$ as $\delta\Phi = \omega\delta$. At optical frequencies a shift of one full fringe ($\delta\Phi = 2\pi$) would represent a time delay $\delta t \sim 10^{-15}$ sec.

The very high sensitivity of this device could be utilized in a variety of precision experiments. For instance, Boyer[9] has recently criticized the interpretation of the Aharanov-Bohm effect[10] as implying physical reality of the electromagnetic potentials in quantum theory. He suggests that classical second-order fields react on the incident electrons to cause a small velocity shift. This "time-lag" effect is entirely classical, and exists whether or not there is de Broglie-wave interference present. We suggest that such a time lag could be observed directly with a laser modulated electron interferometer. This technique would have the advantages of a single beam and great sensitivity.

As another application we can mention the possibility of measuring the gravitational acceleration of electrons and positrons. In principle the interferometer should work as well (although not identically) with positrons as with electrons. By orienting the path L vertically, a velocity change for a particle of mass $\pm m$ appears as a shift in the fringes of $\delta\Phi \cong \pm\, \omega g L^2/2v^3$. This technique would not require a pulsed positron source, but would still be subject to most of the difficulties described by Fairbank and co-workers.[11] Although the numbers appear unfavorable at this time, we believe it merits consideration as a fundamentally new approach to the problem.

Discussion

1. Criticism of the theory. The theory developed above is certainly limited by the simplifying assumptions we have made. In particular, an electron beam is not a plane wave, so some of the interference terms (e.g. $\psi_0^* \psi_1$) are valid only in the forward direction. We have not treated the electrons relativistically, and have not indicated the effects of finite modulator thickness and velocity distribution. In fact, the applicability of perturbation theory itself might be questioned. However, it appears that a more complete theory would merely blur the fringe pattern, but not materially alter the conclusions reached above.

2. Experimental difficulties. In order to maintain the fringe pattern, the energy of the beam must have a spread of $\sigma(E)/E \ll 2\beta\lambda/L$. For E = 50 keV, λ = 5000 Å, L = 10 cm, this means $\sigma(E) \ll 0.3$ eV, which is a quite reasonable requirement with present laboratory techniques. The requirement is actually more stringent than this, since the fringes arise from DeBroglie-wave interference, so coherence must be maintained just as in a normal electron interferometer. This requires energy spreads of the order 10^{-6} eV, an impossible requirement. However, two techniques will help to obviate this requirement: First, at very high energies, the velocity is essentially constant at the velocity of light and the monochromaticity requirement is weakened;[12] second, the use of an achromatic Bragg doublet as in the Marton type electron interferometer[13] makes small velocity shifts irrelevant. We have concluded that these experiments can be performed reasonably at an energy $E_0 \sim 100$ keV and resolution of about 1 meV, and this requirement might be relaxed somewhat if a favorable set of parameters can be determined.

Footnotes and References

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6. In these formulas we use $f_i(\vec{p},\vec{q}) = (-m/2\pi\hbar^2) \times \int d^3\vec{x}\; e^{-i\vec{p}\cdot\vec{x}}\; V_i(\vec{x})\; e^{i\vec{q}\cdot\vec{x}}$, $\vec{R} = (\vec{a} + \vec{b})/2$, $\vec{L} = \vec{b} - \vec{a}$, $\hbar\Omega_0 = \hbar^2 K_0^2/2m_e = E_0$, $\hbar\Omega_n = \hbar^2 K_n^2/2m_e = E_0 + \hbar\omega_n$, $\hbar\Omega_{mn} = \hbar^2 K_{mn}^2/2m_e = E_0 + \hbar\omega_m + \hbar\omega_n$, $\vec{K}_n = K_n\,\hat{R}_1$, $\vec{K}_m = K_m\,\hat{R}_2$, $\vec{K}_{mn} = K_{mn}\,\hat{R}_2$, $\vec{K}_{nm} = K_{nm}\,\hat{R}_1$.
7. We must restrict these terms to the forward direction, since a real electron beam will be a narrow pencil, not a plane wave. This restriction is not necessary for products (e.g. Eq. (4)) not involving ψ_0.
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Figure Caption

Fig. 1. Schematic of proposed laser modulated electron interferometer.

(a) Diagram of a two-crystal transmission interferometer using two lasers;

(b) Structural diagram indicating the vectors referred to in the text.

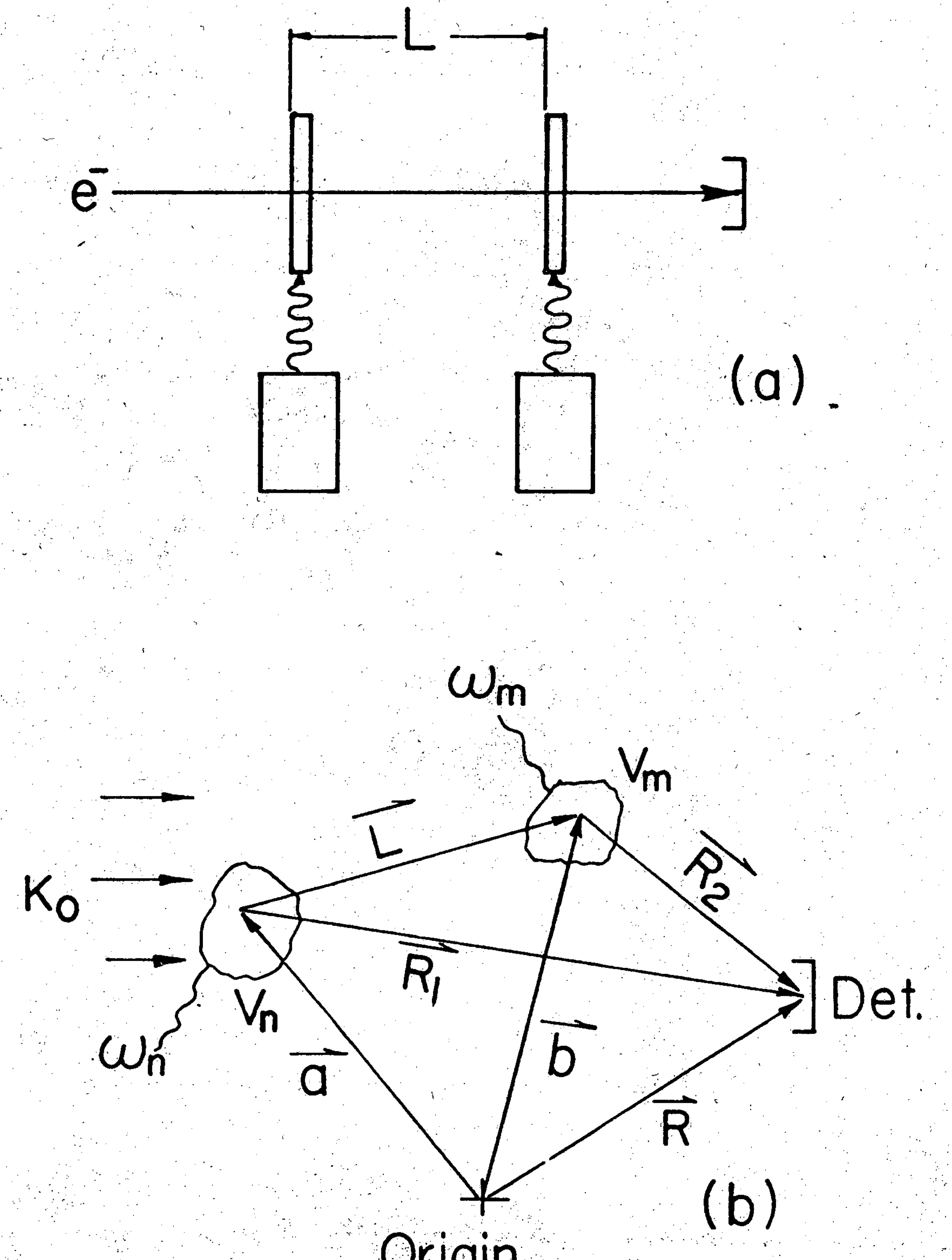

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Fig. 1

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