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GEOMETRICAL ASPECTS OF RELATIVISTIC NUCLEAR COLLISIONS*

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A common feature of many approaches to relativistic nuclear collisions¹⁻⁴ is that the experimentally observed quantities in inclusive measurements of various kinds are calculated by means of a four-dimensional integral of the following form:

$$Q = \int d\vec{s} \int d\vec{\sigma} q_s(\vec{\sigma})$$

where Q is the quantity of interest (a differential cross section, for example), \vec{s} is the two-dimensional vector impact parameter, $\vec{\sigma}$ is the two-dimensional vector position in the plane perpendicular to the beam, and $q_s(\vec{\sigma})$ is the local contribution to Q for impact parameter \vec{s} from the point $\vec{\sigma}$. This integral can be recast in the form,⁵

$$Q = \int 2\pi s_1 ds_1 \int 2\pi s_2 ds_2 q(s_1, s_2) ,$$

where s_1 and s_2 are one-dimensional radial variables measured from the centers of the projections of the target and projectile nuclear density distributions onto a plane. Finally, for the idealization of nuclei as uniform, sharp surface spheres of density ρ , the integral can be written,

$$Q = \int_0^{2R_1} \frac{\pi}{2} \alpha d\alpha \int_0^{2R_2} \frac{\pi}{2} \beta d\beta q(\alpha, \beta) ,$$

where R_1 and R_2 are the sharp radii and α and β are length variables proportional to the number of particles per unit area when the nuclear densities are projected onto a plane.

As an example of how this expression is to be used, consider the question of the total cross section for a particle that is to emerge from a relativistic nuclear collision. Assuming that all the particles come from the overlap region and none from the "spectators," the quantity $q = (\alpha + \beta)\rho$ and the total cross section is calculated to be,² $\sigma_t = \pi[A_1 R_2^2 + R_1^2 A_2]$. Similarly, if we wish to calculate the total cross section assuming that the yield comes only from single knock-on collisions in the overlap region (under the drastic assumption of

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infinite nucleon-nucleon cross section), then

$$q = 2 \cdot \min(\alpha, \beta) \rho \quad \text{and} \quad \sigma_{k.o.} = 2\pi A_1 [R_2^2 - 1/5 R_1^2] .$$

A somewhat more complex application is made in Ref. 4.

To continue, let us choose to measure the lengths α and β in units of $\lambda = 6.9$ fm, which is simply the length of a column one fermi square, which contains one nucleon when the nucleon density $\rho = 0.145$ fm⁻³. Then our integral can be written,

$$Q = C \int_0^{\Omega_1} d\omega_1 \int_0^{\Omega_2} d\omega_2 W(\omega_1, \omega_2) ,$$

where C is a proportionality constant and the density function $W(\omega_1, \omega_2) = \omega_1 \omega_2 (\omega_1 + \omega_2)$. This latter quantity is plotted in Fig. 1 and the boundaries for various target and projectile combinations are indicated. Such plots are useful to illustrate the relative importance of different ω_1, ω_2 combinations and their dependence on the particular target and projectile. The quantity W can be defined with respect to ω and η where $\omega = \omega_1 + \omega_2$ and $\eta = \omega_1/\omega$, in which case $W(\eta, \omega) = \omega \cdot W(\omega_1, \omega_2)$. Finally, the quantity $W(\eta)$ [which is the dimensionless analog of the quantity $Y(\eta)$ in Ref. 1] can then be formed by projecting that part of the $W(\eta, \omega)$ function which is bounded by Ω_1 and Ω_2 , onto the η axis.

These generally useful relations allow us to do completely analytic calculations for various differential cross sections in the firestreak,¹ rows-on-rows,² and knock-on³ models but are limited by the unrealistic assumption of sharp nuclear surfaces. Fortunately, they may be extended by using an approximation in which the projection of the diffuse nuclear density distribution onto a plane is represented by a circle smoothly joined to an exponential as is shown in Fig. 2. The curves show the approximation and the dots are the actual values of the projected density for a particular model.

With this extension we can reconsider such quantities as $W(\omega_1, \omega_2)$ which is shown in Fig. 3. The figure was drawn for the case of Ne+U, and the dashed lines show the boundaries of the region to be considered, just as they did in

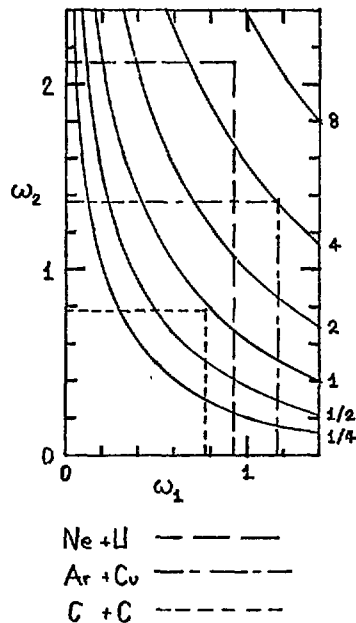


Fig. 1

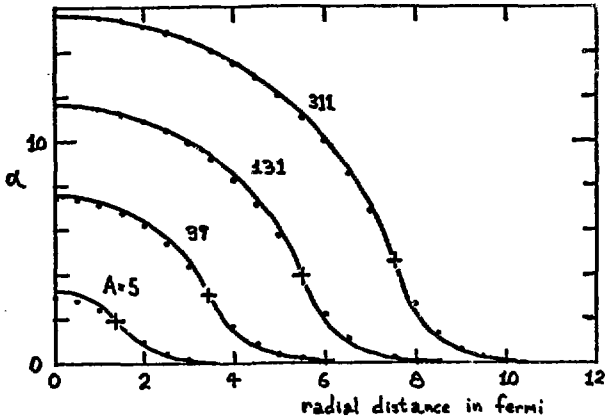


Fig. 2

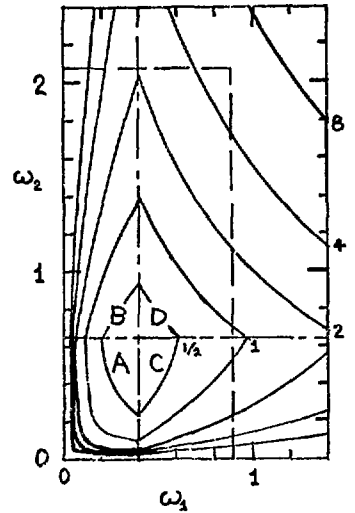


Fig. 3

Fig. 1. However, the presence of the diffuse tails of the nuclear density distributions add a new aspect. The dot-dashed lines divide the surface into four regions. Region A concerns that part of the collision process in which the diffuse fringe around the projectile collides with the fringe around the target. In region B the fringe of the projectile is incident on the massive central part of the target. In C the central part of the projectile is incident on the fringe around the target, and in D the two central regions are incident on each other. Region D, of course, is identical to the corresponding region in Fig. 1, but the boundaries limiting the region have moved in slightly.

As before the weight function $W(\omega_1, \omega_2)$ can be converted to $W(\eta, \omega)$ and then projected onto the η -axis to give $W(\eta)$, which is the exact analog of the $Y(\eta)$ functions that were tabulated in Ref. 1.

These analytic forms of the geometrical weight functions are currently being employed in a generalization of the firestreak model which includes effects of transparency and the contributions of simple two-body knock-on collisions to the inclusive cross sections, but this work is not yet complete.

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