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Study of Differences in Behavior of Asymptotically Distribution Free Test Statistics in Covariance and Correlation Structure Analysis

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Yafei Huang

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Abstract of the Thesis

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Master of Science in Statistics

University of California, Los Angeles, 2013

Professor Peter Bentler, Chair

The asymptotically distributed free (ADF) method is often used to estimate parameters or test models without a normal distribution assumptions on variables, both in covariance structure analysis and in correlation structure analysis. However, little has been done to study the differences in behaviors of the ADF method in covariance structure analysis and correlation structure analysis. In this thesis the behaviors of the ADF method in covariance structure analysis and correlation structure analysis were studied for three test statistics, χ^2 test T_{AGLS} and its small-sample improvements T_{YB} and $T_{F(AGLS)}$. Results showed that the ADF method in correlation structure analysis with test statistic T_{AGLS} performs much better at small sample sizes than the corresponding test for covariance structures. However, test statistics T_{YB} and $T_{F(AGLS)}$ under the same conditions generally perform better with covariance structures than with correlation structures. Results also showed that condition numbers of weight matrices are systematically increased with substantial increase in variance as sample size decreases. Implications for research and practice are discussed.

The thesis of Yafei Huang is approved.

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Table of Contents

List of Figures

List of Tables

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CHAPTER 1

Introduction

Structural equation modeling (SEM) is a set of statistical techniques that examine causal relations between one or more independent variables (IVs), either discrete or continuous, and one or more dependent variables (DVs), either discrete or continuous. Both IVs and DVs can be either latent variables (factors) or measured variables. For comprehensive reviews of SEM, readers can refer to [Byr06, YB07, Ull10, Mul09, Kli11, UB13]. SEM is based on the general linear model and is widely used in psychology and other social sciences. For its applications in practice, readers can refer to [AG88, Byr06, MA00, Kli11].

In general SEM models, latent variables can be functionally related. A specialized subset of SEM models involve latent variables that may be correlated but have no directional or causal influences on each other. These are called measurement models and can be categorized into exploratory factor analysis (EFA) and confirmatory factor analysis (CFA). EFA techniques are typically used in the preliminary phase of research where the researcher has observed data and hypotheses of the underlying structure but needs to discover the exact structure. Through EFA, the researcher explores and determines configuration of factors, the relations among factors, and how observed variables are associated with factors. For reviews of EFA techniques, readers can refer to the aforementioned comprehensive reviews of SEM, as well as [FMT86, FWM99, Tho04, Mul10].

In CFA, the researcher already knows the most probable underlying structure: how many factors, the relations among factors, and the relations between factors and observed variables. The main goal of CFA research is to verify and test the hypothesized structure, including estimation of model parameters. For reviews of CFA, readers can also refer to [Bro06, SNS06, Har09]. One important difference between EFA and CFA is, EFA is typically performed using correlations, while in CFA researchers mostly analyze covariance matrices. A covariance, as a non-standardized correlation, also contains scale information of variables. For a many sets of data, it is possible to use either correlation or covariance structures to analyze the data since the key parameters of interest are related by a rescaling. Thus either approach would be used to evaluate a model. This study compares test statistics in covariance v.s. correlation structures.

Parameter estimation and test statistics are important research areas in CFA. After the structure of a model is specified, the researcher needs to estimate parameters of the model. Typically these are estimated by minimizing the difference between measured unstructured sample covariance matrix and the estimated structured population covariance matrix. There are two important aspects of this estimation: the function to be minimized should have a statistical interpretation, and the associated test statistic should provide a test of the covariance structure. The function to be minimized is usually chosen to be

$$F = (s - \sigma(\theta))' W (s - \sigma(\theta)), \tag{1.1}$$

where s is the vectorized sample covariance matrix of the observed variables, σ is the vectorized population covariance matrix, θ is the vector of parameters, and W is the weight matrix. Different methods choose different weight matrices.

Early development of estimation methods were mostly under the assumption of multivariate normality. Maximum likelihood (ML) [Jor69] is one of the most commonly employed techniques. It will yield the most precise estimates when the multivariate normal assumption of observed variables is satisfied. Generalized least square (GLS) [Bro84b] has the same optimal properties as the ML under the same normal assumption, and is obtained as a special case of (1.1).

However, in practice, the multivariate normal assumption is not always valid [BC87, Rus02]. When the multivariate normal assumption is violated, researchers either use other methods under different distribution assumptions or make corrections to statistics resulting from normal methods. Among various methods of the first approach, the asymptotically distribution free (ADF) method [Bro84a, Dij81] is the one that does not have any distributional assumptions. But it is often impractical when the number of variables is large and can also be inaccurate when the number of samples is not large [HBK92, CWF96]. Various adjustments to test statistics have been proposed to correct the behavior of estimators in such situations. This topic will be discussed in Chapter 2 after further details on estimation methods is presented.

There are other estimation methods with distribution assumptions more general than multivariate normal but more restricted than arbitrary. Among them, multivariate elliptical theory (ERLS) [Bro82, BB86] introduces an additional kurtosis parameter to control the distribution of a variable such that it can be heavier tailed or lighter tailed than normal. Some more recent studies, including a geodesic discrepancy function for use in covariance structure analysis [BOB97], have been proposed to extend the elliptical theory family. Later, an extension of elliptical theory, heterogeneous kurtosis theory (HK) [KBB90], was proposed to use heterogeneous kurtoses for different variables. HK can be as easily computed as ERLS, yet applied to a wider class of multivariate distribution with different kurtosis. There are also studies of comparing different methods under different conditions such as non-normality or factor-error dependence situations [HBK92, FC04].

When distributional assumptions are false or sample sizes are too small for some estimation methods, test statistics that are based on these assumptions, or that are only accurate when sample sizes are large, can be corrected to cope with these problems. One of the examples is a scaling correction method (SCALED) developed in [SB88, Kan90]. The SCALED method calculates a scaling correction based on model, estimation method, and sample fourth-order moments, and test statistics are corrected by the scaling factor. Some more recent statistical corrections include a YB-correction in [YB97], an F-statistics in [YB99], and a Barlett-corrected adjusted statistic [Fou00]. There have been studies of comparisons of these corrected statistics under different conditions in [CWF96, YB99, Fou00, NH04].

Although correlation structure analysis can be traced back as early as in [Wri21, Wri34, Jor78], for the past quarter century, most of the structure analysis methods were developed using covariance matrices since the asymptotic distribution of covariance secured to be better understood. However, psychological theories or data have been specified in terms of correlation coefficients, and it is important to be able to perform correlation structure analysis as well. In correlation structures, the vectors in (1.1) is the vector of correlations rather than covariances, and model σ and weight matrix W are adjusted accordingly.

In the past, some correlation structure analysis was done with correlation matrices being treated as special covariance matrices and covariance structure analysis was used [KM78, Cud89, SB90]. If the model is fully scale-invariant, this procedure may yield correct statistics. However, if the model is not fully scale-invariant, it is very likely that parameter estimation or test statistics will be incorrect. The statistical theories for correct analysis of correlations were developed rather slowly for the past quarter century [SH82, Lee83, Moo85, Mel00, Ben07, BS10]. Some of the estimation methods are already incorporated in EQS [Ben06].

There has been very little research to evaluate the performance of different parameter estimation methods in correlation structure analysis [Fou00, Mel00], and very little is known about the performance of covariance structure analysis and correlation structure analysis using similarly justified parameter estimation methods and test statistics. In this thesis, we directly compare the performance of covariance structure analysis and correlation structure analysis with a Monte Carlo confirmatory factor analysis, using the ADF method and several test statistics, aiming to understand the differences in performance. We study the different behaviors of covariance and correlation structure test statistics when theoretical multivariate normal conditions are violated. Three ways of violating theoretical conditions are investigated: normal distribution assumption violation, independence assumption violation, and/or asymptotic sample size requirement violation.

CHAPTER 2

Test Statistics

Suppose the model has p continuous variables and q parameters. Let θ be the parameter vector (q by 1), S be the unbiased sample covariance matrix (p by p), Σ be the population covariance matrix (p by p): $\Sigma = \Sigma(\theta)$. Let s (p^* by 1, where $p^* = \frac{p(p+1)}{2}$) be the vector of elements in the lower triangular of S, σ (p^* by 1) be the vector of elements in the lower triangular of $\Sigma : \sigma = \sigma(\theta)$. The ADF method minimizes the generalized least squares function

$$Q = (s - \sigma(\theta))' W (s - \sigma(\theta)) \tag{2.1}$$

to get the optimal parameter estimator $\hat{\theta}$, and W is an optimal weight matrix defined in (2.5) below. Then a goodness-of-fit χ^2 test is given by

$$T_{AGLS} = n\hat{Q} = (N-1)Q(\hat{\theta}), \tag{2.2}$$

where N is sample size. The χ^2 test has degrees of freedom $d = p^* - q + r$, where r is the number of nondependent constraints ($r = 0$ if there is no constraints).

Yuan and Bentler [YB97] improved the statistic (2.2) and proposed

$$T_{YB} = \frac{T_{AGLS}}{1 + \frac{T_{AGLS}}{n}}. \tag{2.3}$$

T_{YB} becomes T_{AGLS} as sample size gets larger, but appears to perform better than T_{AGLS} at smaller sample sizes.

Another statistics developed by Yuan and Bentler [YB99] is

$$T_{F(AGLS)} = \frac{N-d}{nd} T_{AGLS}. \tag{2.4}$$

This is referred to an F distribution with d and $(N-d)$ degrees of freedom. It may perform even better than T_{YB} at the smallest sample sizes.

As for the weight matrix W, a consistent estimator is

$$W = \hat{V}^{-1} = V(\hat{\theta})^{-1}. \tag{2.5}$$

The matrix V is defined by the asymptotic distribution of the residual

$$\sqrt{n}(s - \sigma) \xrightarrow{\text{D}} \text{Normal}(0, V). \tag{2.6}$$

In covariance structures, the typical element of V are usually given by

$$v_{ij,kl} = \sigma_{ijkl} - \sigma_{ij}\sigma_{kl}, \tag{2.7}$$

where σ_{ij} is sample covariance and

$$\sigma_{ijkl} = E(z_i - \mu_i)(z_j - \mu_j)(z_k - \mu_k)(z_l - \mu_l) \tag{2.8}$$

is the multivariate product moment for four variables i, j, k, and l.

In correlation structure analysis, the generalized least squares function is given by

$$Q = (r - \rho(\theta))'\tilde{W}(r - \rho(\theta)) \tag{2.9}$$

instead of (2.1), where r and ρ are the vector of elements of the sample correlation matrix and the vector of elements of the population correlation matrix, respectively. The optimal choice of $\tilde{W}$ is $\tilde{W} = \widehat{\tilde{V}^{-1}}$ in EQS [Ben06] and $\tilde{V}$ is defined by the asymptotic distribution of the residual

$$\sqrt{n}(r - p) \xrightarrow{\text{D}} \text{Normal}(0, \tilde{V}). \tag{2.10}$$

The elements of the matrix $\tilde{V}$ were given by Steiger and Hakstian [SH82] and de Leeuw [Lee83] separately as

$$\begin{aligned}\tilde{v}_{ij,kl} =& \rho_{ijkl} + 0.25\rho_{ij}\rho_{kl}\left\{\rho_{iikk} + \rho_{jjkk} + \rho_{iill} + \rho_{jjll}\right\} \\ &- 0.5\rho_{ij}\left\{\rho_{iikl} + \rho_{jjkl}\right\} - 0.5\rho_{kl}\left\{\rho_{ijkk} + \rho_{ijll}\right\},\end{aligned} \tag{2.11}$$

where ρ_{ij} is sample correlation and

$$\rho_{ijkl} = \frac{\sigma_{ijkl}}{\sqrt{\sigma_{ii}\sigma_{jj}\sigma_{kk}\sigma_{ll}}}. \tag{2.12}$$

The three test statistics from covariance structure analysis can also be applied to correlation structure analysis. However, almost nothing is known about their behavior in practice, i.e., under conditions of violation of distributional assumptions and less than asymptotic sample sizes.

CHAPTER 3

Method

We used the confirmatory model in [HBK92]: $x = \Lambda\xi + \epsilon$, where x is a vector of measured variables, ξ is a vector of latent variables (factors), ϵ is a vector of unique errors, and Λ is a factor loading matrix. Typically, ξ is assumed to be normally distributed and uncorrelated from ϵ, ϵ's are assumed to be independent from each other, in which case the covariance matrix $\Sigma = \Sigma(\theta) = \Lambda\Phi\Lambda' + \Psi$, where Φ and Ψ are the covariance matrices of ξ and ϵ, respectively. We tested all 7 conditions in [HBK92]: factors and errors are normally distributed, errors are independent from factors (Case 1); factors and errors are non-normally distributed, errors are independent from factors, and factor covariance matrix is fixed (Case 2); factors and errors are non-normally distributed, errors are independent from factors (Case 3) ; factors are normally distributed, errors are non-normally distributed, errors are independent from factors (Case 4); factors and errors are normally distributed, but division by a random variable creates dependence (Case 5); factors are normally distributed, errors are non-normally distributed, with factors and errors dependent (Case 6); factors and errors are non-normally distributed, and factors and errors are dependent (Case 7).

More specifically, the confirmatory factor model we used contains 15 measured variables, 3 latent variables, and 15 unique errors. A simple Λ is used such that each measured variables is dependent on one and only one latent variable, as shown in (3.1).

$$\Lambda' = \begin{pmatrix} 0.7 & 0.7 & 0.75 & 0.8 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.7 & 0.7 & 0.75 & 0.8 & 0.8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7 & 0.7 & 0.75 & 0.8 & 0.8 \end{pmatrix} \tag{3.1}$$

Variances of the factors are 1.0, and the covariances among the three factors are 0.3, 0.4, and 0.5. The variances of the errors are set to values such that the variances of the measured variables are 1.0 under normality.

After the population covariance matrix Σ is generated for each case (condition), random samples of a specific sample size are drawn from the population. We set different sample sizes to be 150, 250, 500, 1000, 2500, 5000, and 10000. In each sample set, the parameters of the model are estimated using ADF with covariance structure analysis and correlation structure analysis. In estimation, $\Lambda_{1,5}$, $\Lambda_{2,10}$, and $\Lambda_{3,15}$ are fixed to 0.8, and the rest of non-zero parameters are free to be estimated. In the covariance structure, there are a total of 33 parameters to be estimated, while in correlation structures, there are 18 since $\hat{\Psi} = I - \text{diag}(\hat{\Lambda}\hat{\Phi}\hat{\Lambda}')$.

For each sample size, we replicate the process 200 times. The performance of the three test statistics across the 200 replications are then evaluated with mean, standard deviation, and rejection rate at α=0.05 level. We also recorded and evaluated the condition numbers of weight matrices W for both covariance and correlation structure analysis, where a condition number is the ratio of largest to smallest eigenvalue. We expect better condition numbers when test statistics perform well, and hope to understand the relationship between the size of condition numbers and other conditions such as non-normality or dependence. All these statistics are the main results of this study.

The experimental design of each case is as follows:

Case 1: all factors and all errors are normally distributed, errors are independent from factors.

Case 2: all factors and all errors are non-normally distributed, with kurtosis of the 3 factors -1.0, 2.0, and 5.0, and the kurtoses of the 15 errors -1.0, 0.5, 2.5, 4.5, 6.5, -1.0, 1.0, 3.0, 5.0, 7.0, -0.5, 1.5, 3.5, 5.5, and 7.5. Errors are independent from factors. All elements in the factor covariance matrix are fixed at their true values.

Case 3: all factors and all errors are non-normally distributed, with kurtoses of the 3 factors -1.0, 2.0, and 5.0, and the kurtoses of the 15 errors -1.0, 0.5, 2.5, 4.5, 6.5, -1.0, 1.0, 3.0, 5.0, 7.0, -0.5, 1.5, 3.5, 5.5, and 7.5. Errors are independent from factors.

Case 4: all factors are normally distributed. All errors are non-normally distributed, with the kurtoses -1.0, 0.5, 2.5, 4.5, 6.5, -1.0, 1.0, 3.0, 5.0, 7.0, -0.5, 1.5, 3.5, 5.5, and 7.5. Errors are independent from factors.

Case 5: the factors and errors are normally distributed variables divided by a common random variable $Z = \sqrt{\frac{\chi^2_{(5)}}{3}}$ that is independent from all the factors and errors. Errors and factors are dependent because of Z.

Case 6: the factors are normally distributed, the errors are non-normally distributed as specified in Case 4. They are both divided by a common random variable Z as specified in Case 5. Errors and factors are dependent because of Z.

Case 7: the factors and errors are non-normally distributed as specified in Case 3. They are both divided by a common random variable Z as specified in Case 5. Errors and factors are dependent because of Z.

In all cases but Case 2, the degrees of freedom is 87. Thus in theory E(T_{AGLS})=87 for all cases but Case 2. In Case 2, the degree of freedom is 93. Thus in theory E(T_{AGLS})=93 for Case 2. E(T_{YB}) is smaller than E(T_{AGLS}) but they are asymptotically the same ($T_{YB} \rightarrow T_{AGLS}$ as N$\rightarrow \infty$). E($T_{F(AGLS)}$) is smaller than 1 and

$E(T_{(F(AGLS))}) \rightarrow 1$ as $N \rightarrow \infty$. The expected rate of rejections at α=0.05 level would be 5%.

CHAPTER 4

Results

The simulation results are summarized in Tables 1-7, each case (condition) in one table. The statistical results of covariance structure analysis with T_{AGLS} agree with the corresponding results in [HBK92]. Thus validates the simulation methodology.

Table 4.1 summarizes the results in Case 1, with covariance structure results in the top half, and correlation results in the bottom half of the table. This is the baseline model. The factors, errors, as well as measured variables are all multivariate normally distributed. Asymptotically, $E(T_{AGLS})= E(T_{YB})$=87, SD(T_{AGLS})= SD(T_{YB})=13.19, E($T_{F(AGLS)}$)=1, SD($T_{F(AGLS)}$)=0.1516, and the expectated rejection count at α=0.05 level would be 10. Due to their nature, E(T_{YB}) and E($T_{F(AGLS)}$) are always smaller than their asymptotically value: the smaller sample size is, the smaller their expectations are.

Although the ADF method in covariance structure and correlation structure analyses yield similar behaviors when the sample size is large, these tests perform differently when the sample size is small. In general, T_{AGLS} performs better in correlation structure analysis than in covariance structure analysis. There are far too many model rejections in covariance structure analysis at all but the largest sample sizes, while the rejection rates for correlation structures are much close to nominal levels with only a small amount of model over rejection. The statistics T_{YB} and $T_{F(AGLS)}$ that were proposed to reduce the amount of model over rejection in covariance structures actually do so, with T_{YB} performing closer to nominal level than $T_{F(AGLS)}$. In contrast, these two test statistics substantially overcorrect in correlation structures so that models are accepted more frequently than expected under the null hypothesis of model structure.

We also compared the condition numbers of the weight matrices associated with both methods. As can be seen in the bottom section of Table 4.1, the means and standard deviations of the condition numbers for both covariances and correlations decrease as sample size increases. In general, better performance of test statistics occurs when the condition numbers are smaller. The condition numbers of weight matrices in correlation structure analysis are roughly similar in size to those in covariance structure analysis, though they are a bit smaller for the latter.

Table 4.2 and table 4.3 summarize the results in Case 2 and Case 3. Factors and errors in both cases are non-normally distributed, while the factor covariance matrix is fixed in Case 2. The covariance structure analysis performance of T_{AGLS} is similar in both cases and is also similar to that in Case 1, although the performance in Case 3 is generally better than in Case 2. In contrast, the correlation structure analysis performance of T_{AGLS} in the two cases is vastly different with small sample sizes: the performance in Case 3 is much better than the performance in Case 2 and very similar to that in Case 1. Thus the ADF method in covariance and correlation structure analyses generally performs under these conditions of non-normality as it had under normality, though the fixed factor covariance matrix (Case 2) does cause it trouble in correlation structure analysis with smaller sample sizes. The performance of T_{YB} and $T_{F(AGLS)}$ in covariance and correlation structure analyses in Case 3 is similar to their performance in Case 1. Results in Case 2 are a bit more complex, with covariance structure performance somewhat worse at small samples and correlation structure performance somewhat better at larger sample sizes as compared to Case 1.

Since the distributions of factors and errors in Case 2 and Case 3 are the

Table 4.1: Summary of simulation results for Case 1.

		Sample size						
Method	Statistics	150	250	500	1000	2500	5000	10000
ADF, covariance structure analysis	T_{AGLS}							
	M	224.649	146.8563	110.1098	98.9225	89.9213	87.8146	88.154
	SD	49.8764	28.9769	19.263	16.4164	13.1901	12.7041	13.1948
	Count	200	184	92	46	16	10	10
	T_{YB}							
	M	88.5393	91.5565	89.8005	89.8083	86.7361	86.2683	87.3667
	SD	7.9981	11.2973	12.8646	13.5554	12.2573	12.2627	12.9567
	Count	0	11	12	16	10	9	8
	$T_{F(AGLS)}$							
	M	1.0918	1.105	1.0475	1.0392	0.998	0.992	1.0045
	SD	0.2424	0.218	0.1833	0.1725	0.1464	0.1435	0.1504
	Count	15	25	20	18	10	9	8
ADF, correlation structure analysis	T_{AGLS}							
	M	93.1574	93.3013	90.1351	89.9235	86.9441	86.383	87.3545
	SD	12.5419	14.7537	13.6532	13.8974	12.5499	12.3512	12.9292
	Count	18/199	30	16	16	10	8	9
	T_{YB}							
	M	57.0763	67.5344	76.1201	82.349	83.9644	84.8867	86.5818
	SD	4.7399	7.8378	9.7516	11.6855	11.6851	11.9296	12.6973
	Count	0/199	0	0	3	4	4	8
	$T_{F(AGLS)}$							
	M	0.4527	0.702	0.8575	0.9446	0.965	0.9758	0.9954
	SD	0.061	0.111	0.1299	0.146	0.1393	0.1395	0.1473
	Count	0/199	0	0	3	4	4	8
		Weight matrix condition numbers						
ADF, covariance	M	25544.12	2007.359	629.4618	384.9135	277.3411	249.4422	233.8382
	SD	11260.63	518.0769	111.837	55.7553	28.73396	22.35056	17.72888
ADF, correlation	M	31140.09	2377.839	742.2598	436.6897	313.2534	281.8307	266.2201
	SD	13110.74	510.1428	118.5445	50.31107	25.48614	20.33156	19.1209

M: means of statistics; SD: standard deviations of statistics; Count: rejection count at α=0.05 level.
All results are based on 200 replications unless two numbers are given in the Count box,
in which case the second number is the number of replications that converged,
and the results are based on only the converged replications.

same and we used the same seed for the simulations, the condition numbers of the weight matrices are the same in both cases. We observe the same decreasing trend in means and standard deviations as the sample size increases, with roughly equal values for covariance and correlation structures. In both cases, these means and standard deviations are significantly larger than in Case 1. At $N = 10000$, the coefficients of variation (SD/M) of these condition numbers are roughly double in Cases 2 and 3 as compared to Case 1. Stated differently, the signal/noise ratio M/SD of the condition numbers is about twice as large with normal distributions as with the non-normal ones.

Table 4.4 summarizes the results for Case 4, in which factors are normally distributed, errors are non-normally distributed, and they are independent of each other. By comparing the results with the previous ones, we can see that for covariance structure analysis, the performance of all 3 statistics is similar to that of Case 1 and Case 3. For correlation structure analysis, the performance of T_{AGLS} in Case 4 is slightly better than in Case 3, especially at the smallest sample size. The performance of T_{YB} and $T_{F(AGLS)}$ is very similar across the 3 cases, and, as usual, they perform worse in correlation than covariance structures at most sample sizes.

The condition numbers of the weight matrices in Case 4 are smaller than those in Case 3 but somewhat larger than those in Case 1. Their coefficients of variation are only marginally larger, and signal/noise slightly smaller, under this type of non-normality than under normality. As in previous cases, the condition numbers for covariance and correlation structures are highly similar.

Table 4.5, 4.6, and 4.7 summarize the results for Cases 5, 6, and 7, where factors and errors are divided by a common random number resulting in dependent observations. In general, as in previous cases, T_{AGLS} statistics in covariance structures perform nominally asymptotically but seriously over reject the true model at small and medium sample sizes, while the correlational T_{AGLS} performs much bet-

Table 4.2: Summary of simulation results for Case 2.

		Sample size						
Method	Statistics	150	250	500	1000	2500	5000	10000
ADF,								
covariance	T_{AGLS}							
structure	M	286.3002	169.2534	121.4485	107.5488	97.4112	94.8508	94.8147
analysis	SD	68.4868	32.8661	21.3566	16.801	14.3258	13.8507	13.5709
	Count	200	190	115	58	19	15	11
	T_{YB}							
	M	96.7809	99.8579	97.2067	96.8897	93.6836	93.0483	93.906
	SD	7.9676	11.6177	13.7109	13.6509	13.2313	13.3281	13.3087
	Count	0	15	19	15	11	13	8
	$T_{F(AGLS)}$							
	M	1.1777	1.1475	1.0651	1.0499	1.0089	1.0011	1.0101
	SD	0.2817	0.2228	0.1873	0.164	0.1484	0.1462	0.1446
	Count	32	34	24	19	13	13	8
ADF,								
correlation	T_{AGLS}							
structure	M	202.1393	137.5241	109.2368	100.8685	94.2044	92.4154	92.7229
analysis	SD	51.3794	27.7071	19.6659	16.7556	14.2426	13.3547	13.3638
	Count	197	155	66	39	15	8	9
	T_{YB}							
	M	84.521	87.7857	89.1964	91.4088	90.7101	90.7043	91.8535
	SD	8.8089	11.3668	13.1356	13.7784	13.1842	12.8661	13.1113
	Count	0	1	4	10	5	7	8
	$T_{F(AGLS)}$							
	M	0.8315	0.9324	0.958	0.9847	0.9757	0.9754	0.9878
	SD	0.2113	0.1879	0.1725	0.1636	0.1475	0.141	0.1424
	Count	1	3	6	11	7	7	8
		Weight matrix condition numbers						
ADF,	M	76172.62	5834.362	1718.458	1029.257	712.7037	632.0674	575.7587
covariance	SD	50905.41	2916.986	640.2147	352.0613	208.1045	163.8565	90.60651
ADF,	M	73558.64	5943.245	1779.606	1044.091	713.614	619.9113	555.5487
correlation	SD	43936.93	2752.86	619.3706	347.362	214.9624	149.7043	81.73356

M: means of statistics; SD: standard deviations of statistics; Count: rejection count at $\alpha = 0.05$ level.
All results are based on 200 replications unless two numbers are given in the Count box,
in which case the second number is the number of replications that converged,
and the results are based on only the converged replications.

Table 4.3: Summary of simulation results for Case 3.

		Sample size						
Method	Statistics	150	250	500	1000	2500	5000	10000
ADF,								
covariance	T_{AGLS}							
structure	M	219.9541	146.0257	110.1443	98.6583	90.4165	88.1948	88.5836
analysis	SD	47.5195	27.9936	18.5762	16.3763	13.3616	12.9406	13.331
	Count	199/199	180	99	46	16	10	12
	T_{YB}							
	M	87.8554	91.2699	89.851	89.5898	87.1955	86.6344	87.7885
	SD	7.7019	11.0474	12.4379	13.5222	12.4152	12.4883	13.09
	Count	0/199	11	14	13	7	7	11
	$T_{F(AGLS)}$							
	M	1.069	1.0987	1.0478	1.0364	1.0035	0.9963	1.0094
	SD	0.2309	0.2106	0.1767	0.172	0.1483	0.1462	0.1519
	Count	11/199	22	17	14	7	8	11
ADF,								
correlation	T_{AGLS}							
structure	M	95.2845	93.5815	90.7471	89.9615	87.6806	86.7636	87.8856
analysis	SD	15.121	15.7935	14.0571	14.2183	12.9792	12.4982	13.1776
	Count	31	35	19	15	10	8	12
	T_{YB}							
	M	57.7759	67.6367	76.546	82.3744	84.6482	85.254	87.1031
	SD	5.5777	8.3289	10.0192	11.9469	12.0778	12.0709	12.9404
	Count	0	0	0	1	5	4	7
	$T_{F(AGLS)}$							
	M	0.4631	0.7041	0.8633	0.945	0.9731	0.9801	1.0015
	SD	0.0735	0.1188	0.1337	0.1494	0.1441	0.1412	0.1502
	Count	0	0	0	3	5	4	7
		Weight matrix condition numbers						
ADF,	M	76172.62	5834.362	1718.458	1029.257	712.7037	632.0674	575.7587
covariance	SD	50905.41	2916.986	640.2147	352.0613	208.1045	163.8565	90.60651
ADF,	M	73558.64	5943.245	1779.606	1044.091	713.614	619.9113	555.5487
correlation	SD	43936.93	2752.86	619.3706	347.362	214.9624	149.7043	81.73356

M: means of statistics; SD: standard deviations of statistics; Count: rejection count at α=0.05 level.
All results are based on 200 replications unless two numbers are given in the Count box,
in which case the second number is the number of replications that converged,
and the results are based on only the converged replications.

Table 4.4: Summary of simulation results for Case 4.

		Sample size						
Method	Statistics	150	250	500	1000	2500	5000	10000
ADF, covariance structure analysis	T_{AGLS}							
	M	221.4522	146.146	110.3154	98.7184	90.2788	88.1898	88.6902
	SD	48.1452	27.991	18.61	16.3519	13.523	12.9549	13.2714
	Count	200	181	101	49	14	13	13
	T_{YB}							
	M	88.0733	91.3186	89.9649	89.6405	87.066	86.6293	87.8933
	SD	7.8325	11.0431	12.4464	13.5026	12.5627	12.5032	13.0288
	Count	0	10	16	13	9	7	11
	$T_{F(AGLS)}$							
	M	1.0763	1.0997	1.0495	1.037	1.002	0.9962	1.0107
	SD	0.234	0.2106	0.177	0.1718	0.1501	0.1464	0.1512
	Count	10	25	18	17	11	7	11
ADF, correlation structure analysis	T_{AGLS}							
	M	93.8581	93.2639	90.4126	89.8608	87.2889	86.7243	87.917
	SD	13.2846	14.626	13.6436	13.9274	12.8717	12.5695	13.0521
	Count	21	28	17	15	6	7	11
	T_{YB}							
	M	57.3142	67.521	76.3195	82.2958	84.2835	85.2157	87.1344
	SD	4.9783	7.7565	9.7385	11.7018	11.979	12.1403	12.8153
	Count	0	0	0	1	5	4	9
	$T_{F(AGLS)}$							
	M	0.4561	0.7018	0.8601	0.944	0.9688	0.9797	1.0019
	SD	0.0646	0.1101	0.1298	0.1463	0.1429	0.142	0.1487
	Count	0	0	0	3	5	4	9
		Weight matrix condition numbers						
ADF, covariance	M	43419.79	3212.517	929.9867	524.0417	344.056	305.9564	270.5049
	SD	28871.7	1190.623	245.5542	121.1078	50.25963	98.96762	23.26951
ADF, correlation	M	49475.99	3735.899	1064.594	584.8088	382.0124	337.7119	295.306
	SD	28964.63	1276.599	233.8229	134.9129	48.04584	108.9194	24.2551

M: means of statistics; SD: standard deviations of statistics; Count: rejection count at α=0.05 level.
All results are based on 200 replications unless two numbers are given in the Count box,
in which case the second number is the number of replications that converged,
and the results are based on only the converged replications.

ter. However, correlational T_{AGLS} is nowhere near as well performing as in Cases 1 and 3; instead, in Cases 5-7 the performance is similar to that of Case 2 where there was substantial over rejection of the true model at the smaller sample sizes although nowhere near as extremely as with covariance structures. These conditions also have up to 3.5% of replications failing to converge with correlations. The statistics and rejection rates of T_{YB} and $T_{F(AGLS)}$ in covariance structure analysis are generally similar to those of the corresponding independence cases, although in Cases 5-7 their performance at the smaller sample sizes yields greater under rejection. In correlation structure analysis with small sample sizes, these statistics perform even worse. While in Cases 5-7, T_{YB} and $T_{F(AGLS)}$ converge more rapidly to their theoretical limits than in Cases 1, 3, and 4, in absolute terms the under rejection is still very severe at the smaller sample sizes.

The condition numbers of weight matrices in Cases 5-7 are much larger and more widely spread as compared with their corresponding independence cases. Furthermore, the asymptotic coefficients of variation of the condition numbers is 4-5 times as large as the independence cases, and up to 10 times as large as those observed under normality in Condition 1.

In summary, comparing to their covariance structure analysis counterparts, the ADF method with T_{AGLS} statistics performs surprisingly well in correlation structure analysis with small sample sizes in terms of statistics and rejection rates, but not in converging rates, while the ADF method with T_{YB} and $T_{F(AGLS)}$ statistics performs poorly in correlation structure analysis with small sample sizes. With large sample sizes, the ADF methods with all 3 statistics in both covariance structure analysis and correlation structure analysis are able to converge to their corresponding theoretical limits.

As for condition numbers of weight matrices, the condition numbers decrease as sample sizes increase. Moreover, violations of normality increase both the means and standard deviations of the condition numbers but not their relative

Table 4.5: Summary of simulation results for Case 5.

		Sample size						
Method	Statistics	150	250	500	1000	2500	5000	10000
ADF,								
covariance	T_{AGLS}							
structure	M	209.9947	140.4924	108.1344	99.5106	90.6292	90.0573	88.372
analysis	SD	36.0421	23.1144	15.3194	14.1865	12.1619	12.602	13.517
	Count	200	188	85	44	15	14	8
	T_{YB}							
	M	86.5444	89.2643	88.6157	90.346	87.4046	88.4338	87.5801
	SD	6.2016	9.3792	10.3345	11.6987	11.3179	12.1515	13.2708
	Count	0	3	4	10	4	11	8
	$T_{F(AGLS)}$							
	M	1.0206	1.0571	1.0287	1.0453	1.0059	1.0173	1.007
	SD	0.1752	0.1739	0.1457	0.149	0.135	0.1424	0.154
	Count	2	10	7	13	5	11	8
ADF,								
correlation	T_{AGLS}							
structure	M	115.8832	100.3471	93.8847	92.6419	88.6876	89.0765	87.7855
analysis	SD	29.0865	20.8819	18.0115	14.0728	12.869	13.3677	13.9243
	Count	109/196	59	27	24	8	15	7
	T_{YB}							
	M	64.2154	70.9137	78.6472	84.6288	85.5887	87.4836	87.0026
	SD	8.9876	10.2931	12.293	11.7609	11.9808	12.8942	13.6712
	Count	0/196	0	2	3	5	11	7
	$T_{F(AGLS)}$							
	M	0.5632	0.755	0.8932	0.9732	0.9843	1.0063	1.0003
	SD	0.1414	0.1571	0.1713	0.1478	0.1428	0.151	0.1587
	Count	0/196	1	4	6	5	13	7
		Weight matrix condition numbers						
ADF,	M	306730.2	22284.95	5149.106	2125.579	1247.086	866.5005	530.0225
covariance	SD	798649.8	56694.84	11658.51	3998.007	3932.683	1303.571	323.2938
ADF,	M	285996.8	22360.56	5508.089	2252.228	1369.207	881.9237	556.9175
correlation	SD	662600.5	44623.85	12794.05	4061.555	4743.763	1193.424	317.2301

M: means of statistics; SD: standard deviations of statistics; Count: rejection count at α=0.05 level.
All results are based on 200 replications unless two numbers are given in the Count box,
in which case the second number is the number of replications that converged,
and the results are based on only the converged replications.

Table 4.6: Summary of simulation results for Case 6.

		Sample size						
Method	Statistics	150	250	500	1000	2500	5000	10000
ADF, covariance structure analysis	T_{AGLS}							
	M	209.6036	139.3892	107.8177	98.7268	90.4895	89.6663	88.8373
	SD	33.9494	23.2638	14.5975	13.48	12.2549	12.3921	13.5384
	Count	200	180	88	41	11	16	11
	T_{YB}							
	M	86.5356	88.7991	88.4246	89.7113	87.2737	88.0575	88.0373
	SD	5.9368	9.5043	9.8825	11.1461	11.4059	11.9533	13.2873
	Count	0	2	2	11	5	10	8
	$T_{F(AGLS)}$							
	M	1.0187	1.0488	1.0257	1.0371	1.0043	1.0129	1.0123
	SD	0.165	0.175	0.1389	0.1416	0.136	0.14	0.1543
	Count	1	12	4	14	6	10	8
ADF, correlation structure analysis	T_{AGLS}							
	M	123.012	101.7469	94.4171	92.4639	88.8611	88.67	88.2718
	SD	31.0884	20.8587	16.6102	13.7616	14.2385	13.1088	13.9074
	Count	126/195	60/199	32/199	24	10	17	12
	T_{YB}							
	M	66.3465	71.6239	79.0739	84.4863	85.7377	87.0922	87.4806
	SD	9.2147	10.301	11.5273	11.5162	13.1721	12.6444	13.6489
	Count	0/195	0/199	2/199	3	6	11	11
	$T_{F(AGLS)}$							
	M	0.5978	0.7656	0.8982	0.9713	0.9862	1.0017	1.0059
	SD	0.1511	0.1569	0.158	0.1446	0.158	0.1481	0.1585
	Count	0/195	0/199	2/199	4	6	11	11
		Weight matrix condition numbers						
ADF, covariance	M	448858.5	35152.88	6894.126	2635.108	1473.717	1214.176	645.8547
	SD	911396.6	96209.51	14815.99	4828.256	3727.372	2788.326	440.7358
ADF, correlation	M	440517	37361.53	7195.832	2620.814	1551.541	1272.651	673.7769
	SD	784565.3	102924.1	14857.71	3925.581	3964.034	3042.005	443.1857

M: means of statistics; SD: standard deviations of statistics; Count: rejection count at α=0.05 level.
All results are based on 200 replications unless two numbers are given in the Count box,
in which case the second number is the number of replications that converged,
and the results are based on only the converged replications.

Table 4.7: Summary of simulation results for Case 7.

		Sample size						
Method	Statistics	150	250	500	1000	2500	5000	10000
ADF,								
covariance	T_{AGLS}							
structure	M	207.4921	138.9543	107.7686	98.6843	90.4124	89.7547	88.928
analysis	SD	34.1429	22.1298	14.7087	13.6724	12.3778	12.2541	13.5543
	Count	200	184	86	36	14	14	12
	T_{YB}							
	M	86.1429	88.6714	88.3882	89.6724	87.201	88.1434	88.1264
	SD	6.1423	9.0729	9.9485	11.2993	11.5234	11.8189	13.3027
	Count	0	2	2	10	5	8	8
	$T_{F(AGLS)}$							
	M	1.0084	1.0455	1.0252	1.0367	1.0035	1.0139	1.0134
	SD	0.1659	0.1665	0.1399	0.1436	0.1374	0.1384	0.1545
	Count	0	8	4	13	5	9	10
ADF,								
correlation	T_{AGLS}							
structure	M	125.8541	103.7095	95.3557	93.1508	88.9243	89.0635	88.3685
analysis	SD	32.5554	23.1856	18.0662	14.7465	14.7668	13.0089	13.926
	Count	129/193	68	39/199	28	9	13	11
	T_{YB}							
	M	67.1289	72.4905	79.6824	85.0405	85.7908	87.4726	87.5755
	SD	9.4418	11.1433	12.4123	12.3014	13.6751	12.5443	13.667
	Count	0/193	1	2/199	6	8	11	9
	$T_{F(AGLS)}$							
	M	0.6116	0.7803	0.9071	0.9785	0.9869	1.0061	1.007
	SD	0.1582	0.1745	0.1719	0.1549	0.1639	0.147	0.1587
	Count	0/193	2	2/199	6	8	11	10
		Weight matrix condition numbers						
ADF,	M	498903.4	42058.49	9841.297	3632.087	2299.307	1567.962	1072.239
covariance	SD	871533.2	82849.18	22723.32	4972.402	5452.845	2549.437	806.0849
ADF,	M	504899.2	43326.45	9876.272	3776.686	2485.859	1618.726	1105.31
correlation	SD	1073513	87457.93	20975.29	5525.001	6581.523	2850.889	834.1139

M: means of statistics; SD: standard deviations of statistics; Count: rejection count at α=0.05 level.
All results are based on 200 replications unless two numbers are given in the Count box,
in which case the second number is the number of replications that converged,
and the results are based on only the converged replications.

ratio, although violation of independence conditions greatly increases the mean and even more greatly increases the standard deviation of condition numbers, causing the condition numbers to be more widely spread.

CHAPTER 5

Discussion

The behaviors of the asymptotically distributed free (ADF) methods in covariance and correlation structure analysis were studied for three test statistics, two of which are referred to the χ^2 distribution for evaluation, and one of which is referred to the F distribution. The main statistic studied is the statistics T_{AGLS} originally developed by Browne [Bro84a] for covariance structure analysis, but adaptable to correlation structures through the results of Steiger and Hakstian [SH82] or de Leeuw [Lee83]. The secondary statistics studied, T_{YB} and $T_{F(AGLS)}$, are based on T_{AGLS} and are meant to improve model evaluation with small samples. Results for T_{AGLS} in covariance structures essentially replicate those obtained previously by [HBK92], while those for T_{YB} and $T_{F(AGLS)}$ are consistent with those of their developers. Specifically, T_{YB} results are consistent with those of [YB97] and $T_{F(AGLS)}$ results are consistent with those of [YB99]. Thus for covariance structures, the simulations are largely confirmatory.

Little is known about the empirical performance of T_{AGLS} in correlation structure analysis. The most thorough available studies are those of Fouladi [Fou00] and Mels [Mel00], which provided somewhat contradictory results. Fouladi compared 8 covariance structure analysis techniques and 4 correlation structure analysis techniques, among which 2 covariance and 2 correlation structure analysis techniques are asymptotically distribution free methods, including T_{AGLS} for covariance and correlation structures, and T_{YB} for covariance structures. Fouladi tested each method over a variety of cases, including different covariance matrix structures, non-normalities, and sample sizes. Her results on asymptotically distribution free statistics are consistent with our results, showing that at less than asymptotic samples sizes the performance of T_{AGLS} in correlation structures is better than that of T_{AGLS} in covariance structures. She also verified the generally good performance of T_{YB} in covariance structures, but did not evaluate the parallel performance in correlation structures. In some ways, Mels [Mel00] obtained results similar to those of Fouladi and this study, showing that in a CFA model with 12 non-normal variables, a correlation structure ADF methodology with an adjusted asymptotic covariance matrix yielded a two-stage test statistic T_{TAGLS} that performed much better at small and intermediate sized samples than the corresponding T_{AGLS} from covariance structure analysis. Elements of the adjusted asymptotic covariance matrix of correlations used by Mels are given by our (2.11) with least-squares model-based estimates of correlations ρ_{ij} instead of sample correlations as used here and by Fouladi. In fact, Mels also reported on the test T_{AGLS} using (2.11) with sample correlations and fourth-order moments, i.e., a statistic that used the same asymptotic covariance matrix of correlations as used by Fouladi and this study. Hence, Mels' results for this statistic should be consistent with Fouladi's and our results. However, he found that this test performed very poorly. We have two reasons to suspect that there is a problem with Mels' results on this statistic. First, his statistic does not behave correctly asymptotically, as it should. For example, at $N = 5000$ the mean of the test statistic across simulation replications is nowhere near its expectation, the degrees of freedom of the model. Second, in large samples under the correct model the adjusted weight matrix used in his two-stage ADF test statistic (his eq. (4.15)) should be essentially identical to that of the ordinary ADF correlation weight matrix, since sample based and model based estimates of ρ_{ij} both converge to the population value. Hence, his one-stage ADF test (his eq. (4.13)) should have behaved essentially identically to that of his two-stage test in large samples, yet it did not.

In this paper, we studied parallel ADF statistics for covariance and correlation structure analysis and made direct comparisons between their performances under various conditions (different sample sizes, normality and non-normality, and/or dependence conditions). Results show that, compared to the ADF method in covariance structure analysis, T_{AGLS} in correlation structure analysis performed surprisingly well at small sample sizes. More specifically, compared with their covariance structure analysis counterparts, with small sample sizes the means of the T_{AGLS} statistics of ADF correlation structure analysis are closer to the theoretical values, and the rejection rates of the correlational T_{AGLS} statistics under the model are closer to nominal levels as well.

As for statistics T_{YB} and $T_{F(AGLS)}$, Bentler and Savalei [BS10] proposed that these two statistics could be adapted to use in correlation structure analysis, and in fact the methodology has been available in the EQS program for a decade [Ben06]. However, they noted that the conditions utilized in the derivations of these statistics for covariance structure analysis are less well met in correlation structures. For example, T_{YB} arises from a substitute computation for the asymptotic covariance matrix of covariances that does not have a parallel in correlation structures, and the Hotelling T^2 rationale for $T_{F(AGLS)}$ seems less well justified. Nonetheless, they expected these statistics to perform well in practice although they provided no evidence in this regard. The results observed in this study are quite contradictory to these expectations. Speaking generally, T_{YB} and $T_{F(AGLS)}$ based on correlations perform much worse than their covariance structure counterparts. As an explanation for these differences, we note that T_{AGLS} rejects the true model with smaller samples with covariance structure analysis, and T_{YB} and $T_{F(AGLS)}$ substantially reduce excessive model rejections. With covariance structures, this reduction is generally quite good. In contrast, in correlation structures, T_{AGLS} already performs quite well without any corrections. The further reduction in test values obtained via T_{YB} and $T_{F(AGLS)}$ thus overcorrect, leading to model acceptance far beyond nominal levels.

Some graphs provide a convenient way to compare an aspect of the performance of the various test statistics. Figure 5.1 shows the means of T_{AGLS} across replications at different sample sizes for both covariance and correlation structure analysis under the various distributional conditions of our simulation. The solid lines give results from covariance structures, and the dashed lines give the results from correlation structures. The results of both covariance and correlation structures under the same condition are plotted in the same color. Figure 5.2 shows the means of T_{YB} at different sample sizes in both covariance and correlation structure analysis using the same notation as in Figure 5.1, while the same notation is also used in Figure 5.3 that shows the means of $T_{F(AGLS)}$ at different sample sizes in both covariance and correlation structure analysis.

From the figures we can see that the means of test statistics in correlation structure analysis are generally lower than the means of corresponding statistics in covariance structure analysis. When the sample sizes are small, the T_{AGLS} test statistics in covariance structure analysis are very far away from the theoretical value. Under these conditions, the lower values of means of T_{AGLS} statistics in correlation structure analysis are closer to their theoretical values, thus making the correlation structure analysis ADF method a better choice than its covariance structure analysis counterpart, especially at smaller sample sizes. On the other hand, while the means of T_{YB} and $T_{F(AGLS)}$ test statistics in covariance structure analysis are close to their theoretical values at most sample sizes, in correlation structure analysis their lowered means compared to T_{AGLS} taken them further away from their expected values.

We can also compare the results across different cases. For example, the T_{AGLS} results in cases 1, 3, and 4 are quite similar with both covariance and correlation structure analysis. The differences between cases 1, 3, and 4 involve whether normality of factors or errors is violated. These types of non-normality create little

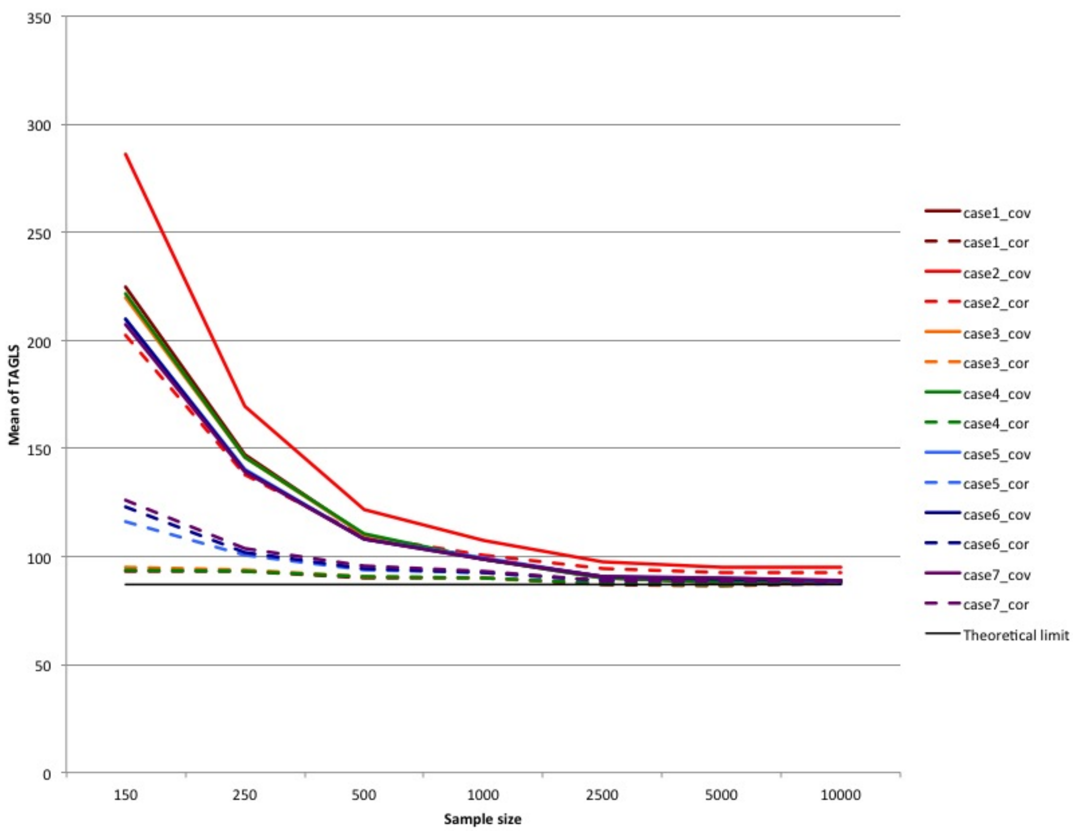

Figure 5.1: The means of T_{AGLS} of covariance structure analysis and correlation structure analysis of all 7 cases as a function of sample sizes. Note that the theoretical limit of Case 2 should be 93.

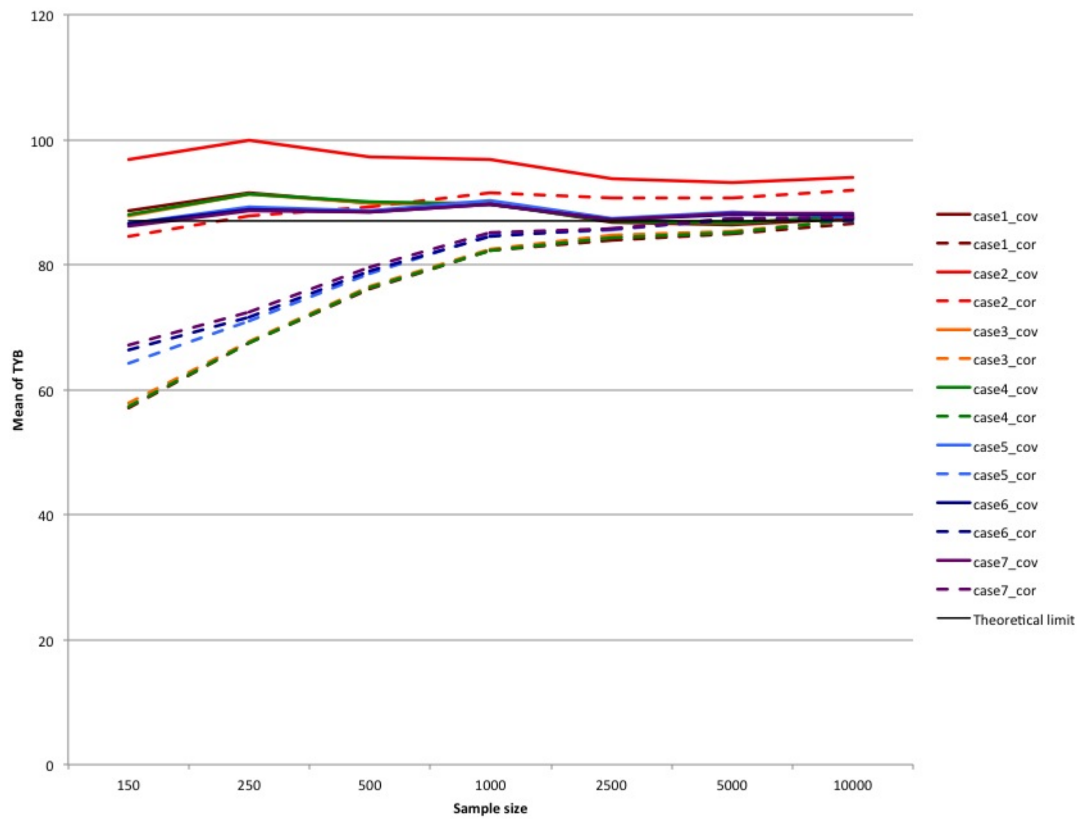

Figure 5.2: The means of T_{YB} of covariance structure analysis and correlation structure analysis of all 7 cases as a function of sample sizes. Note that the theoretical limit of Case 2 should be 93.

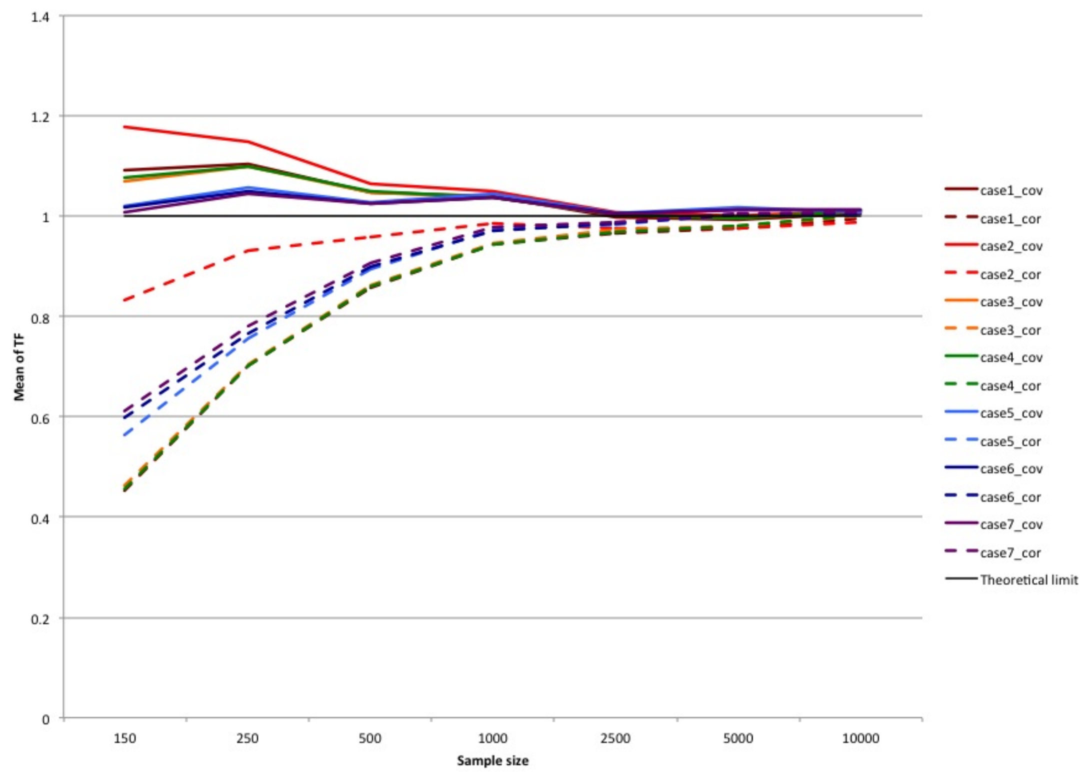

Figure 5.3: The means of $T_{F(AGLS)}$ of covariance structure analysis and correlation structure analysis of all 7 cases as a function of sample sizes.

trouble for either ADF method, which is to be expected from the distribution-free nature of ADF statistics. On the other hand, the T_{AGLS} results in cases 1, 3, and 4 (independence conditions) are better than those of cases 5, 6, and 7 (dependence conditions) in correlation structure analysis, while the opposite occurs in covariance structure analysis. In other words, violation of the independence assumption creates a larger problem for T_{AGLS} in correlation structures at small sample sizes while the same violation has little effect in the corresponding covariance structure analysis. In contrast, the results for cases 5, 6, and 7 are slightly better than the results of case 1, 3, and 4 in terms of statistics and rejection rates with T_{YB} and $T_{F(AGLS)}$ in correlation structure analysis. This differential performance implies the use of different statistics in practice, but it is hard to make recommendations for practice that depend on conditions, since practical methods for evaluating independence versus dependence of observations are not available. Nonetheless, the figures imply that when the independence assumption is not violated, T_{AGLS} is a better choice than T_{YB} and $T_{F(AGLS)}$ in correlation structure analysis at small sample sizes; when the independence assumption is violated, T_{YB} and $T_{F(AGLS}$ statistics produce better results than T_{AGLS}. This is quite different than covariance structure analysis, where in all cases T_{YB} and $T_{F(AGLS)}$ perform better than T_{AGLS} at small sample sizes.

Finally, we turn to the condition numbers of the weight matrices. Those associated with correlational and covariance structure analysis behaved remarkably similarly. At small sample sizes, the condition numbers were far larger and had substantially greater variability than at the largest sample sizes. A decline in the size of the condition numbers with increasing sample size is to be expected since it is known that sample eigenvalues are more extreme than their population counterparts. However, we were surprised that in some conditions the average condition numbers for the smallest sample size could be hundreds of times as large as those for the largest sample size (see e.g., Table 4.6). Similarly, since condition numbers become less extreme with larger sample sizes, one might expect standard deviations of condition numbers across replications to decrease with increasing sample size. Nonetheless, the tremendous variation in condition numbers within any case or condition, with hugely increased variability at the smallest sample sizes compared to the largest, was unexpected. For example, in Table 4.6, the SD for condition numbers in covariance structures at the smallest sample size is over 2000 times larger than that at the largest sample size. These observations suggest that some of the poor behavior of ADF test statistics may be associated with excessively extreme condition numbers of estimates of the population weight matrix, and that research might be directed at methods to robustify these weight matrices with an eye to improved condition numbers. Among new approaches that might be considered are those that create the weight matrix as a convex combination of the usual weight matrix and the identity, similar to a methodology proposed in another context by Yuan [Yua]; those that shrink the eigenvalues of a sample weight matrix towards their geometric mean, similar to a methodology proposed by Dey [Dey88], or estimate all the eigenvalues more effectively as proposed by Ledoit and Wolf [LW13]; or, since a weight matrix is just a covariance matrix, those that robustify the weight matrix by downweighting extreme cases, e.g., using Campbell's methodology [Cam80] or any other as reviewed by Maronna, Martin & Yohai [MMY06]. Of course, the field of covariance matrix estimation under difficult conditions is a huge and expanding one (e.g., [LW12, XMZ12]), so a lot of research remains to be done to adapt this work to structural modeling with ADF.

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