Title
Maturity Effects in Futures Markets: Evidence from Eleven Financial Futures Markets

Author
Madarassy Akin, Rita

Publication Date
2003-04-01

Maturity Effects in Futures Markets:
Evidence from Eleven Financial Futures Markets

Rita Madarassy Akin*
SCCIE, UC Santa Cruz
and Westminster College

April 2003

Abstract

This essay examines the volatility dynamics of the financial futures returns. Samuelson (1965) demonstrated theoretically that the conditional variance of changes in futures prices should increase as the time-to-maturity decreases. Interestingly, the empirical evidence on the Samuelson hypothesis is mixed. This essay revisits that issue, applying a unified GARCH framework to a unique data set of daily data, spanning 19 years up to 2000, and eleven types of financial contracts (currencies, S&P500, Nikkei 225, Eurodollar, Treasury Bills). The conditional variance equation is augmented by time-to-maturity, open interest and trading volume variables. I detect evidence for a role of the time-to-maturity in currency futures, and mixed evidence in equity index and interest rate futures. Lagged trading volume and open interest are positively related to volatility in most of these financial futures but they do not fully account for the estimated conditional variance.

Keywords: futures, volatility, GARCH, time-to-maturity

JEL Classification: G13, G14, G15

Acknowledgements: I am deeply indebted to Yin-Wong Cheung, Menzie Chinn, Kenneth Kletzer for their advice, encouragement, and insights. In addition, conversations with Binbin Guo have been helpful. Naturally, all errors remain my responsibility.

***Correspondence**: Santa Cruz Center for International Economics, University of California, Santa Cruz, CA 95064. From Sept. 1, 2003: 217 Old Main, Dept. of Economics and Business, Westminster College, New Wilmington, PA 16172. Email: rita_madarassy@yahoo.com

1 Introduction

The aim of this essay is to better understand the process governing the evolution of volatility in futures prices. By volatility, I mean the second moment in changes of futures prices. Volatility in price changes is a variable of interest because futures prices reflect information regarding the market participants' expectations about subsequent commodity price changes.[1] Hence, examining the behavior of the variance in price changes may provide evidence regarding how information is assimilated in a large and rapidly growing derivatives market.

Assuming that spot prices follow a stationary autoregressive process, Samuelson (1965) defines futures prices as the expected spot prices at maturity of the contract. He shows theoretically that the conditional variance of futures price changes per unit of time monotonically increases as the time-to-maturity decreases.[2] If the variance of futures prices decreases with time-to-maturity, then that has certain implications about the information assimilation of futures markets – either the market is more sensitive, may be overreacting to the new information, or the rate of information flow and its transmission increases, in other words, the resolution of uncertainty is higher near maturity, or both these channels might be at work.

Some previous studies examining a range of financial futures find evidence of the Samuelson effect, while others find the reverse. Anderson (1985) and Kenyon et al. (1987) claim that seasonality is a better explanation for the maturity effect and suggest that the observed maturity effect is likely to be a proxy for what actually is the consequence of the movements in some underlying fundamental variables such as information flows[3].

My contribution is that I use a unifying GARCH framework to model the persistence in volatility. In addition, I augment the conditional variance equation by a time-to-maturity variable, to capture any maturity effect, and by the open interest and the trading volume to determine if they explain any of the observed GARCH effect. Although the relationship between trading volume and price changes have been identified[4] mainly in spot markets, the role of open interest has been explored only by a few. I test three types of financial futures contracts, with a unique daily data set that is significantly longer than previous data sets, 19 years of daily data, starting on 4 January 1982, and ending on 31 December 2000, originating from the Chicago Mercantile Exchange. I augment the basic GARCH model to incorporate a maturity variable and two other economic variables into the

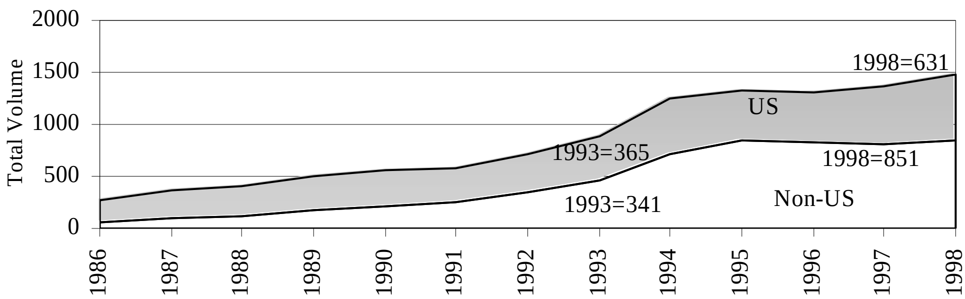
---

[1] Indeed futures prices will equal the expected future spot price when agents are risk neutral.
[2] This is known as the time-to-maturity effect, or the Samuelson hypothesis. See Samuelson (1965).
[3] Also known as the state variable hypothesis.
[4] See Karpoff (1987) for a survey of studies. Bessembinder and Seguin (1992), Jones et al. (1994).

conditional variance equation, so they are allowed to play a role in the volatility of futures. While most researchers find time-varying futures price volatility, previous empirical studies on futures prices of various commodities have found mixed evidence of the time-to-maturity effect. This analysis sheds more light on the possible sources and patterns of volatility of futures returns, the validity of the Samuelson hypothesis, and how uniformly these results hold in various financial futures markets. These results are of particular relevance, in light of the extraordinary growth of financial futures markets over the past decade.

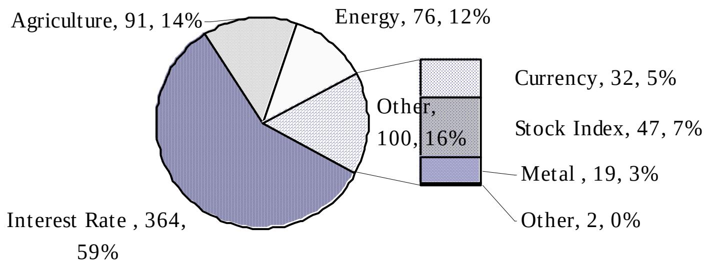
Figures 1.A and 1.B demonstrate the dramatic growth and the distribution of futures markets by trading volume, respectively. Over the decade ending 1998, world contract volume grew on the average by 15% per annum, and in that year 71% of the total volume represents financial futures contracts. The Chicago Mercantile Exchange, the source of the data set used in this study and also one of the two largest exchanges in the US, almost doubled its contract volume per year in the last 3 years. Over the first nine months of 2001, the Chicago Mercantile Exchange had a record number of contracts traded, nearly 295.5 million valued at $210.2 trillion.

Figure 1.A World Volume of Exchange Traded Futures and Options on Futures
(in millions of contracts)



Source: Commodity Futures Trading Commission

Figure 1.B Distribution of U.S. Exchange Traded Futures & Options
Annual Volume 1998 - Total Volume 630 million



Source: Commodity Futures Trading Commission

The rest of the paper is organized as follows. Section 2 sets the stage for the analysis. I discuss the previous literature, and outline the models for futures return dynamics, and the empirical findings. Section 3 describes the data I use. Section 4 presents the methodology and the various GARCH specifications used to examine the volatility of futures returns. Section 5 presents the empirical results. I conclude in section 6.

2 Literature Review of Maturity Effect and Related Literature

2.1. Theoretical Approaches to the Behavior of Futures Prices

A theoretical hypothesis introduced by Samuelson (1965), known as the Samuelson hypothesis (SH) or the time-to-maturity effect (TTM), develops a model predicting a rise in the volatility of futures prices as maturity nears. The intuition behind his theory is that our view of a distant future environment, which includes our opinion of distant futures prices, will not change much in the next month since few of the disturbances[5] affecting the distant future environment will change this month; it stays close to the general level given by the so-called law of averages[6]. As time passes and we approach the maturity date, and our future becomes our present, we become more and more sensitive to information that influence the final level of the futures price. When the maturity date arrives, arbitrage forces the futures price to equal the actual spot price.

Anderson & Danthine (1983)[7] propose the state variable hypothesis which they claim is compatible with the Samuelson hypothesis. They introduce information flows into their theoretical model and demonstrate that the resolution of uncertainty is the source of increased volatility in futures prices, which can be used to explain both the Samuelson hypothesis and their own state variable hypothesis under a unified framework. They show that the ex ante variance of futures prices tends to be high (low) when the amount of economic uncertainty tends to be large (small). The Samuelson hypothesis is a special case, where large amounts of uncertainty are being resolved toward the maturity date and thus the ex ante futures price variance tends to be higher as the maturity date nears. In other words, there are no information flows that resolve uncertainty about futures prices in the far distant future. Anderson and Danthine (1983) and Anderson (1985) argue that the SH is generally not true unless we have information flows incorporated in the model.[8]

Hong (2000) develops an equilibrium model of a competitive futures market where investors trade to hedge positions and to speculate on their private information. He examines the equilibrium

[5] Supply and demand factors such as weather, crop plantings, crop yields, pest population, income, taste changes, population.
[6] See Samuelson (1965).
[7] See also Richard and Sundaresan (1980).
[8] Stein's (1979) model does imply an increase in the variance as the maturity date approaches.

return and trading patterns and concludes that in markets where the information asymmetry among investors is small, the return volatility of futures contract decreases with time-to-maturity, that is, the Samuelson hypothesis holds. However, when the asymmetry is large, the Samuelson effect need not hold. He is also able to show nonlinear time-to-maturity patterns when information flow is heterogeneous unlike Anderson and Danthine. Although one could argue that these models are not mutually exclusive. Volatility can increase when uncertainty is resolved any time during the life of the contract. In addition, returns can be much more elastic or sensitive to the resolution of uncertainty near expiration. For example, 50 basis points fall in interest rates one month before expiration can have as much effect on the futures price as a 200 basis points fall six months before. Hong explains that the Samuelson effect is actually like an elasticity effect, and gives the example of a one-year silver futures. Suppose there is a mean-reverting, negative supply shock today with a half-life of two months. Holding all else the same, the spot price of silver will rise today but the one-year futures price is mostly unaffected since much of the shock dies away before its maturity date. If the same shock would occur one month before, the shock will not have died away by the maturity date, and thus would have a much greater price elasticity effect.

2.2 Empirical Papers on the Time-To-Maturity Effect in Financial Futures

Milonas (1986) found general support for the maturity effect for 10 commodities out of 11, three of which were financial assets and the rest agricultural commodities. For the three interest rate futures he found evidence although somewhat weaker than for the agricultural and metal futures.

For currency futures Han et al. (1999) and Galloway and Kolb (1996) do not find maturity effect in major currency futures with respect to both standard deviation and number of price changes. Galloway and Kolb (1996) conclude that they found support for SH for commodities with seasonal supply or demand such as agricultural commodities but also noted that it would not hold for commodities for which the cost-of-carry model works well.[9]

Another approach to studying the volatility of futures prices is via the basis, which is defined as the current cash price of a particular commodity minus the price of a particular futures contract for the same commodity, as in the following equation:

$$B = P - F$$

where B is the basis, P is the current spot price, and F is the futures price. Usually the futures price is the price of the nearby futures contract. However, there is a basis for each outstanding futures contract, and this basis will vary for contracts with different maturities.

[9] See Working (1949).

Beaulieu (1998) tests the maturity effect, indirectly using the basis instead of the futures prices alone, in two stock market equity indices and she does find supportive evidence consistent with the Samuelson's (1965) hypothesis. Only three months of data for each contract are considered rather than the more commonly used continuous futures series artificially linking nearby contracts. The sample period includes data from September 30, 1985 to December 31, 1991. The paper utilizes the GARCH model to estimate the model of the basis since there is heteroscedasticity and leptokurtosis present in the basis like in other financial series. The results indicate that maturity effect exists; the size of the variance of the basis decreases as the futures contract approaches expiration. The results are robust across the two equity indices and across time to maturity specification.

Chen, Duan, and Hung (1999) look at the Samuelson effect and compare hedge ratios under scenarios with and without maturity effect in equity index futures and test the Nikkei-225 empirically. The data of daily Nikkei-225 index spot and futures series traded on the Osaka exchange start November 24, 1988 and end June 6, 1996. Their finding of decreasing volatility as maturity approaches contradicts the Samuelson's (1965) hypothesis. However, they find also that optimal hedging and its effectiveness depend on maturity and GARCH effects.

2.3 Market Depth and Information Flows

Market depth might be another factor affecting the degree of sensitivity of volatility to levels of trading volume. Kyle (1985), in his theoretical model, proposes that market depth helps to create more favorable conditions, speed transactions and reduce price pressures when trading provides new information. In a more precise definition Kyle suggested that market depth is the order flow required to move prices by one unit. As order flow changes, open interest also changes endogeneously, thus it makes it a good measure of market depth.

Bessembinder and Seguin (1992, 1993) find that market depth, measured by open interest, has inverse relationship with volatility, as market depth increases volatility decreases. They also conclude that information flow, measured by trading volume, has a positive relationship with volatility – as trading volume increases, volatility also increases. Thus market depth and trading volume have opposite relationships with return volatility. They argue that open interest is a good proxy for market depth because open interest reflects the current willingness of futures investors to risk their capital in futures contracts, which is an indicator of market depth. Bessembinder and Seguin (1993) use the Schwert(1990) procedure for computing unbiased estimates of the conditional daily return standard deviations which takes into account the heteroscedasticity and leptokurtosis present in the futures data series. They examine eight futures markets, two from four different sectors: 2 currencies, 2 metals, 2 agricultural, 2 financial futures series. The sample period is from May 1982 to March 1990 from several futures data sources. The study finds strong positive contemporaneous relationship between trading volume and return volatility, as well as, a new finding that the unanticipated volume shocks raise return volatility two to 13 times more than expected volume shocks. They also show, consistent with previous studies, that market depth affects return volatility. Market depth, constructed by lagged open interest, decreases when actual order flows are different from anticipated order flows. They also look at open interest, a proxy for the number of traders at the beginning of a trading session, and find that it has a negative relationship with trading volume in all markets, which is consistent with the idea that changes in open interest reflect changes in market depth.

Fung and Patterson (2001) integrated two branches of the return volatility and trading volume literature and examine the effect of market depth in addition to the relationship between return volatility and trading volume. The data, obtained from the Futures Industry Institute, consists of five currency futures and two interest rate futures for the sample period June 1982 to March 1994. Fund and Patterson find that market depth had the strongest relationship with return volatility when the trading volume was high, and this was mainly through its interaction with trading volume. The negative impact of market depth on volatility is relatively marginal and is dependent on its interaction with trading volume. They are also able to show that when the market is characterized by low trading volume, the return volatility of nontrading-periods exceeds the return volatility of the trading period. Thus they conclude that the nontrading information flow mostly from offshore has a greater impact on return volatility in low-volume markets and suggest that this provides evidence of greater financial market integration.

Karpoff (1987), in his review of the price-volume literature, establishes that empirically there is a positive relationship between trading volume and magnitude of the price change, and to the price change itself. He points out that mixture distribution hypothesis[10] is supported by the price-volume tests and that price series seems to be generated by a conditional stochastic process with a changing variance parameter that can be proxied by trading volume. Lamoureux and Lastrapes (1990) explore this hypothesis that the ARCH process is capturing the time series properties of the mixing variable, that is, the trading volume using daily return and volume data for 20 actively traded stocks. They conclude that ARCH effects disappear when trading volume is included as an explanatory variable in the conditional variance equation.

Cornell (1981), Grammatikos and Saunders (1986) also find evidence of a positive volume-volatility relationship in futures contracts, the lead and lagged trading volume is insignificant. Wang & Yau (2000) look at futures prices specifically and find evidence of positive relationship between

[10] The mixture distribution hypothesis states that the daily price changes are sampled from a set of distributions that are characterized by different variance.

price volatility and trading volume, and negative relationship with lagged trading volume. The intuition behind their result is that as trading volume increases, there is more opportunity for prices to move into higher or lower levels. On the other hand, the intuition works the other way as well - there is more opportunity for the market to offset the undesirable positions of their inventories and, hence, reduce the price risk and thus observe lower volatility.

3 Data Description

The sample of daily futures data traded on the Chicago Mercantile Exchange spans the period 4 January 1982 through 31 December 2000 (or shorter if the commodity futures market did not exist yet or as long)[11]. There are 11 commodities from 3 groups of futures markets: 1.) Foreign Exchange: Australian dollar, British pound, Canadian dollar, German mark, Japanese yen, and Swiss franc; 2.) Equity Indices: Nikkei-225, S&P midcap, and S&P 500; 3.) Interest Rates: 3-month Eurodollar, and 90-day T-bill. A more detailed description of the futures contracts data is in Table 1. The variables of interest are the settlement price, the open interest and the total trading volume. The settlement price is used instead of the close price since they are usually identical, but when they are not, the settlement price may be a more accurate representation of the current market price.

As futures prices are typically collected daily during the life of the contract, the individual futures series are limited to the lifespan of the contract which is usually less than 24 months. This complicates testing hypothesis of the underlying structure of futures prices. In order to expand futures prices time series, many studies link price series of individual contracts through time to form a longer artificial price history.[12] To obtain a spliced series of each type of futures contract, following the most common practice, I track a particular contract until the last day of the pre-expiration month, at which point the series switch to the next nearby contract. The constructed series starts three months before the expiration of the first contract, then switches to the next contract a day before the last month begins. I do not include the observations during the maturity month to avoid biases caused by the unusual market activities near maturity. The period included from each contract is the mostly highly traded period, with high open interest, and high trading volume also, and it imitates well the actions of a market participant who decides not to complete the transaction but instead to roll over to the next contract when maturity nears.

In the GARCH estimations I use futures returns, which are obtained by taking the difference of log of futures prices, $R_t = \ln(F_t / F_{t-1})$, where F_t represents the settlement price on day t. The settlement price is the official daily close price, typically set at last trade price of the day or the mid

[11] The data are obtained from the Futures Industry Institute, Washington D.C.

point of closing range, the range of prices during a period designated as the official close by the settlement committee.

The other two economic variables, open interest and trading volume, used in the analysis are also generated by the futures markets. Open interest is the total number of futures contracts outstanding for each maturity month, in other words, the number of futures contracts for which delivery is currently obligated. When trading begins and one contract is bought, it also means that another agent sold, and this creates one contract of open interest. Thus open interest is a measure of liquidity in the market. Trading volume is the number of contracts traded for each delivery month during the trading period and is a measure of market activity. Time-to-maturity variable measures the number of days to maturity date. Since the delivery date is flexible and can take place anytime during the maturity month, I defined the last day of the maturity month as zero days to maturity. The days to maturity ranges from 90 days to 30 days since the maturity month is not considered as it might distort the results as discussed above.

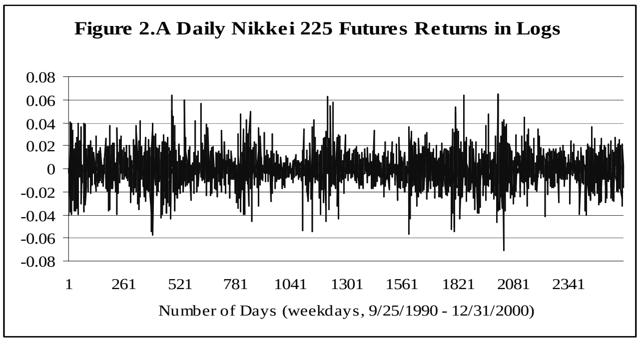
3.1 Descriptive Statistics

Table 1 describes the futures contracts for the 11 financial series used in this study. Table 2.A summarizes the descriptive statistics of futures series, futures return series, total trading volume, and open interest. The futures return series are characterized by higher peakedness and fat tails relative to a normal distribution, they are leptokurtic, which means dramatic movements in futures returns occur with greater frequency than is predicted by the normal distribution. As an example, Figure 2.A below plots the daily Nikkei 225 futures returns for the period 25 September 1990 to 31 December 2000, which is also representative of the other futures return series. Clearly, the time series in not homoscedastic, returns are not independent identically distributed (i.i.d.) through time, but are characterized by periods of tranquility followed by periods of more turbulent movements in future returns. Such volatility clustering is also found in other financial time series. At the same time, in Figure 2.B. below, there appears to be little or no serial dependence in the level of futures prices. These visual observations are also borne out by more formal tests for serial correlation.

Table 2.B presents the formal Ljung-Box (1978) tests of the autocorrelation of futures returns and squared futures returns series. With the exception of three series the residuals have no serial correlation, however, the squared residuals are serially correlated for all the series as suggested by the significant Ljung-Box Q-statistics. For example, the Ljung-Box test for Nikkei 225 for up to the twentieth order of serial correlation in the returns is 32.0, whereas the same test statistic for the serial correlation in the squared returns equals 604.7. These characteristics are empirical regularities of asset

[12] See Ma, Nercer and Walker (1992) for review of rollover methods.

returns, where the second moments are predictable and can be characterized by volatility clustering – the variance of the forecast errors depends on the size of the preceding disturbance. Modeling the series using Autoregressive Conditional Heteroscedasticity model by Engle (1982) or the generalized version by Bollerslev (1986) is more appropriate than standard statistical models since GARCH models can capture the fat-tailed nature of the distribution and, furthermore, allow presence of time-varying volatility.



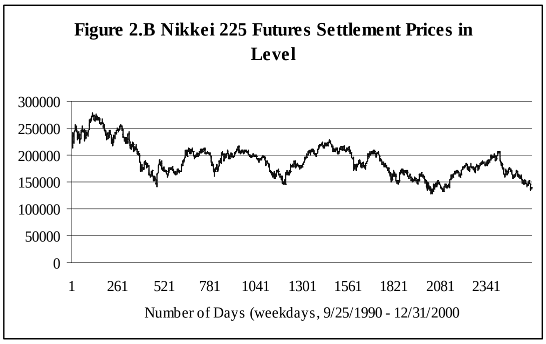


Table 2.C describes the contract specification, measurement unit, mean contract value and the mean daily dollar volume for the futures series in the sample. For example, the daily volume is large for all financial futures, especially for the interest rate futures and the S&P 500, with the Eurodollar interest rate futures in the lead with an average value of \$37 billion daily, approximately 40,000 contracts, followed by the S&P 500 with mean daily value of \$6.9 billion and the Treasury Bills with \$4.5 billion over the full sample period. The trading volume in British pound and Japanese yen are the highest among the currencies futures, near \$1.6 billion average daily value.

4 Methodology

In this section, I outline an approach to measuring the time to maturity effect that extends the previous literature. In particular, I will examine if the conditional variance of futures price changes depends upon time-to-maturity, total trading volume, and open interest after accounting for GARCH effects. In order to do so, it is first necessary to describe the implementation of the GARCH methodology.

4.1 Basic GARCH Specification

GARCH models successfully account for the heteroscedasticity and the leptokurtosis characteristics of financial time series, which characterize these financial futures data as shown in Table 2.A. The ARCH process by Engle (1982) and the GARCH process by Bollerslev (1986) have become standard tools in modeling these empirical features but they have not been utilized much for estimating futures price volatility.

The basic GARCH(p,q) model with ARMA dynamincs in the mean can be written as follows:

$$
\begin{aligned}
&R_t = (\log F_t - \log F_{t-1}) = f_t - f_{t-1} \\
&R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t \\
&\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t) \qquad\qquad (4.1.1) \\
&h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}
\end{aligned}
$$

where F_t, is the futures price, R_t is the futures return, h_t is the conditional variance, α_i is the coefficient of the moving average component of order q, β_j is the autoregressive component of order p. In order to ensure that the conditional variance is never negative, zero, or infinite, it is necessary to constrain α_i and β_j to be between 0 and 1. The model is covariance stationary if and only if all the roots of $\alpha_i(x) + \beta_j(x) = 1$ lie outside the unit circle.[13] As the sum of α_i and β_j approaches unity, a shock to the conditional variance is persistent in the sense that it remains important for future forecasts of all horizons. The degree of persistence depends on the magnitude of these two parameters. Conditioned on an information set at time t, denoted Ω_t, the distribution of the disturbance is assumed to be normal with mean zero and conditional variance h_t. The general model contains an ARCH in mean term, θh_t, which allows the mean of a series to depend on its own conditional variance. The intuition is that risk-averse agents demand compensation for holding risky assets. Thus when variance, a measure of riskiness, of an asset increases, it is necessary that risk premium increases also to induce investor to hold the asset, thus we expect θ to be positive.

Even in linear statistical models, the problem of selecting the appropriate model is non-trivial. Here models for each financial futures series is carefully selected based on the significance of the explanatory variables and the autocorrelation of the standardized residuals and standardized squared residuals using the Ljung-Box Q-statistics. The technique to construct the correlogram for the squared residuals is as follows. First the returns series is estimated and the best fitting ARMA model is selected. The squares of those fitted errors are obtained, and the sample autocorrelations of the squared residuals are calculated. The Ljung-Box statistics can be used to test for groups of significant coefficients. Rejecting the null hypothesis that the squared residuals are not correlated is equivalent to rejecting the null hypothesis of no GARCH errors.

If our model is adequate the autocorrelation of the residuals should be indicative of a white-noise process, thus no evidence of autocorrelation between residuals. The autocorrelation function of the squared residuals can help to identify the order of the GARCH process and the residuals can help

[13] See Bollerslev (1986) for a formal proof.

to find the order of the mean equation; any lag with a high Q-stat, thus low p-value, suggests that an autoregressive term with that lag might be needed to eliminate the observed correlation in the residuals or squared residuals. The goal is to minimize the Q-stats of both the standardized residuals and standardized squared residuals, so that they are indistinguishable from white noise, since then the model adequately captures the futures return mean and variance process.

4.2 Augmented GARCH specification

The basic GARCH specification represented by the set of equations numbered (4.1.1) is augmented by a time-to-maturity variable and two economic variables, open interest and trading volume, in order to determine their relative contribution to the conditional variance. The following equation represents the augmented conditional variance:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt} \qquad (4.2.1)$$

where ϕ_k is the list of weighting series in the heteroscedastic variance, and g is the set of control variables. The non-negativity constraint on ϕ_k is relaxed, so that it can be negative as we would expect in the case of some of the weighting variables, for example, the coefficient of the time-to-maturity variable.

The time-to-maturity variable measures the time in days until the maturity, the last day of the maturity month is defined as zero days to maturity. The sign on time-to-maturity is expected to be negative according to the Samuelson hypothesis – as the number of days left to expiration decreases, the changes in the futures returns increase.

Theory suggests that variables such as total trading volume, open interest, number of transactions, or market liquidity, are related to the return volatility process. An intuitive explanation for the presence for ARCH in the futures returns is based on the hypothesis that daily returns are generated by a mixture of distributions where the mixing variable can be the rate of information arrival. Since information cannot be observed directly the standard proxy for it is total trading volume. This hypothesis has been documented by Lamoureux and Lastrapes (1990) but others like Bessembinder and Seguin (1993), and Wang and Yau (2000) only found contradictory evidence. To test the importance of some of these economic variables in determining the underlying time series characteristics of the conditional variance and the robustness of the maturity effect, I added two of economic variables – open interest and total trading volume – in different combinations, to the conditional variance term. Open interest, defined as the total number of futures or options on futures contracts that have not yet been offset or fulfilled for delivery, is often used as a proxy for market depth or liquidity. High open interest indicates more trades likely in the future, more opportunity for prices to move into higher or lower levels in the future, thus increasing changes in futures returns in the future relative to present movements. To summarize, the sign on the open interest variables has been found to be negative by Bessembinder (1993) and Wang & Yau (2000). The positive correlation between contemporaneous trading volume and return volatility is the result of the majority of research as discussed by Karpoff (1987). The reverse effect is found for lagged trading volume by Wang & Yau (2000). As described above, the intuition behind the positive correlation is that as trading volume increases, there is more opportunity for the prices to move into higher or lower levels.

5 Estimation Results

Table 3 Panel I through VIII present the basic and augmented GARCH(p,q) estimation results using the full sample period for the each type of financial futures contract based on the set of equations (4.1.1) and (4.2.1). Panel I of Table 3 reports the results of the basic GARCH specification omitting all of the economic control variables. Panel II, III, IV augment this basic specification by time to maturity (TTM), trading volume (TV), and open interest (OI), respectively. Panel V reports the results including all three economic variables, while Panels VI, VII, and VIII contain results for various permutations. Table 3.1 contains currency results and Table 3.2 contains the equity index and interest rate results. Briefly, the organization of the Panels can be summarized in the following matrix:

	GARCH	TTM	TV	OI
Panel I	X			
Panel II	X	X		
Panel III	X		X	
Panel IV	X			X
Panel V	X	X	X	X
Panel VI	X	X	X	
Panel VII	X	X		X
Panel VIII	X		X	X

5.1 Estimation Results for the Basic GARCH Specification

Panel I presents the estimation results for the basic GARCH models for each type of futures contract without any explanatory variable added to the conditional variance equation. The optimal GARCH specification varies among different spliced futures contracts but the process seems to be well represented by GARCH models as suggested by the diagnostic checks of the residuals and the squared residuals which appear to be white noise or close to it. There appears to be a pattern across GARCH specifications in financial futures. The GARCH(2,1) specification dominates in the currency futures with some variation in the ARMA terms in the mean equation. In contrast, the GARCH(1,1) is selected for two of the three equity index futures, and for the two interest rate futures with an AR(1) term in three of their mean equation. Only the S&P 500 futures equity index has a significant ARCH in mean term which allows the mean of the series to depend on its own conditional variance.[14] This class of models is suited to study some asset markets where risk averse agents require compensation for holding risky assets. Since the riskiness of an asset can be measured by the variance of returns, the risk premium is an increasing function of the conditional variance of the returns.

5.2 Estimation Results for the Augmented GARCH Specification

For the full sample, the time-to-maturity variable for currency futures is partly negative and partly positive. However, at closer inspection, all the ones with a positive sign are insignificant and those that are negative are all significant, with one borderline significant. This result is in contrast with Galloway and Kolb (1996) and Han et al. (1999), and in some agreement with Leistikow (1987). This might not be the prediction of the Anderson & Danthine (1983) theoretical model if one argues that there are no information flows, thus no supply and demand uncertainties resolved near the expiration of the contracts such that exist in agricultural commodities. However, Hong's model can explain, even in face of heterogeneous information flows, this time-to-maturity pattern that we observe in these financial futures as long as information asymmetry among market participants is small. In other words, Hong's model does not depend on the characteristics of information flows but on the degree of asymmetry of information among agents or relative asymmetry of information endowments and flows. In this model, it is the sensitivity to information flows that can explain the increase in the volatility of futures returns. Near maturity, agents react more to news, hence prices are more elastic. In other words, information is assimilated faster into the markets as reflected by more volatile returns. Agents attempt to quickly incorporate the new and more heavily weighted information into their objective function.

This result conflicts with much of the literature on currency futures but not all, however, I do test a significantly longer data set than all previous studies, and use the GARCH estimation method to try to better account for the characteristics of the data. This pattern of negative coefficient on the time-to-maturity remains for the full specification, when both additional economic variables are in the variance equation, total trading volume and open interest. Thus for currency futures, the Samuelson hypothesis appears to remain robust with respect to additional variables in the conditional variance equation.

The evidence for interest rate futures, by far the biggest market by the number of contracts traded, is that time-to-maturity enters as negative and significant, consistent with existing empirical literature, such as Milonas (1986) and Leistikow (1987), although it is not robust to dropping the open

---

[14] See Engle, Lilien, and Robins (1987).

interest. In contrast, and interestingly, the results for the equity indices stand out in that time-to-maturity does not appear to matter in any specification similar to Chen, Duang, and Hung (1999) results, but unlike Beaulieu (1998) results. In fact, the only thing that appears to matter for these is open interest, except for the Nikkei 225 where total trading volume matters as well. (See Table 3.2)

The GARCH estimation with the two economic variables individually and jointly are in Table 3, Panel II, III, and VIII, respectively. Similar to Bessembinder and Seguin (1993), Wang & Yau (2000) but unlike Lamoureux and Lastrapes (1990), we do not find the GARCH effects disappearing when the trading volume, a common measure of information flow, is included as an explanatory variable in the variance equation, whether on its own or with other variables. Trading volume is significant only about 50% of the time, and it is has a very small magnitude.

Open interest, used to measure market depth or liquidity, has not been examined extensively in the literature. Similarly to trading volume, the signs are mixed, however, only the positively signed coefficients are significant in the case of the currencies. The sign of open interest for equity indices and interest rates is mostly positive when significant, but the pattern is fainter than it is for currencies. This is opposite of the result we expected based on the few studies on open interest, however, intuitively it still has some logical appeal - it implies that the higher the open interest, the more future trade expected, the higher is the current futures price volatility. The more future trade, the more opportunity there is for the prices to move into higher or lower levels, similarly to trading volume.

5.3 Estimation Results for the Sub-samples

The sample is also divided into two periods: 1980s & 1990s. The results are not reported here for brevity but are available upon request. As financial futures markets expanded rapidly throughout the 1990s, with increasing liquidity, one might reasonably conjecture that the idealized conditions of the Samuelson model might be better approximated by actual market conditions. Indeed, it turns out that the hypothesized time-to-maturity effect is more apparent during the 1990's subsample. The results across the two periods are very close for the currency futures, for the equity indices and interest rates less so. As expected, the 1990s futures series give a clearer pattern for all three categories, with a stronger time-to-maturity effect, more mixed results for trading volume, and mostly positive signs for open interest. In the 1980s, all three categories, currency, equity index, and interest rate futures have mixed results, with both positive and negative significant coefficients for open interest and lagged trading volume, which might be partly due to lower levels of activity.

6 Conclusion

This study has examined the dynamics of volatility in several, increasingly important, financial futures markets. Several new patterns have been identified in this unique data set.

First, a strong time-to-maturity effect is detected for currency futures. This variable has a less prominent role in equity index and interest rate futures. The result is somewhat puzzling for the former category, since one does not necessarily expect an increase in information flows near the maturation of currency futures. Second, as markets have become larger and more liquid, it appears that the time-to-maturity hypothesis of Samuelson has become increasingly relevant. This suggests that earlier studies may have failed to find a role for time-to-maturity because markets did not completely fulfill the conditions outlined by Samuelson. Third, one policy implication is that if agents fail to incorporate time-to-maturity in making their hedging decisions, then they may be failing to optimize. Fourth, the empirical modeling of the second moments of futures returns will need to incorporate economic as well as GARCH effects.

One finding at variance with some of the earlier studies is the mixed role of lagged trading volume and open interest for equity index and interest rate futures. However, those earlier studies did not examine such a comprehensive data set; nor did they necessarily account for GARCH effects. Hence, these findings should be considered as establishing new stylized facts.

References

Adrangi, B., and A. Chatrath, 1999, "Margin Requirements and Futures Activity: Evidence from the Soybean and Corn Market," *Journal of Futures Markets*, 19(4), 433-55.

Adrangi, B., and A. Chatrath, K. K. Dhanda, and K. Raffiee, 2001, "Chaos in Oil Prices? Evidence from Futures Markets," *Energy Economics*, 23, 405-25.

Andersen, T. G., 1996, "Return Volatility and Trading Volume: An Information Flow Interpretation of Stochastic Volatility," *Journal of Finance*, 51(1), 169-204.

Anderson, R. W., 1985, "Some Determinants of the Volatility of Futures Prices," *Journal of Futures Markets,* 5(3), 331-48.

Anderson, R. W., and J. Danthine, 1983, "The Time Pattern of Hedging and the Volatility of Futures Prices," *Review of Economic Studies* 50, 249-266.

Antoniou, A., P. Holmes, and R. Priestley, 1998, "The Effects of Stock Index Futures Trading on Stock Index Volatility: An Analysis of the Asymmetric Response of Volatility to News," *Journal of Futures Markets*, 18(2), 151-166.

Beaulieu, M.-C., 1998, "Time to maturity in the basis of stock market indices: Evidence from the S&P 500 and the MMI," *Journal of Empirical Finance,* 5, 177-95.

Bessembinder, H. and P. J. Seguin, 1992, "Futures Trading Activity and Stock Price Volatility," *Journal of Finance*, 47(5), 2015-34.

Bessembinder, H. and P. J. Seguin, 1992, "Price Volatility, Trading Volume, and Market Depth: Evidence from Futures Markets," *Journal of Financial and Quantitative Analysis*, 28(1), 21-39.

Bessembinder, H., J. Coughernour, P. Seguin, and M. Smoller, 1995, "Futures Price Volatility and Spot Price Stationarity: Reevaluating the Samuelson Hypothesis," manuscript.

Black, J. and I.. Tonks, 2000, "Time Series Volatility of Commodity Futures Prices," *Journal of Futures Markets* forthcoming.

Bollerslev, T., 1986, "Generalized autoregressive conditional heteroscedasticity," *Journal of Econometrics*, 31, 307-27.

Bollerslev, T., R.Y. Chou, and K.F. Kroner, 1992, "ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence," *Journal of Econometrics*, 52, 5-59.

Bracker, K., and K. L. Smith, 1999, "Detecting and Modeling Changing Volatility in the Copper Futures Market," *Journal of Futures Markets*, 19(1), 79-100.

Castelino, M., and J. Francis, 1982, "Basis Speculation in Commodity Futures: the Maturity Effect," *Journal of Futures Markets*, 2(2), 195-206.

Chatrath, A., and F. Song, 1998, "Information and Volatility in Futures and Spot Markets: The Case of the Japanese Yen," *Journal of Futures Markets*, 18(2), 201-23.

Chen, Y.J., J.C. Duan, and M.W. Hung, 1999, "Volatility and Maturity Effects in the Nikkei Index Futures," *Journal of Futures Markets*, 19(8), 895-909.

Chu, S.H., and S. Freund, 1994, "Volatility Estimation for Stock Index Options: A GARCH Approach," *Quarterly Review of Economics and Finance*, 431-50.

Clark, P.K., 1973, "A Subordinated Stochastic Process Model with Finite Variances for Speculative Prices," *Econometrica*, 41, 135-55.

Copeland, T.E., 1976, "A Model of Asset Trading under the Assumption of Sequential Information Arrival," *Journal of Finance*, 31, 1149—68.

Cornell, B., 1981, "The relationship between volume and price variability in futures markets," *Journal of Futures Markets*, 1(3), 304-16.

Cox, C.C., 1976, "Futures Trading and Market Information," *Journal of Political Economy*, 84, 1215-37.

DeCoster, G.P., W.C. Labys, and D.W. Mitchell, 1992, "Evidence of Chaos in Commodity Futures Prices," *Journal of Futures Markets*, 12(3), 291-305.

Engle, R., 1982, "Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation," *Econometrica*, 50, 987-1007.

Engle, R., D. Lilien, and R. Robins, 1987, "Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M model," Econometrica, 55, 391-407.

Epps, T.W., and M.L. Epps, 1976, "The Stochastic Dependence of Security Price Changes and Transaction Volumes: Implications for the Mixture-of-Distribution Hypothesis," *Econometrica*, 44, 305-25.

Fujihara, R.A., and M. Mougoue, 1997, "An Examination of Linear and Nonlinear Causal Relationships between Price Variability and Volume in Petroleum Futures Markets," *Journal of Futures Markets*, 17(4), 385-416.

Fung, H.G., and G.A. Patterson, 2001, "Volatility, Global Information, and Market Conditions: A Study in Futures Markets," *Journal of Futures Markets*, 21(2), 173-96.

Galloway, T. M., and R.W. Kolb, 1996, "Futures Prices and the Maturity Effect," *Journal of Futures Markets*,16(7), 809-28.

Garman, M.B., and M. J. Klass, 1980, "On the Estimation of Security Price Volatilities from Historical Data," *Journal of Business*, 53(1), 67-78.

Grammatikos, T., and A. Saunders, 1986, "Futures Price Variability: A Test of Maturity and Volume Effects," *Journal of Business*, 59:319-30.

Harvey, C. R. and R.D. Huang, 1991, "Volatility in the Foreign Currency Futures Market," *Review of Financial Studies*, 4(3), 543-69.

Harris, L., 1986, "Cross-Security Tests of the Mixture of Distribution Hypothesis," *Journal of Financial and Quantitative Analysis,* 21, 39-46.

Herbert, J. H., 1995, "Trading Volume, maturity, and natural gas futures price volatility," *Energy Economics*, 17(4), 293-99.

Hennessy, D. and T. Wahl, 1996, "The effects of decision making on futures price volatility," *American Journal of Agricultural Economics,* 78(3), 591-604.

Han, L., J. Kling, and C. Sell, 1999, "Foreign Exchange Futures Volatility: Day-of-the-Week, Intraday, and Maturity Patterns in the Presence of Macroeconomic Announcements," *Journal of Futures Markets*, 19(6), 665-693.

Hong, H., 2000, "A Model of Returns and Trading in Futures Markets," *Journal of Finance*, 55(2), 959-988.

Hudson, D. and K. Coble, 1999, "Harvest Contract Price Volatility for Cotton," The *Journal of Futures Markets,* 19(6), 717-733.

Jennings, R.H., and C. Barry, 1983, "Information Dissemination and Portfolio Choice," *Journal of Financial and Quantitative Analysis,* 18, 1-19.

Jones, C. M., G. Kaul, and M. L. Lipson, 1994, "Transactions, Volume, and Volatility," *Review of Financial Studies*, 7(4), 631-51.

Karpoff, J. M., 1987, "The Relation between Price Changes and Trading Volume: A Survey," *Journal of Financial and Quantitative Analysis* 22, 109-26.

Kenyon, D., K. Kling, J.Jordan, W. Seale, and N. McCabe, 1987, "Factors Affecting Agricultural Futures Price Variance," *Journal of Futures Markets,* 7(1), 73-91.

Khoury, N., and P. Yourougou, 1993, "Determinants of Agricultural Futures Prices Volatilities: Evidence from Winnipeg Commodity Exchange," *Journal of Futures Markets*, 13(4), 345-56.

Kolb, R., 1985, *Understanding Futures Markets*, London: Scott, Foresman and Company.

Kyle, A.S., 1985, "Continuous Auctions and Insider Trading," *Econometrica,* 53, 1315-35.

Lamoureux, C.G., and W.D. Lastrapes, 1990, "Heteroscedasticity in Stock Return Data: Volume versus GARCH Effects," *Journal of Finance*, 45, 221-30.

Lestikow, D., 1989, "Announcements and Futures Price Variability," *Journal of Futures Markets*, 9(6), 477-486.

Ljung, G., and G. Box, 1978, "On a Measure of Lack of Fit in Time Series Models," *Biometrica*, 65, 297-303.

Ma, K.C., J.M. Mercer, and M.A. Walker, 1992, "Rolling Over Futures Contracts: A Note," *Journal of Futures Markets*, 12(2), 203-217.

Milonas, N. 1986, “Price Variability and the Maturity Effect in Futures Markets,” *Journal of Futures Markets*, 6, 443-460.

Montalvo, J.G., 1999, “Volume Versus GARCH Effects Reconsidered: An Application to the Spanish Government Bond Futures Market,” *Applied Financial Economics,* 9, 469-75.

Morse, D., 1981, “Price and Trading Volume Reaction Surrounding Earnings Announcements: A Closer Examination,” *Journal of Accounting Research,* 19, 374-83.

Nelson, D.J., 1991, “Conditional Heteroscedasticity in Asset Returns: A New Approach,” *Econometrica*, 59, 347-70.

Richard, S., and M. Sundaresan, 1981, “A Continuous Time Equilibrium Model of Forward Prices and Futures Prices in a Multigood Economy,” *Journal of Financial Economics*, 9, 347-371.

Rutledge, D.J.S., 1976, "A Note on the Variability of Futures Prices," *Review of Economics and Statistics*, 58,118-20.

Samuelson, P. A., 1965, “Proof that properly anticipated prices fluctuate randomly,” *Industrial Management Review*, 6, 41-63.

Samuelson, P. A., 1976, “Is Real-World Price a Tale Told by the Idiot of Chance?” *Review of Economics and Statistics*, 58, 120-123.

Schwert, G.W., 1990, “Stock Volatility and the Crash of ’87,” *Review of Financial Studies*, 3, 77-102.

Stein, J., 1979, “Spot, Forward, and Futures,” *Research in Finance*, 1, 225-310.

Stoll, H. R., and R.E. Whaley, 1990, “Stock Market Structure and Volatility,” *Review of Financial Studies*, 3(1), 37-71.

Streeter, D., and W. G. Tomek, 1992, "Variability in Soybean Futures Prices: An Integrated Framework," *The Journal of Futures Markets*, 12 (6), 705-728.

Tauchen, G.E. and M.Pitts, 1983, “The Price Variability-Volume Relationship on Speculative Markets,” *Econometrica*, 51, 485-505.

Wang, G., J. Yau, 2000, “Trading Volume, Bid-Ask Spread, and Price Volatility in Futures Markets,” *Journal of Futures Markets*, 20(10), 943-970.

Working, H. 1949, “The Theory of Price Storage,” *American Economic Review*, 12, 1254-1262.

Table 1. Description of Futures Prices Data from the Chicago Mercantile Exchange

Commodity Description	NOBS	Years available	Start Trade Date	Start Date of Contract Delivery	Ending Date of Contract Delivery	Number of Contracts	Possible Delivery Month
Currencies (FX)							
Australian dollar (AD)	3534	1987-2000 (14)	1/13/1987	03/87	12/01	60	3,6,9,12
British pound (BP)§	4598	1982-88, 1988-2000 (19)	1/6/1982	03/82	12/00	77	3,6,9,12
Canadian dollar (CD)	4725	1982-2000 (19)	1/4/1982	03/82	3/02	81	3,6,9,12
German mark (DM)	4785	1982-2000 (19)	1/4/1982	03/82	06/01	79	3,6,9,12
Japanese Yen (JY)	4810	1982-2000 (19)	1/4/1982	01/82	12/01	84	3,6,9,12
Swiss franc (SF)	4502	1982-2000 (19)	1/4/1982	03/82	12/01	72	3,6,9,12
Equity Indices (EQ)							
S&P Midcap 400 (MD)	2247	1992-2000 (9)	2/13/1992	03/92	6/01	38	1,2,3,4,5,6,7,8,9,12
Nikkei 225 (NK)	2598	1990-2000 (11)	9/25/1990	12/90	6/01	42	3,6,9,12
S&P 500 (SP)	4718	1982-2000(19)	4/21/1982	06/82	9/02	83	3,6,9,12
Interest Rates (IR)							
Eurodollar (ED)	4812	1982-2000(19)	1/4/1982	01/82	12/01	159	3,6,9,12
90-Day Treasury Bills (TB)	4605	1982-2000(19)	1/4/1982	01/82	06/00	76	1,3,4,6,9, 10,12

§ British Pound (BP) changed to the New British Pound (NB) in 1988.

Table 2.A Descriptive Statistics of Futures, Futures Returns, Total Trading volume, Open Interest

	AUSTRALIAN DOLLAR (No.of obs: 3530)				BRITISH POUND (No.of obs: 4595)				CANADIAN DOLLAR (No.of obs: 4722)			
	Futures Series	**Futures Return Series**	**Total Trading volume**	**Open Interest**	**Futures Series**	**Futures Return Series**	**Total Trading volume**	**Open Interest**	**Futures Series**	**Futures Return Series**	**Total Trading volume**	**Open Interest**
Mean	71915.9	-0.00004	1040	8580	15960.8	-4E-05	9150	28700	76256.6	-4.9E-05	4590	26600
Std Dev	6732.3	0.00625	1480	8750	1565.2	0.00707	5490	13600	6093.1	0.00305	3710	18900
Min	51120	-0.037	0	0	10445	-0.045	0	0	63250	-0.021	0	0
Max	88720	0.0517	22100	45000	19890	0.0455	47600	76600	88980	0.0199	33700	82300
Skewness	- 0.61182	-0.339	3.915	1.265	- 0.33394	0.012	1.509	0.566	0.12384	-0.224	1.91	0.91
Kurtosis	0.04589	4.101	26.431	1.111	0.87892	3.528	4.19	-0.099	- 0.99887	3.998	5.814	-0.058

	GERMAN MARK (No.of obs: 4782)				JAPANESE YEN (No.of obs: 4802)				SWISS FRANC (No.of obs: 4499)			
	Futures Series	**Futures Return Series**	**Total Trading volume**	**Open Interest**	**Futures Series**	**Futures Return Series**	**Total Trading volume**	**Open Interest**	**Futures Series**	**Futures Return Series**	**Total Trading volume**	**Open Interest**
Mean	54022.2	1.54E-05	23000	51000	73973.5	0.000136	17200	49200	64625.3	2.74E-05	16600	32000
Std Dev	10412.5	0.00714	16100	33700	20221.3	0.00739	10800	26800	12471.2	0.00799	7760	14800
Min	29060	-0.033	0	0	36100	-0.042	0	0	34290	-0.04	0	0
Max	74130	0.0483	102000	170000	124910	0.0827	82600	145000	90030	0.0497	59500	102000
Skewness	- 0.49176	0.19	0.74	0.795	- 0.30557	0.7	1.08	0.536	- 0.41755	0.189	0.521	0.787
Kurtosis	- 0.66155	2.111	0.494	0.488	- 0.73244	7.149	1.774	0.061	- 0.49358	1.885	0.712	1.065

The sample includes daily data traded on the Chicago Mercantile Exchange from January 4, 1982 to December 29, 2000.

(continued on next page)

Table 2.A (Continued) Descriptive Statistics of Futures, Futures Returns, Total Trading volume, Open Interest

	S&P MIDCAP 400 (No. of obs: 2244)				NIKKEI 225 (No. of obs: 2595)				S&P 500 (No. of obs: 4715)			
	Futures Series	Futures Return Series	Total Trading volume	Open Interest	Futures Series	Futures Return Series	Total Trading volume	Open Interest	Futures Series	Futures Return Series	Total Trading volume	Open Interest
Mean	27483.3	0.00055	783	9920	189853	-0.00018	1350	17600	52071.3	0.00052	53400	146000
Std Dev	11427.7	0.0109	547	4100	29793.4	0.0152	1000	7390	38887.9	0.0123	29200	108000
Min	13650	-0.095	0	42	127500	-0.071	0	0	10205	-0.337	0	0
Max	55800	0.0548	5230	17800	277650	0.0647	8460	36900	155540	0.177	218000	469000
Skewness	0.69058	-0.515	1.991	-0.725	0.54582	0.062	2.312	-0.071	1.25129	-3.926	0.689	0.982
Kurtosis	-0.71411	5.776	6.478	-0.345	0.02184	1.639	7.545	-0.264	0.36347	132.999	1.046	0.223

	EURODOLLAR (No. of obs: 4812)				90-DAY T-BILL (No. of obs:4605)			
	Futures Series	Futures Return Series	Total Trading volume	Open Interest	Futures Series	Futures Return Series	Total Trading volume	Open Interest
Mean	928566	2.1E-05	40336.5	179817	937700	1.6E-05	4755.55	18140.4
Std Dev	25712.4	0.0012	44418.7	163224	21687.5	0.00114	5397.41	10406.6
Min	832900	-0.02355	0	0	856300	-0.02353	0	0
Max	969900	0.01272	357463	618679	973500	0.01396	33954	48737
Skewness	-1.04955	-1.99911	1.90829	0.62605	-0.93884	-1.95183	1.98769	0.03599
Kurtosis	1.33384	56.4456	5.23695	-0.87079	0.91041	60.0781	4.49097	-0.76665

The sample includes daily data traded on the Chicago Mercantile Exchange from January 4, 1982 to December 29, 2000.

Table 2.B Summary Statistics on Futures Returns

Autocorrelation of Futures Returns

	AD **Australian dollar**	BP **British pound**	CD **Canadian dollar**	DM **German mark**	JY **Japanese yen**	SF **Swiss franc**
Ljung-Box test						
Q(1)	0.06	0.44	11.28**	0.00	0.11	0.00
	(0.810)	(0.508)	(0.001)	(0.951)	(0.741)	(0.999)
Q(5)	13.32**	0.91	19.56**	2.53	0.87	3.08
	(0.021)	(0.970)	(0.002)	(0.773)	(0.973)	(0.687)
Q(10)	25.68**	3.33	22.53**	9.01	6.54	7.82
	(0.004)	(0.973)	(0.013)	(0.531)	(0.768)	(0.646)
Q(20)	32.88**	17.10	53.21**	24.74	22.65	15.07
	(0.035)	(0.646)	(0.000)	(0.212)	(0.307)	(0.772)

Autocorrelation of Squared Futures Returns

	AD **Australian dollar**	BP **British pound**	CD **Canadian dollar**	DM **German mark**	JY **Japanese yen**	SF **Swiss franc**
Ljung-Box test						
Q2(1)	19.70**	48.53**	111.75**	24.31**	43.23**	33.52**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Q2(5)	58.94**	253.86**	332.68**	94.38**	100.43**	69.11**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Q2(10)	111.63**	477.69**	418.16**	224.46**	124.81**	191.98**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Q2(20)	142.49**	823.30**	608.97**	335.41**	204.49**	278.66**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

The sample period includes daily data from January 4, 1982 to December 29, 2000. Q(q) where q=1,5,10 and 20 is the Ljung-Box statistics for testing the joint significance of autocorrelations of returns for the first q lags. Q2(q) is the same statistic for testing the joint significance of autocorrelations of squared returns. Under the null hypothesis of zero correlations, each of the Q-statistics is distributed as a chi-square variable with q degrees of freedom. (p-value in parentheses). * and ** denote statistical significance at 10% and 5% respectively.

(continued on next page)

Table 2.B (Continued) Summary Statistics on Futures Returns

Autocorrelation of Futures Returns

	MD	NK	SP	ED	TB
	S&P Midcap 400	**Nikkei 225**	**S&P 500**	**Euro-dollar**	**90-day T-bill**
Ljung-Box test					
Q(1)	0.25	10.74**	5.15**	10.97**	7.17**
	(0.616)	(0.001)	(0.023)	(0.001)	(0.007)
Q(5)	11.79**	14.72**	56.66**	37.65**	19.51**
	(0.038)	(0.012)	(0.000)	(0.000)	(0.002)
Q(10)	28.46**	27.60**	69.45**	56.27**	56.04**
	(0.002)	(0.002)	(0.000)	(0.000)	(0.000)
Q(20)	57.76**	31.96**	75.76**	95.01**	82.95**
	(0.000)	(0.044)	(0.000)	(0.000)	(0.000)

Autocorrelation of Squared Futures Returns

	MD	NK	SP	ED	TB
	S&P Midcap 400	**Nikkei 225**	**S&P 500**	**Euro-dollar**	**90-day T-bill**
Ljung-Box test					
Q2(1)	92.07**	55.79**	33.48**	7.26**	15.10**
	(0.000)	(0.000)	(0.000)	(0.007)	(0.000)
Q2(5)	251.16**	236.08**	382.92**	102.73**	55.45**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Q2(10)	395.93**	428.63**	396.36**	203.44**	136.20**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Q2(20)	595.16**	604.69**	398.39**	354.76**	268.93**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

The sample period includes daily data from January 4, 1982 to December 29, 2000. Q(q) where q=1,5,10 and 20 is the Ljung-Box statistics for testing the joint significance of autocorrelations of returns for the first q lags. Q2(q) is the same statistic for testing the joint significance of autocorrelations of squared returns. Under the null hypothesis of zero correlations, each of the Q-statistics is distributed as a chi-square variable with q degrees of freedom. (p-value in parentheses). * and ** denote statistical significance at 10% and 5% respectively.

Table 2.C Characteristics of Futures Markets in Sample[a]

	Deliverable Good	Contract Unit	Mean Futures Price	Mean Contract Value ($)[b]	Mean Daily Dollar Volume ($ Millions)[c]
FX	AD	100000 Australian dollars	0.71915000	71,915	75
	BP	62,500 British pounds	1.59600000	99,750	913
	CD	100,000 Canadian dollars	0.76256000	76,256	350
	DM	125,000 German marks	0.54022000	67,528	1,553
	JY	12,500,000 Japanese yens	0.00739730	92,466	1,590
	SF	125,000 Swiss franks	0.64625000	80,781	1,341
EQ	MD	$500 times MD index	274.830	137,415	108
	NK	$5 times NK average	18985.299	94,926	128
	SP	$250 times SP index	520.713	130,178	6,952
IR	ED	$1,000,000 face value	0.92856644	928,566	37,456
	TB	$1,000,000 face value	0.93770000	937,700	4,460

[a]All futures contracts in sample are traded on the Chicago Mercantile Exchange.
[b]Contract size times mean nearby futures price (Jan 4, 1982 to Dec 29, 2000)
[c]Contract size times mean nearby futures price times mean volume (Jan 4, 1982 to Dec 29, 2000)

Table 3.1 Maximum Likelihood Estimates of GARCH(p,q) Models

Panel I	**CURRENCIES**					
	AD	**BP**	**CD**	**DM**	**JY**	**SF**
	Australian dollar	**British pound**	**Canadian dollar**	**German mark**	**Japanese yen**	**Swiss franc**
	AR(3)-GARCH (2,1)	GARCH (2,1)	AR(1),AR(2), AR(3)-GARCH (2,1)	AR(3)-GARCH (1,1)	GARCH (2,1)	GARCH (2,1)
c	-0.000028519		-0.000059309	-0.00004439		
	(0.765)		(0.098)*	(0.638)		
γ_1			0.045494			
			(0.004)**			
γ_2			-0.025404			
			(0.084)**			
γ_3	-0.056074		-0.036542	0.021213		
	(0.001)**		(0.015)**	(0.162)		
γ_4						
γ_5						
θ						
α_0	3.55E-07	3.42E-07	1.13E-07	1.00E-06	2.12E-06	1.55E-06
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_1	0.036853	0.043177	0.086082	0.048088	0.092635	0.053398
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_2						
β_1	0.53815	0.53636	0.32821	0.93274	0.21513	0.48801
	(0.034)**	(0.004)**	(0.000)**	(0.000)**	(0.000)**	(0.007)**
β_2	0.41625	0.41329	0.57604		0.65558	0.43428
	(0.092)*	(0.023)**	(0.000)**		(0.000)**	(0.012)**
φ_{TTM}						
$\varphi_{TV(-1)}$						
φ_{OI}						

NOTE: The sample period is 1 Jan. 1982 through 31 Dec. 2000 (except for AD, NK, MD). * & ** denote statistical significance at 10% & 5% respectively. P-values are in parenthesis. The following GARCH estimation is used to generate the results in this table:

$$R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt}$$

Table 3.1 Maximum Likelihood Estimates of GARCH(p,q) Models

Panel II	CURRENCIES					
	AD	**BP**	**CD**	**DM**	**JY**	**SF**
ttm	**Australian dollar**	**British pound**	**Canadian dollar**	**German mark**	**Japanese yen**	**Swiss franc**
	AR(3)-GARCH (2,1)	GARCH (2,1)	AR(1),AR(2), AR(3)-GARCH (2,1)	AR(3)-GARCH (1,1)	GARCH (2,1)	GARCH (2,1)
c	-0.00001891		-0.000043837	-0.000043116		
	(0.843)**		(0.228)	(0.648)		
γ_1			0.046982			
			(0.003)**			
γ_2			-0.024623			
			(0.098)*			
γ_3	-0.055513		-0.036261	0.021545		
	(0.001)**		(0.015)**	(0.155)		
γ_4						
γ_5						
θ						
α_0	1.10639E-06	3.40E-07	3.73E-07	1.59E-06	1.88E-06	2.20E-06
	(0.000)**	(0.037)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_1	0.040542	0.043226	0.084035	0.049014	0.093048	0.05333
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_2						
β_1	0.59104	0.53645	0.40168	0.92991	0.2131	0.51719
	(0.022)	(0.004)	(0.000)**	(0.000)**	(0.000)**	(0.006)**
β_2	0.35491	0.41316	0.50026		0.65733	0.40393
	(0.152)	(0.023)**	(0.000)**		(0.000)**	(0.024)**
φ_{TTM}	-8.01E-09	4.20E-11	-3.13E-09	-6.73E-09	3.22E-09	-7.85E-09
	(0.003)**	(0.983)	(0.000)**	(0.048)**	(0.483)	(0.173)
$\varphi_{TV(-1)}$						
φ_{OI}						

NOTE: The sample period is 1 Jan. 1982 through 31 Dec. 2000 (except for AD, NK, MD). * & ** denote statistical significance at 10% & 5% respectively. P-values are in paranthesis. The following GARCH estimation is used to generate the results in this table:

$$R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt}$$

Table 3.1 Maximum Likelihood Estimates of GARCH(p,q) Models

Panel III	**CURRENCIES**					
	AD	**BP**	**CD**	**DM**	**JY**	**SF**
tv(-1)	**Australian dollar**	**British pound**	**Canadian dollar**	**German mark**	**Japanese yen**	**Swiss franc**
	AR(3)-GARCH (2,1)	GARCH (2,1)	AR(1),AR(2), AR(3)-GARCH (2,1)	AR(3)-GARCH (1,1)	GARCH (2,1)	GARCH (2,1)
c	-0.000027577		-0.000062324	-0.000045182		
	(0.776)		(0.092)**	(0.633)		
γ_1			0.048744			
			(0.003)**			
γ_2			-0.02355			
			(0.120)			
γ_3	-0.056187		-0.035047	0.02122		
	(0.001)**		(0.023)**	(0.161)		
γ_4						
γ_5						
θ						
α_0	2.86E-07	4.35E-07	7.04E-08	9.49E-07	1.78E-06	1.29E-06
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_1	0.036744	0.04323	0.088893	0.048096	0.095933	0.052463
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_2						
β_1	0.52291	0.53971	0.32912	0.9319	0.20139	0.48167
	(0.043)**	(0.004)**	(0.000)**	(0.000)**	(0.000)**	(0.007)**
β_2	0.43154	0.40932	0.55734		0.65802	0.44096
	(0.086)*	(0.025)**	(0.000)**		(0.000)**	(0.011)**
φ_{TTM}						
$\varphi_{TV(-1)}$	8.34E-11	-8.53E-12	4.40E-11	4.21E-12	4.65E-11	1.86E-11
	(0.007)**	(0.383)	(0.000)**	(0.341)	(0.000)**	(0.339)
φ_{OI}						

NOTE: The sample period is 1 Jan. 1982 through 31 Dec. 2000 (except for AD, NK, MD). * & ** denote statistical significance at 10% & 5% respectively. P-values are in paranthesis. The following GARCH estimation is used to generate the results in this table:

$$R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt}$$

Table 3.1 Maximum Likelihood Estimates of GARCH(p,q) Models

Panel IV	**CURRENCIES**					
	AD	**BP**	**CD**	**DM**	**JY**	**SF**
oi	**Australian dollar**	**British pound**	**Canadian dollar**	**German mark**	**Japanese yen**	**Swiss franc**
	AR(3)-GARCH (2,1)	GARCH (2,1)	AR(1),AR(2), AR(3)-GARCH (2,1)	AR(3)-GARCH (1,1)	GARCH (2,1)	GARCH (2,1)
c	-0.0000231		-0.000059719	-0.000044588		
	(0.812)		(0.104)	(0.637)		
γ_1			0.048117			
			(0.003)**			
γ_2			-0.025			
			(0.099)**			
γ_3	-0.056199		-0.035677	0.021214		
	(0.001)**		(0.020)**	(0.162)		
γ_4						
γ_5						
θ						
α_0	3.24E-07	5.75E-07	6.41E-08	9.89E-07	1.69E-06	1.79E-06
	(0.000)**	(0.001)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_1	0.039248	0.043959	0.087917	0.048128	0.10466	0.053325
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_2						
β_1	0.49956	0.52874	0.38006	0.93275	0.19965	0.49034
	(0.041)**	(0.004)**	(0.000)**	(0.000)**	(0.000)**	(0.007)**
β_2	0.4498	0.41748	0.51184		0.64184	0.43049
	(0.058)*	(0.020)**	(0.000)**		(0.000)**	(0.013)**
φ_{TTM}						
$\varphi_{TV(-1)}$						
φ_{OI}	1.83E-11	-4.18E-12	6.29E-12	1.91E-13	2.89E-11	-4.44E-12
	(0.000)**	(0.121)	(0.000)**	(0.903)	(0.000)**	(0.404)

NOTE: The sample period is 1 Jan. 1982 through 31 Dec. 2000 (except for AD, NK, MD). * & ** denote statistical significance at 10% & 5% respectively. P-values are in paranthesis. The following GARCH estimation is used to generate the results in this table:

$$R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt}$$

Table 3.1 Maximum Likelihood Estimates of GARCH(p,q) Models

Panel V	CURRENCIES					
	AD	**BP**	**CD**	**DM**	**JY**	**SF**
ttm tv(-1) oi	**Australian dollar**	**British pound**	**Canadian dollar**	**German mark**	**Japanese yen**	**Swiss franc**
	AR(3)-GARCH (2,1)	GARCH (2,1)	AR(1),AR(2), AR(3)-GARCH (2,1)	AR(3)-GARCH (1,1)	GARCH (2,1)	GARCH (2,1)
c	-4.88E-06		-0.00004168	-0.000042368		
	(0.960)		(0.253)	(0.655)		
γ_1			0.050096			
			(0.002)**			
γ_2			-0.024684			
			(0.105)			
γ_3	-0.055536		-0.035548	0.021453		
	(0.001)**		(0.019)**	(0.158)		
γ_4						
γ_5						
θ						
α_0	1.46E-06	5.75E-07	3.04E-07	1.65E-06	1.38E-06	2.15E-06
	(0.000)**	(0.023)**	(0.000)**	(0.000)**	(0.004)**	(0.002)**
α_1	0.040709	0.043887	0.081096	0.047081	0.10482	0.050836
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_2						
β_1	0.41016	0.52938	0.4491	0.92921	0.20163	0.51515
	(0.047)**	(0.005)**	(0.000)**	(0.000)**	(0.000)**	(0.007)**
β_2	0.52958	0.41703	0.4458		0.64116	0.40624
	(0.008)**	(0.021)**	(0.000)**		(0.000)**	(0.026)**
φ_{TTM}	-1.11E-08	4.58E-11	-3.34E-09	-7.30E-09	5.60E-09	-9.09E-09
	(0.000)**	(0.986)	(0.000)**	(0.032)**	(0.264)	(0.116)
$\varphi_{TV(-1)}$	-9.08E-10	-1.96E-12	1.21E-11	1.39E-11	-2.28E-11	2.96E-11
	(0.000)**	(0.879)	(0.254)	(0.100)*	(0.165)	(0.182)
φ_{OI}	1.25E-10	-3.89E-12	5.25E-12	-4.01E-12	3.30E-11	-6.54E-12
	(0.000)**	(0.223)	(0.000)**	(0.173)	(0.000)**	(0.242)

NOTE: The sample period is 1 Jan. 1982 through 31 Dec. 2000 (except for AD, NK, MD). * & ** denote statistical significance at 10% & 5% respectively. P-values are in paranthesis. The following GARCH estimation is used to generate the results in this table:

$$R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt}$$

Table 3.1 Maximum Likelihood Estimates of GARCH(p,q) Models

Panel VI	**CURRENCIES**					
	AD	**BP**	**CD**	**DM**	**JY**	**SF**
ttm, tv(-1)	**Australian dollar**	**British pound**	**Canadian dollar**	**German mark**	**Japanese yen**	**Swiss franc**
	AR(3)-GARCH (2,1)	GARCH (2,1)	AR(1),AR(2), AR(3)-GARCH (2,1)	AR(3)-GARCH (1,1)	GARCH (2,1)	GARCH (2,1)
c	-0.000019736		-0.000042626	-0.000043738		
	(0.839)		(0.245)	(0.644)		
γ_1			0.050548			
			(0.002)**			
γ_2			-0.023266			
			(0.125)			
γ_3	-0.05567		-0.035052	0.021528		
	(0.001)**		(0.021)**	(0.156)		
γ_4						
γ_5						
θ						
α_0	9.96171E-07	3.51E-07	3.41E-07	1.51E-06	1.71E-06	1.81E-06
	(0.000)**	(0.044)**	(0.000)**	(0.000)**	(0.000)**	(0.001)**
α_1	0.039237	4.33E-02	0.082414	0.048682	0.096178	0.051193
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_2						
β_1	0.57328	0.52915	0.39077	0.92963	0.20097	0.50832
	(0.026)**	(0.004)**	(0.000)**	(0.000)**	(0.000)**	(0.007)**
β_2	0.37409	0.4204	0.49935		0.65816	0.41515
	(0.132)	(0.020)**	(0.000)**		(0.000)**	(0.021)**
φ_{TTM}	-8.16E-09	1.13E-09	-3.76E-09	-6.83E-09	1.05E-09	-8.30E-09
	(0.002)**	(0.590)	(0.000)**	(0.045)**	(0.827)	(0.133)
$\varphi_{TV(-1)}$	1.20E-10	-1.05E-11	4.82E-11	4.99E-12	4.61E-11	2.54E-11
	(0.003)**	(0.277)	(0.000)**	(0.282)	(0.000)**	(0.214)
φ_{OI}						

NOTE: The sample period is 1 Jan. 1982 through 31 Dec. 2000 (except for AD, NK, MD). * & ** denote statistical significance at 10% & 5% respectively. P-values are in paranthesis. The following GARCH estimation is used to generate the results in this table:

$$R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt}$$

Table 3.1 Maximum Likelihood Estimates of GARCH(p,q) Models

Panel VII	CURRENCIES					
	AD	**BP**	**CD**	**DM**	**JY**	**SF**
ttm, oi	**Australian dollar**	**British pound**	**Canadian dollar**	**German mark**	**Japanese yen**	**Swiss franc**
	AR(3)-GARCH (2,1)	GARCH (2,1)	AR(1),AR(2), AR(3)-GARCH (2,1)	AR(3)-GARCH (1,1)	GARCH (2,1)	GARCH (2,1)
c	-0.000014224		-0.000041748	-0.000043209		
	(0.884)		(0.251)	(0.647)		
γ_1			0.049746			
			(0.002)**			
γ_2			-0.025188			
			(0.097)**			
γ_3	-0.055588		-0.035775	0.021536		
	(0.001)**		(0.018)**	(0.156)		
γ_4						
γ_5						
θ						
α_0	1.16E-06	5.91E-07	2.92E-07	1.57E-06	1.42E-06	2.63E-06
	(0.000)**	(0.020)**	(0.000)**	(0.000)**	(0.004)**	(0.000)**
α_1	0.040869	0.043905	0.080451	0.04905	0.10632	0.053349
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_2						
β_1	0.55544	0.52978	0.465	0.92995	0.1976	0.52558
	(0.026)**	(0.005)**	(0.000)**	(0.000)**	(0.000)**	(0.005)**
β_2	0.38787	0.41642	0.43257		0.64052	0.39324
	(0.106)	(0.021)**	(0.000)**		(0.000)**	(0.030)**
φ_{TTM}	-1.01E-08	-1.54E-10	-3.17E-09	-6.71E-09	4.60E-09	-9.21E-09
	(0.001)**	(0.951)	(0.000)**	(0.049)**	(0.367)	(0.129)
$\varphi_{TV(-1)}$						
φ_{OI}	2.42E-11	-4.23E-12	6.61E-12	1.52E-13	2.94E-11	-6.10E-12
	(0.000)**	(0.118)	(0.000)**	(0.927)	(0.000)**	(0.300)

NOTE: The sample period is 1 Jan. 1982 through 31 Dec. 2000 (except for AD, NK, MD). * & ** denote statistical significance at 10% & 5% respectively. P-values are in paranthesis. The following GARCH estimation is used to generate the results in this table:

$$R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt}$$

Table 3.1 Maximum Likelihood Estimates of GARCH(p,q) Models

Panel VIII	CURRENCIES					
	AD	**BP**	**CD**	**DM**	**JY**	**SF**
tv(-1) oi	**Australian dollar**	**British pound**	**Canadian dollar**	**German mark**	**Japanese yen**	**Swiss franc**
	AR(3)-GARCH (2,1)	GARCH (2,1)	AR(1),AR(2), AR(3)-GARCH (2,1)	AR(3)-GARCH (1,1)	GARCH (2,1)	GARCH (2,1)
c	-0.000014833		-0.000059587	-0.000043452		
	(0.879)		(0.105)	(0.647)		
γ_1			0.048039			
			(0.003)**			
γ_2			-0.025126			
			(0.097)*			
γ_3	-0.056464		-0.035738	0.021135		
	(0.001)**		(0.020)**	(0.164)		
γ_4						
γ_5						
θ						
α_0	5.80E-07	5.78E-07	6.36E-08	1.03E-06	1.72E-06	1.50E-06
	(0.000)**	(0.001)**	(0.000)**	(0.000)**	(0.000)**	(0.001)**
α_1	0.040778	0.043845	0.087805	0.046689	0.10352	0.051828
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_2						
β_1	0.34693	0.52942	0.38197	0.93152	0.20318	0.48348
	(0.045)**	(0.004)**	(0.000)**	(0.000)**	(0.000)**	(0.008)**
β_2	0.59528	0.41702	0.51064		0.64192	0.4378
	(0.000)**	(0.020)**	(0.000)**		(0.000)**	(0.012)**
φ_{TTM}						
$\varphi_{TV(-1)}$	-9.00E-10	-1.88E-12	-2.75E-12	1.20E-11	-1.81E-11	2.47E-11
	(0.000)*	(0.884)	(0.788)	(0.143)	(0.267)	(0.249)
φ_{OI}	1.22E-10	-3.91E-12	6.58E-12	-3.46E-12	3.18E-11	-5.91E-12
	(0.000)**	(0.220)	(0.000)**	(0.223)	(0.000)**	(0.262)

NOTE: The sample period is 1 Jan. 1982 through 31 Dec. 2000 (except for AD, NK, MD). * & ** denote statistical significance at 10% & 5% respectively. P-values are in paranthesis. The following GARCH estimation is used to generate the results in this table:

$$R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt}$$

Table 3.2 Maximum Likelihood Estimates of GARCH(p,q) Models

Panel I	**INDICES**			**INTEREST RATES**	
	MD	**NK**	**SP**	**ED**	**TB**
	S&P Midcap 400	**Nikkei 225**	**S&P 500**	**Eurodollar**	**90-day T-bill**
	AR(1),AR (4)-GARCH (1,2)	AR(1)-GARCH (1,1)	AR(1)-AR(5)-GARCH (1,1)- M	AR(1)-GARCH (1,1)	AR(1)-GARCH (1,1)
c	0.0007	0.0000	-0.0004	0.0000	0.0000
	(0.000)**	(0.899)	(0.487)	(0.449)	(0.980)
γ_1	0.0554	-0.0599	-0.0275	0.0648	0.0683
	(0.016)**	(0.004)**	(0.082)*	(0.000)**	(0.000)**
γ_2			-0.0008		
			(0.956)		
γ_3			-0.0253		
			(0.109)		
γ_4	-0.0310		-0.0353		
	(0.164)		(0.024)**		
γ_5			-0.0342		
			(0.021)**		
θ			0.1155		
			(0.039)**		
α_0	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_1	0.0657	0.0696	0.0576	0.0175	0.0194
	(0.001)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_2	0.0582				
	(0.002)**				
β_1	0.8480	0.9154	0.9303	0.9825	0.9804
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
β_2					
φ_{TTM}					
$\varphi_{TV(-1)}$					
φ_{OI}					

NOTE: The sample period is 1 Jan. 1982 through 31 Dec. 2000 (except for AD, NK, MD). * & ** denote statistical significance at 10% & 5% respectively. P-values are in parenthesis. The following GARCH estimation is used to generate the results in this table:

$$R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt}$$

Table 3.2 Maximum Likelihood Estimates of GARCH(p,q) Models

Panel II	**INDICES**			**INTEREST RATES**	
	MD	**NK**	**SP**	**ED**	**TB**
ttm	**S&P Midcap 400**	**Nikkei 225**	**S&P 500**	**Eurodollar**	**90-day T-bill**
	AR(1),AR (4)-GARCH (1,2)	AR(1)-GARCH (1,1)	AR(1)-AR(5)-GARCH (1,1)- M	AR(1)-GARCH (1,1)	AR(1)-GARCH (1,1)
c	0.0007	0.0000	-0.0003	0.0000	0.0000
	(0.000)**	(0.879)	(0.530)	(0.770)	(0.036)
γ_1	0.0554	-0.0603	-0.0275	0.1516	0.0700
	(0.016)**	(0.004)**	(0.085)*	(0.000)**	(0.000)**
γ_2			-0.0010		
			(0.948)		
γ_3			-0.0254		
			(0.110)		
γ_4	-0.0310		-0.0349		
	(0.164)		(0.027)**		
γ_5			-0.0338		
			(0.023)**		
θ			0.1118		
			(0.045)**		
α_0	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.001)**	(0.007)**	(0.003)**	(1.000)	(1.000)
α_1	0.0657	0.0683	0.0578	0.4256	0.4516
	(0.001)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_2	0.0583				
	(0.002)**				
β_1	0.8478	0.9165	0.9305	0.6349	0.5889
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
β_2					
φ_{TTM}	-4.00E-11	-2.08E-08	5.05E-09	1.62E-09	1.66E-09
	(0.997)	(0.361)	(0.223)	(0.000)**	(0.000)**
$\varphi_{TV(-1)}$					
φ_{OI}					

NOTE: The sample period is 1 Jan. 1982 through 31 Dec. 2000 (except for AD, NK, MD). * & ** denote statistical significance at 10% & 5% respectively. P-values are in paranthesis. The following GARCH estimation is used to generate the results in this table:

$$R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt}$$

Table 3.2 Maximum Likelihood Estimates of GARCH(p,q) Models

Panel III	**INDICES**			**INTEREST RATES**	
	MD	**NK**	**SP**	**ED**	**TB**
tv(-1)	**S&P Midcap 400**	**Nikkei 225**	**S&P 500**	**Eurodollar**	**90-day T-bill**
	AR(1),AR (4)-GARCH (1,2)	AR(1)-GARCH (1,1)	AR(1)-AR(5)-GARCH (1,1)- M	AR(1)-GARCH (1,1)	AR(1)-GARCH (1,1)
c	0.0007	0.0000	-0.0004	0.0000	0.0000
	(0.000)**	(0.938)	(0.485)	(0.738)	(0.551)
γ_1	0.0558	-0.0597	-0.0276	0.0673	0.0684
	(0.015)**	(0.004)**	(0.086)*	(0.000)**	(0.000)**
γ_2			-0.0009		
			(0.955)		
γ_3			-0.0253		
			(0.109)		
γ_4	-0.0306		-0.0353		
	(0.171)		(0.024)**		
γ_5			-0.0342		
			(0.021)**		
θ			0.1158		
			(0.039)**		
α_0	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.000)**	(0.000)**	(0.000)**	(1.000)	(0.000)**
α_1	0.0651	0.0647	0.0576	0.0283	0.0457
	(0.001)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_2	0.0554				
	(0.004)**				
β_1	0.8509	0.9221	0.9304	0.9709	0.8991
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
β_2					
φ_{TTM}					
$\varphi_{TV(-1)}$	9.34E-10	-2.06E-09	3.67E-13	6.75E-14	1.07E-11
	(0.319)	(0.001)**	(0.935)	(0.000)**	(0.000)**
φ_{OI}					

NOTE: The sample period is 1 Jan. 1982 through 31 Dec. 2000 (except for AD, NK, MD). * & ** denote statistical significance at 10% & 5% respectively. P-values are in paranthesis. The following GARCH estimation is used to generate the results in this table:

$$R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt}$$

Table 3.2 Maximum Likelihood Estimates of GARCH(p,q) Models

Panel IV	INDICES			INTEREST RATES	
	MD	**NK**	**SP**	**ED**	**TB**
oi	**S&P Midcap 400**	**Nikkei 225**	**S&P 500**	**Eurodollar**	**90-day T-bill**
	AR(1),AR (4)-GARCH (1,2)	AR(1)-GARCH (1,1)	AR(1)-AR(5)-GARCH (1,1)- M	AR(1)-GARCH (1,1)	AR(1)-GARCH (1,1)
c	0.0007	0.0000	-0.0003	0.0000	0.0000
	(0.000)**	(0.893)	(0.498)	(0.615)	(0.778)
$\boldsymbol{\gamma_1}$	0.0545	-0.0607	-0.0275	0.0635	0.0683
	(0.018)**	(0.003)**	(0.083)*	(0.000)**	(0.000)**
$\boldsymbol{\gamma_2}$			-0.0009		
			(0.955)		
$\boldsymbol{\gamma_3}$			-0.0253		
			(0.109)		
$\boldsymbol{\gamma_4}$	-0.0287		-0.0353		
	(0.201)		(0.024)**		
$\boldsymbol{\gamma_5}$			-0.0342		
			(0.021)**		
$\boldsymbol{\theta}$			0.1145		
			(0.040)**		
$\boldsymbol{\alpha_0}$	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.007)**	(0.000)**	(0.000)**	(1.000)	(0.000)**
$\boldsymbol{\alpha_1}$	0.0624	0.0663	0.0576	0.0270	0.0205
	(0.002)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
$\boldsymbol{\alpha_2}$	0.0613				
	(0.001)**				
$\boldsymbol{\beta_1}$	0.8464	0.9158	0.9304	0.9738	0.9753
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
$\boldsymbol{\beta_2}$					
$\boldsymbol{\varphi_{TTM}}$					
$\boldsymbol{\varphi_{TV(-1)}}$					
$\boldsymbol{\varphi_{OI}}$	1.93E-10	-1.83E-10	-3.52E-13	6.97E-15	1.38E-13
	(0.002)**	(0.000)**	(0.718)	(0.000)**	(0.000)**

NOTE: The sample period is 1 Jan. 1982 through 31 Dec. 2000 (except for AD, NK, MD). * & ** denote statistical significance at 10% & 5% respectively. P-values are in paranthesis. The following GARCH estimation is used to generate the results in this table:

$$R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt}$$

Table 3.2 Maximum Likelihood Estimates of GARCH(p,q) Models

Panel V	**INDICES**			**INTEREST RATES**	
	MD	**NK**	**SP**	**ED**	**TB**
ttm tv(-1) oi	**S&P Midcap 400**	**Nikkei 225**	**S&P 500**	**Eurodollar***	**90-day T-bill**
	AR(1),AR (4)-GARCH (1,2)	AR(1)-GARCH (1,1)	AR(1)-AR(5)-GARCH (1,1)- M	AR(1)-GARCH (1,1)	AR(1)-GARCH (1,1)
c	0.0007	0.0000	-0.0003	0.0000	0.0000
	(0.000)**	(0.940)	(0.529)	(0.532)	(0.263)
γ_1	0.0539	-0.0602	-0.0276	-0.0211	0.0719
	(0.019)**	(0.003)**	(0.087)*	(0.008)**	(0.000)**
γ_2			-0.0011		
			(0.942)		
γ_3			-0.0253		
			(0.111)		
γ_4	-0.0284		-0.0350		
	(0.206)		(0.027)**		
γ_5			-0.0338		
			(0.024)**		
θ			0.1119		
			(0.045)**		
α_0	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.092)**	(0.003)**	(0.013)**	(1.000)	(0.000)**
α_1	0.0622	0.0659	0.0574	0.2272	0.0197
	(0.002)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_2	0.0639				
	(0.001)**				
β_1	0.8452	0.9180	0.9310	0.0796	0.9607
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
β_2					
φ_{TTM}	4.68E-09	2.24E-09	5.09E-09	1.22E-08	-5.77E-10
	(0.719)	(0.931)	(0.221)	(0.000)**	(0.000)**
$\varphi_{TV(-1)}$	-1.03E-09	-1.35E-09	2.48E-12	-6.81E-13	4.00E-12
	(0.338)	(0.068)**	(0.657)	(0.000)**	(0.000)**
φ_{OI}	2.38E-10	-1.28E-10	-8.30E-13	-6.26E-13	-3.82E-13
	(0.001)**	(0.028)**	(0.497)	(0.000)**	(0.000)**

NOTE: The sample period is 1 Jan. 1982 through 31 Dec. 2000 (except for AD, NK, MD). * & ** denote statistical significance at 10% & 5% respectively. P-values are in paranthesis. The following GARCH estimation is used to generate the results in this table:

*ED did not converge

$$R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt}$$

Table 3.2 Maximum Likelihood Estimates of GARCH(p,q) Models

Panel VI	INDICES			INTEREST RATES	
	MD	**NK**	**SP**	**ED**	**TB**
ttm, tv(-1)	**S&P Midcap 400**	**Nikkei 225**	**S&P 500**	**Eurodollar**	**90-day T-bill**
	AR(1),AR (4)-GARCH (1,2)	AR(1)-GARCH (1,1)	AR(1)-AR(5)-GARCH (1,1)- M	AR(1)-GARCH (1,1)	AR(1)-GARCH (1,1)
c	0.0007	0.0000	-0.0003	0.0000	0.0000
	(0.000)**	(0.951)	(0.533)	(0.489)	(0.410)
γ_1	0.0557	-0.0595	-0.0275	0.1672	0.1061
	(0.015)**	(0.004)**	(0.087)*	(0.000)**	(0.000)**
γ_2			-0.0010		
			(0.948)		
γ_3			-0.0254		
			(0.110)		
γ_4	-0.0310		-0.0349		
	(0.165)		(0.027)**		
γ_5			-0.0338		
			(0.023)**		
θ			0.1117		
			(0.046)**		
α_0	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.002)**	(0.009)**	(0.013)**	(1.000)	(1.000)
α_1	0.0649	0.0648	0.0578	0.5712	0.0959
	(0.001)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_2	0.0561				
	(0.003)**				
β_1	0.8495	0.9226	0.9305	0.4500	0.0000
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(1.000)
β_2					
φ_{TTM}	-5.98E-09	1.05E-08	5.07E-09	3.93E-09	2.75E-09
	(0.668)	(0.653)	(0.230)	(0.000)	(0.000)
$\varphi_{TV(-1)}$	1.12E-09	-2.16E-09	-9.18E-14	-1.34E-12	1.66E-10
	(0.287)	(0.000)**	(0.983)	(0.000)**	(0.000)**
φ_{OI}					

NOTE: The sample period is 1 Jan. 1982 through 31 Dec. 2000 (except for AD, NK, MD). * & ** denote statistical significance at 10% & 5% respectively. P-values are in paranthesis. The following GARCH estimation is used to generate the results in this table:

$$R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt}$$

Table 3.2 Maximum Likelihood Estimates of GARCH(p,q) Models

Panel VII	**INDICES**			**INTEREST RATES**	
	MD	**NK**	**SP**	**ED**	**TB**
ttm, oi	**S&P Midcap 400**	**Nikkei 225**	**S&P 500**	**Eurodollar***	**90-day T-bill**
	AR(1),AR (4)-GARCH (1,2)	AR(1)-GARCH (1,1)	AR(1)-AR(5)-GARCH (1,1)- M	AR(1)-GARCH (1,1)	AR(1)-GARCH (1,1)
c	0.0007	0.0000	-0.0003	0.0000	0.0000
	(0.000)**	(0.886)	(0.547)	(0.140)	(0.179)
γ_1	0.0544	-0.0609	-0.0275	-0.0334	0.0644
	(0.018)**	(0.003)**	(0.085)*	(0.000)**	(0.000)**
γ_2			-0.0010		
			(0.945)		
γ_3			-0.0254		
			(0.110)		
γ_4	-0.0288		-0.0349		
	(0.201)		(0.027)**		
γ_5			-0.0338		
			(0.023)**		
θ			0.1104		
			(0.047)**		
α_0	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.071)**	(0.001)**	(0.004)**	(1.000)	(0.000)**
α_1	0.0623	0.0667	0.0577	0.2257	0.0307
	(0.002)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_2	0.0613				
	(0.001)**				
β_1	0.8464	0.9146	0.9307	0.0618	0.9647
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
β_2					
φ_{TTM}	-6.63E-10	-1.50E-08	5.35E-09	1.16E-08	-6.38E-10
	(0.950)	(0.546)	(0.200)	(0.000)**	(0.000)**
$\varphi_{TV(-1)}$					
φ_{OI}	1.94E-10	-1.86E-10	-4.39E-13	-6.40E-13	1.71E-14
	(0.002)**	(0.001)**	(0.648)	(0.000)**	(0.023)**

NOTE: The sample period is 1 Jan. 1982 through 31 Dec. 2000 (except for AD, NK, MD). * & ** denote statistical significance at 10% & 5% respectively. P-values are in paranthesis. The following GARCH estimation is used to generate the results in this table:

*ED did not converge

$$R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt}$$

Table 3.2 Maximum Likelihood Estimates of GARCH(p,q) Models

Panel VIII	**INDICES**			**INTEREST RATES**	
	MD	**NK**	**SP**	**ED**	**TB**
tv(-1) oi	**S&P Midcap 400**	**Nikkei 225**	**S&P 500**	**Eurodollar**	**90-day T-bill**
	AR(1),AR (4)-GARCH (1,2)	AR(1)-GARCH (1,1)	AR(1)-AR(5)-GARCH (1,1)- M	AR(1)-GARCH (1,1)	AR(1)-GARCH (1,1)
c	0.0007	0.0000	-0.0004	0.0000	0.0000
	(0.000)**	(0.937)	(0.482)	(0.789)	(0.954)
γ_1	0.0538	-0.0602	-0.0276	0.0691	0.0788
	(0.019)**	(0.003)**	(0.085)*	(0.000)**	(0.000)**
γ_2			-0.0010		
			(0.949)		
γ_3			-0.0252		
			(0.111)		
γ_4	-0.0289		-0.0354		
	(0.198)		(0.024)**		
γ_5			-0.0342		
			(0.022)**		
θ			0.1160		
			(0.039)**		
α_0	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.008)**	(0.000)**	(0.000)**	(1.000)	(0.000)**
α_1	0.0619	0.0660	0.0571	0.0296	0.0289
	(0.002)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
α_2	0.0633				
	(0.001)**				
β_1	0.8456	0.9176	0.9309	0.9690	0.8835
	(0.000)**	(0.000)**	(0.000)**	(0.000)**	(0.000)**
β_2					
φ_{TTM}					
$\varphi_{TV(-1)}$	-7.57E-10	-1.32E-09	2.90E-12	1.14E-13	2.37E-11
	(0.376)	(0.059)*	(0.608)	(0.000)**	(0.000)**
φ_{OI}	2.32E-10	-1.30E-10	-7.90E-13	-5.19E-15	-2.17E-12
	(0.001)**	(0.019)**	(0.521)	(0.000)**	(0.000)**

NOTE: The sample period is 1 Jan. 1982 through 31 Dec. 2000 (except for AD, NK, MD). * & ** denote statistical significance at 10% & 5% respectively. P-values are in paranthesis. The following GARCH estimation is used to generate the results in this table:

$$R_t = c + \sum_{i=1}^{NR} \gamma_i R_{t-i} + \theta h_t + \varepsilon_t$$

$$\varepsilon_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j}^{p} \beta_j h_{t-j} + \sum_{k=1}^{NGT} \phi_k g_{kt}$$