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Solving Out Loud: Using discourse as a means to promote problem solving, motivation, and metacognition in a mathematics classroom

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by

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DEDICATION

I dedicate this work to my students; thank you for teaching me.

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ABSTRACT OF THE THESIS

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Megan E. King

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Classroom communication can often be a teacher-centered discussion. Due to the teacher centered format of discussions students are not engaging in meaningful discourse in mathematics classroom, which is part of the NCTM 2000 Standards as well as a necessary component to learning. Students can only learn communication skills when discourse is a central feature from the classroom. In addition, students must explicitly learn problem-solving skills. Unfortunately, many of these features are absent from today's classrooms.

This research investigates the connection between discourse and problem solving in a ninth and tenth Geometry classroom.

Solving Out Loud is a curriculum that was developed to increase students' confidence and ability in problem solving as well as students' mathematical discourse skills, students' motivation, and finally students' metacognition of mathematical learning. Students participating in this study were involved in large student-centered discussions either based on a single question, such as "what motivates you to learn?" or based on the different procedures to solving particular problems. *Solving Out Loud* was evaluated with data from the students' work, recorded class conversations, teacher field notes, and pre and post surveys. This data showed an increase in students' problem solving skills and students' confidence in their ability to discuss their problem solving strategies. The findings imply that student-centered conversations benefit the development of students' problem solving and discourse skills.

Chapter I. Introduction

Mathematical proficiency is a necessary skill for the sake of passing standardized tests as well as a skill necessary for life. Reyna and Brainerd (2007, p. 147) state, “mathematical proficiency is essential for tasks of everyday living, beyond those required in the workplace. In health and medical decision making, in particular, understanding numerical information (e.g., about risks and outcomes of treatments) is literally a matter of life and death.” NCTM Standards show that communication is a key competent for students to become proficiency in mathematics. Unfortunately, students spend very little time each day talking in school. I have observed that many of my students talk for only 4% of the class time. This may explain why classroom conversations can be boring, dull, meaningless, and useless if students are not accustomed to this form of learning. On the contrary, classroom conversation can be meaningful, engaging, thought provoking, and informative if students are given the opportunity to practice and hone this skill.

From my own experience, classroom discourse can fall apart quickly with lackluster involvement. Discourse should be an opportunity for students to share ideas and concerns and to learn from one another. Fortunately, at Mountain View Academy, an independent school in San Diego County (all names in this paper are pseudonyms) classroom discussions are a cornerstone of the philosophy and teaching instruction. However, students often fail to transfer their communications skills from a humanities course such as English or History, where they openly debate and ponder various thought

provoking questions to a math or science course. In my Advanced Algebra class, students are so eager to please me, the teacher; they do not listen to their peers and often repeat each other. In my Geometry classes students are so unsure of their mathematical ability, and lack enthusiasm and motivation, that they do not engage in discourse even with the most basic of questions. In addition, in my Honors Pre-calculus class the students who are confident in their mathematical skills share ideas and problem solving strategies but other less confident students look to me as the real voice of authority and have trouble learning from their peers.

In addition, my Geometry students struggle with learning problem-solving skills. They bring in countless homework assignments that are incomplete and often filled with question marks. Students are not able to transfer the skills they are learning to other problems requiring the same skills. Many students receive a 3-3.8 out of 4 points for homework due to the lack of completion and unfortunately, because of curriculum pressures and standardized tests, we teachers need to continue forward and teach new content to students lacking certain skills because of the breadth of content that needs to be covered.

Not only are my Geometry students missing key problem solving skills but they are also lacking the skills to communicate their ideas about mathematics with their peers in an open conversation. Students are not talking about math and are not problem solving. Yet, it is possible that

discourse in a math class can be engaging, meaningful, motivating, and full of problem-solving strategies and learning powerful experiences.

Chapter II: Assessment of Needs

Many secondary school students in seventh through twelfth grade have not developed a sense of intrinsic motivation when it comes to schoolwork. They not excited about the sheer joy of learning for learning sake; they often do not find the value in learning state mandated curriculum. Schools today are often focused on preparing students for state standardized tests and they fail to focus on the skills that are needed for life beyond school. Preparing for a purposeful life requires formal schooling as well as skills necessary for employment; "mathematical proficiency is increasingly recognized as fundamental to economic success for individuals and for nations" (Reyna and Brainerd, 2007, p.148)

Mathematical proficiency is needed for life and the workplace. The Principals and Standards for School Mathematics by National Council of Teachers of Mathematics (NCTM) states that "knowing mathematics can be personally satisfying and empowering. The underpinnings of everyday life are increasingly mathematical and technological" (NCTM, 2000, p. 4). NCTM (2000) also states that the value of math needed in the workplace has increased across a variety of employment sectors from graphic design to health care.

With the need for students to have a high level of mathematical proficiency, there is evidence that a change needs to occur in our education system in order for students to reach this high level of proficiency. This chapter will explore mathematical achievement at the international level,

national level, and state level and show the need for reform to facilitate improvement of mathematical proficiency levels.

Mathematical Achievement at the International Level

TIMSS shows the average mathematics scores for fourth and eighth grade students in the United States have increased from 1995 to 2007. When compared to the results from other countries the data suggest that current mathematics curriculum in the United States has merely retained the status quo. Unfortunately, the average scores from 2007 show that at least five countries had higher averages than the United States. These countries include Chinese Taipei, Hong Kong SAR, Japan, Korea, and Singapore (TIMMS, 2007). Globalization has made the world a much smaller place and therefore more competitive. American students need the opportunity to compete at the same level as their foreign counterparts.

When comparing the percentage of students who score at the advanced level, it is clear to see that the United States needs an alternate method for teaching students mathematics and allowing students the opportunity to reach the advanced proficiency level. In 2007, only 6% of eighth grade students from the United States reached the TIMSS advanced international benchmark in mathematics. The five countries listed above have many more students who reached this benchmark. Chinese Taipei has 45% of eighth grade students at this level and Korea and Singapore each has 40%. It is clear that at the international level American students are not performing at the same ability level as students from other countries. Mathematical

curriculum and the teaching of problem-solving skills in the United States needs to be reviewed and altered in order for students to be competitive with to other students across the globe.

Mathematical Achievement at the National Level

When comparing these scores, it is important to keep the meaning of aptitude levels clear. According to the NAEP (2009), the mathematical achievement levels for eighth grade are defined as:

> ***Basic:*** *Eighth-grade students performing at the Basic level should exhibit evidence of conceptual and procedural understanding in the five NAEP content areas. This level of performance signifies an understanding of arithmetic operations—including estimation—on whole numbers, decimals, fractions, and percents.*
> ***Proficient:*** *Eighth-grade students performing at the Proficient level should apply mathematical concepts and procedures consistently to complex problems in the five NAEP content areas.*
> ***Advanced:*** *Eighth-grade students performing at the Advanced level should be able to reach beyond the recognition, identification, and application of mathematical rules in order to generalize and synthesize concepts and principles in the five NAEP content areas. (NAEP, 2009, p. 34)*

The national average for eighth grade students for 2009 was as follows: 29% below basic, 71% at or above basic, 33% at or above proficient, and 7% at advanced (NAEP, 2009). With only a third of the nations eighth graders reaching at the proficient level this means only a third of eighth grade students are reaching the NCTM's proposed standards of problem solving skills. A student at the basic level will not have mastered problem-solving skills or be prepared for high school level math.

The NAEP shows that between 2004 and 2008, 13-year old students who scored at or above proficiency levels in moderately complex procedures and reasoning rose from 27.8% to 30% (NAEP, 2009). Only 59.4% of 17-year olds were able to complete moderately complex procedures and reasoning questions and only 6.2% were proficient in answering multistep problem solving and algebra questions. This data shows that the average American student completes high school without the skills to be proficient in problem solving.

Mathematical Achievement at the State Level

The trend in mathematical achievement is even worse for Californians. Forty-one percent of California's eighth grade students are below basic, 36% are at basic, 18% proficient, and only 5% are advanced (NAEP, 2009). When comparing the scores from California to the national averages, it is clear that California needs an alternate method to teach problem solving in mathematics. California students scored below the national average in all levels of ability: below basic, basic, proficient, and advanced. Not only are students unable to engage in meaningful discourse but they also lack problem-solving skills. Again, these tests do not include the method of instruction but if discourse was not part of the instruction it may suggest the lower scores are related to the lack of discourse that helps students internalize mathematical concepts and build meaning (NCTM, 2000).

Conclusion

Improving mathematical proficiency may be a daunting task but small

steps taken by classroom teachers can make the difference for any student struggling with math. As a classroom teacher for seven years, I ask myself "how do I utilize classroom time so students are actively engaged in problem solving instead of note-taking or completing investigation activities that do not allow enough time for practice?" I want my students to really understand their assignments and the mathematical application of these outside the classroom. I am motivated to develop curriculum so that my students can communicate verbally about mathematical strategies and use reason to solve challenging problems. I want them to take ownership of their learning, and to verbally communicate their mathematical thinking and strategies.

Chapter III. Review of Literature

International and national tests suggest that American students are not proficient in problem solving and even though discourse and problem solving are part of the NCTM standards, communication skills are not part of any national assessment. Discourse can provide students with an opportunity to learn problem-solving strategies from peers. Ideally, students must be engaged in class conversations and their own learning. Students must be motivated to learn the problem solving strategies and be aware of their learning process. Most importantly, teachers need to be aware of how these different educational constructs affects students' learning and understanding. This paper reviews the relationship between discourse and problem solving, motivation, and metacognition.

Problem Solving

Problem solving is a key life skill and a skill that can be learned throughout schooling in a variety of ways. As a teacher, it can be frustrating to hear students say, "I don't get it" or "we never learned to do a problem like this". This situation could be avoided if students had more confidence in their problem solving skills. Problem solving strategies involve the ability to consider a variety of ways to solve a problem when the original attempt did not work and to carry out the plan and find a solution (Dossey, 2006). When assessing problem solving skills, the Trends in International Mathematics and Science Study (TIMMS) test considers the following: "the context allows students to be engaged, students do not have a known strategy to

immediately apply, and the situation calls for a solution" (Dossey, 2006, p. 13).

Although problem solving is a standard for NCTM, the NAEP report (2009) states that many eighth grade students enter high school with limited proficiency in these skills. How do students learn to problem solve in math? How can students learn problem solving through discourse? According to NCTM standards, students should be able to:

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving. (NCTM, 2000, p. 334)

Research has shown that there are key elements to teaching problem solving strategies. According to Polya (1945), there are four facets to problem solving. First, are the characteristics of the problem solver, second are the conditions for harder and easier problems, third are the effects of different instructional methods, and finally there are the effects of classroom-related conditions. Hembree's (1992) analysis suggests that a successful problem solver poses diverse approaches to problem solving and is good at basic math. As seen in my own classroom and in the NAEP scores, many students lack the concept of various approaches and are not at a proficient level of mathematical understanding. Students must reach this basic level of understanding before problem solving can be perfected.

One of the most effective ways for teaching problem solving is the heuristics model introduced by Polya (1945) and supported by researchers like

Schoenfeld (1980). Heuristics involves four steps. Students must: (1) understand the problem, (2) obtain a plan for the solution, (3) carry out the plan, and (4) examine the solution. In teaching this model to students, diagrams are useful in allowing students to see the translation from words to math.

Teachers must teach problem solving strategies with the same seriousness that applies to other mathematics and to teach practical structures for proficient problem solving (Schoenfeld, 1980). Schoenfeld (1980) taught two different groups of students: with one group, he explicitly talked about the heuristic training and with the other he did not. Each student in the first group and only one the student in the latter group properly answered the problem.

It is possible that students can learn these problem-solving strategies by discussing the problems in a supportive class conversation, but students still need time for adequate practice. Many mathematics students are not receiving enough time in class to master new concepts. In addition to lacking time for practice in class, many students do not complete homework that can provide time for mastery. Students who do not do their homework are not able to master new ideas resulting in an inability for higher-level problem solving and conceptual understanding. Hembree (1992) states that classroom-related conditions that positively affect problem-solving performance include modeling the heuristics model and the intense study of problems. Practice is a key element to subject mastery (Bransford, 2000; Ericcson et al., 2003). Unfortunately, students are not always given this necessary practice.

The method for student mastery/practice time is not defined as simply ‘let’s start our homework’, where the teacher sits in the front of the room and students silently begin their homework exercise. Instead, this time should be filled with discussions and collaboration. Bransford (2000) describes a math class in which the students spend time discussing strategies with each other and the whole class. This model illustrates that “important ideas in mathematics are developed as students explore solutions to problems” (Bransford, 2000, p. 169). Students should be sharing ideas and strategies and engaging in a class discussion. Most importantly, students should be motivated to learn and aware of their own learning.

Problem solving and understanding.

Wiggins and McTighe (2005) write that application is another facet of understanding. Students show their understanding of a concept or idea by using it and adapting it. This can be applied to problem solving because true problem solving, which requires students to tackle a problem they have never seen before, requires that students apply their understanding of concepts to this new situation.

In order for students to really apply their knowledge and understanding, problems should be similar to real situations. For some, this has developed into problem-based learning. Unfortunately for the scale of this project, problem-based learning was not a viable route. Instead, this paper will focus on how students can strengthen problem-solving skills through discourse, which will build understanding.

Discourse

Merriam-Webster (2010) defines discourse as a *"verbal interchange of ideas; especially: conversation*". Discourse in a classroom is a conversation about the subject at hand, whether it is a novel, the ethics of a scientific topic, or strategies for solving a problem. Discourse takes on a different approach in various classes. In a math classroom, it can be used as a method for sharing problem solving strategies so that students focus on understanding the process of solving the problem instead of the final answer.

Teacher-centered discourse is often the most common form of communication that occurs in a classroom setting. It is the responsibility of the teacher to control the negative and positive communication that occurs in the classroom (Cazden, 1988). According to Cazden (1988), the most common pattern of classroom discourse is IRE meaning teacher initiation (I), student response (R), teacher evaluation (E). Even though the students are involved in this type of discourse, it is strongly teacher centered. The teacher is in control of the conversations, knows the questions that are going to be asked and students only speak after the teacher has spoken.

Shifting to a student-centered discussion can be a challenge for teachers and students because the IRE pattern is so prevalent in all grade levels (Cazden, 1988). This pattern is hard to break because students have been conditioned not to listen or respond to their peers. The IRE pattern is only an interaction between the teacher and student and this interaction is rewarded. When I have tried to limit my involvement in class conversations

many students still looked to me, the teacher, when others were talking. Even if the pattern of conversation has less teacher involvement, the physical actions show that the teacher is still the authority figure in the conversations because students are still watching them instead of the student talking.

From my own experiences, students do not openly engage in classroom discourse. Discourse can be a key component to instruction but it can be difficult. Most students look to the teacher as the main authority figure because students see the teacher as the expert; this creates a teacher-centered discussion that lacks active listening and engagement. In contrast, a student-centered dialogue can allow students to share their problem solving strategies and learn from each other. The National Council for Teachers of Mathematics (NCTM) has stressed the need for mathematical communication. The NCTM standards state that the process of communication supports understanding and meaning (NCTM, 2000). Research has also shown the value of mathematical discourse and the need for students to communicate their ideas verbally (Beasley, 1995). Teachers must train students to listen to each other and engage in discourse that is student centered and not teacher centered.

Research conducted by Beasley (1995) in a third-grade class describes discourse using two possible outcomes: convergent and divergent tasks. Convergent tasks lead students to one answer even if there are various methods for reaching this answer. Students listening to these various methods may be one reason why discourse is so important and why it leads to

problem solving strategies. Divergent tasks allow for many answers and for students to explore many different avenues (Beasley, 1995). Both tasks hold mathematical value and offer students time for critical analysis of their problem solving abilities. The teacher in this study had to manage the conversations and make sure the discussion was moving in the right direction and yet the students were able to remain 'leaders in their learning' because the conversations involved all students and were 'enhancing students' understanding of important ideas in mathematics (Beasley, 1995).

Karen Heinz (2010) suggests that in order to shift the conversation to one that is student-centered, teachers allow students small sections of information to discuss and a small amount of time to discuss it. One reason is for students to feel supported in this model so that if they came up with an incorrect strategy the classroom discussion would let them know of the proper methods without significant time wasted. Students often felt more willing to explore when the time was limited because they could afford to make a mistake (Heinz, 2010). This shift to student-centered discourse promotes the opportunity for students to provide explanations to each other, which mediates student learning, and understanding (Webb, 1992).

Two types of instructional strategies that incorporate discourse are Harkness discussions and Socratic Seminars. Harkness discussions are large group discussions prompted by a debatable intriguing question. A further description of Harkness discussion is provided in Chapter VI. Socratic Seminars is "a form of structured discourse about ideas and moral dilemmas"

(Tredway, 1995, p. 26). Socratic Seminars and Harkness discussions are very similar in that they involve compelling questions and require students to reason, predict, and to collect and analyze information. Both types of discourse offer students the opportunity to learn through the practice of class conversations. They are more popular in humanities classes but they can be very useful in math and science courses as well.

Discourse and understanding.

According to Wiggins and McTighe (2005), the six facets of understanding start with the first facet: explanation. Explanation is a key component of understanding. One must be able to explain the process and theories behind ideas in order to show they understand the idea and are not just reciting a generic answer. Students may be able to state the quadratic formula but they may not understand its origin or the importance of the x values it generates. For students to show understanding through explanation they must "provide an explanation on their own, not simply recall; to link specific facts with larger ideas and justify the connections" (Wiggins & McTighe, 2005, p. 88).

Explanations can be verbal or written as long as students are actively involved and support their conclusions. Incorporating discourse into the classroom can improve students' understanding of mathematics. Students will be able to share their knowledge, ask questions, and converse with peers. Discourse can help a teacher assess students' understanding versus their memorization skills.

Motivation

Motivating factors behind academic success can vary from personal accomplishment to monetary rewards to parental approval. Indeed, students are motivated by a variety of intrinsic and extrinsic rewards. Some students are motivated by the prospect of improving and learning while others are may be motivated by extrinsic rewards offered by parents, teachers, and coaches. Unfortunately, not all rewards are created equally. When developing self-motivation, extrinsic rewards can actually dampen motivation instead of supporting and developing intrinsic motivation (Deci, 1996).

Teachers and parents are often the main source for affecting the intrinsic and extrinsic motivation of students. A study conducted by Elliot, Hufton, Willis, and Illushin (2005) compared several visits to schools in Kentucky, United States; Sunderland, England; and St. Petersburg, Russia for a transnational comparison of motivation, engagement, and educational performance. Some general findings include the involvement of parents in supporting educational practices and well as the teachers' role and attitude in the classroom.

The study showed that the more motivated students are more likely to have parents at home that value education and show the importance education has played in their own personal success. In addition, parents who have the time and the organization to support their child's learning will most likely have more motivated students. If the parents are too disorganized or

unengaged, the students are often less motivated (Elliot, Hufton, Willis, & Illushin, 2005).

In addition to parental involvement, the teacher plays a vital role in shaping motivation as well as providing opportunities for developing students' intrinsic motivation. Teachers must consider a variety of factors when dealing with motivation. A teacher's attitude towards learning and towards their students can affect motivation; students are more motivated to like a subject if they like the teacher (Elliot, Hufton, Willis, & Illushin, 2005). The "expert" role teachers take on in the class can affect motivation. An "expert" teacher may only want to provide information that students must memorize and absorb however another "expert" teacher may want their students to become experts themselves and learn and understand the content (Erickson, 1974). The latter can support motivation since the learning and understanding is being expected of the students beyond memorization.

As previously mentioned, offering rewards can have a negative effect on motivation. Examples of external rewards offered by teachers or schools for students to perform higher on high-stakes standardized tests can backfire because it does not promote self-regulation or self-motivation within the students (Elliot, Hufton, Willis, & Illushin, 2005). However, symbolic rewards such as praise can be an important factor in motivating low-achieving students (Elliot, Hufton, Willis, & Illushin, 2005; Deci, 1996). Teachers much arrange tasks so students are successful, then teachers can offer the necessary praise and encouragement. When a teacher is trying to motivate students, it can be

much more productive to influence the students' intrinsic motivation versus offering extrinsic rewards. Students can be intrinsically motivated in the classroom through choice, rewards of acknowledgement, use of language, students as teachers, and non-evaluative quizzes (Deci, 1996).

Motivation and understanding.

Most importantly, each individual student has the ability to change their own motivation even when peers, teachers, and parents can influence motivation. Many students have an "I'm good at this and I'm bad at that" attitude. Student motivation can play a role in changing this mindset to one of growth (Dweck, 2006). If a student believes that they can get better, they will get better at math but they have to be motivated to work on mastering the skills and believe in their ability to learn. Students should be motivated to learn math and improve their understanding, they should be excited about the task itself and have the desire to problem solve.

Metacognition

Confident learners are motivated to learn and are self-aware of their learning. Not all students are able to develop these skills on their own. Metacognition is the self-knowledge reflection of an individual's thinking and learning and it can be developed with the help of a teacher (Wiggins & McTighe, 2005). Teachers can help students develop metacognition in math by either directly or indirectly teaching students to "be aware of their own problem solving processes, monitoring their progress, and reflecting on their own thinking" (Zemelman, 2005, p. 116).

Helping students learn the ability to be aware of their learning and develop necessary metacognition skills is not just a math skill but also a life skill. Students need to be aware of their learning and understanding of what they do not yet know. While practicing and mastering mathematical skills, students can also be actively monitoring their learning. An ideal goal is for students to be conscious about their problem solving skills. Reaching this goal will require that students practice skills with support from their teachers and peers. Ideally, a teacher will scaffold metacognitive practice into class lessons and students will begin to ask self-regulatory questions themselves. (Bransford, 2000)

Metacognition and understanding

Students need to understand themselves before they can understand the world they are studying (Wiggins & McTighe, 2005). This belief supports the idea that if a student is going to succeed academically and really master the content they are learning, they must have self-knowledge and be self-regulatory. Metacognition allows students to evaluate their own advancement towards understanding. (Bransford, 2000)

Chapter IV. Review of Existing Geometry Curriculum

Many math courses are taught with the aid of a textbook. The extents to which textbooks are used in the classroom varies between districts and teachers. Textbooks can serve as a guide for pacing, content of lessons, and practice problems. Teachers often decide the instructional method for delivering the content. These methods can vary but are not limited to the following: lecture, collaborative group work, inquiry based activities, and computer generated practice problems. Not only do teachers decide the instructional method they often add supplemental material to fill in curriculum gaps left out of textbooks.

This chapter will describe three curricula and then compare the three curricula against the main educational constructs of problem solving, discourse, motivation, and metacognition. All three curricula align with California state standards however they all lack various aspects of the listed educational constructs. There is little guidance offered on classroom discourse and there are no explicit instructions for teaching problem-solving strategies. The three curricula are also designed for direct instruction, which does not always foster a motivation or self-regulation and metacognition.

Existing Curricula

Discovering Geometry: An investigative approach.

Discovering Geometry (Serra, 2008) provides an engaging student-centered approach to learning while covering the NCTM standards for Geometry. Each section in the book has at least one new conjecture or

property for students to learn from the investigations. These conjectures and properties are fill-in-the-blank with no answer key provided. This leaves students with the freedom to truly investigate the properties instead of proving them true. This setup of discovery may motivate eager students, but does not always motivate struggling math students. From my experience, this textbook can be difficult for students who need more reinforcement and practice because each section has incomplete properties and theorems, if a student did not take good notes or was absent from class they have difficulty solving the related homework problems.

Discovering Geometry incorporates the best teaching practice of small-group investigations, reading as thinking, and representing-to-learn by writing conjectures based on investigations (Zemelman, 2005). Even with these best practices in use, students often have difficulty applying the written conjectures to problem solving tasks. The curriculum does not offer guidance to students on how to complete these applications of knowledge problems. It also leaves out authentic practices for students, and does not offer students a chance to reflect on their learning.

One of the main features of the *Discovering Geometry* curriculum is that students learn concepts through discovering investigations and mainly utilize Geometer's Sketchpad or Patty Paper instead of a compass and straightedge. Geometer's Sketchpad is a computer software program that allows students to create geometry figures such as parallel lines, angle bisectors, and even tessellations. Patty Paper is another method for carrying out investigations.

The paper is a square tracing paper that allows students to duplicate and manipulate figures to visualize various properties. The investigations using these products walk students through steps to construct and identify a geometric property. Both offer a visual and kinesthetic way for students to discover geometry properties. Because the investigations are student-centered, they allow the teacher to facilitate the activities while the students work together.

There is opportunity for students to learn problem-solving strategies in these investigations. However, the author of the text assumes that the students will be able to take this new concept and apply it to practice problems. This can be difficult for students because there is little support for this process. The curriculum jumps straight from the investigation/lesson to exercise problems with little opportunity for students to practice these new concepts.

Indeed *Discovering Geometry* could incorporate more problem solving skill practices, it could also incorporate more discourse strategies and features in the teacher's edition. The *Discovering Geometry* teacher's edition provides a lesson guide and some helpful questions to ask students when working through each lesson. There are also occasional "sharing ideas" scattered throughout the book. These questions can be a starting point for teachers to begin a discussion but there is no other discourse support. The curriculum does not incorporate discussion opportunities beyond these provided questions.

However, *Discovering Geometry* does attempt to incorporate motivation in a variety of ways. Each section has a quote and a real-world connection to the lesson's content. Beyond that, the book assumes that students are interested in learning geometry; it is up to the teacher to keep the intrinsic motivation high. This can be seen in the lessons' investigations. I have seen students more engaged in these activities than they are in my teacher-centered lectures but not overly eager to start each investigation. Unfortunately, they do not offer students any choice in the investigations. If choice were part of the investigations, students may be more motivated to learn the content (Deci, 1996).

With similar attempts to motivate students, each curriculum has a different approach in supporting and developing metacognition. *Discovering Geometry* does very little in developing metacognition skills. Each chapter review contains an "assessing what you've learned" section and a portfolio protocol. However, having these at the end of a chapter does little for students to reflect on their learning while they are learning new material. The supplemental materials offer section quizzes and tests but do not provide teachers with ways to incorporate metacognition skills into each chapter to support student learning.

Geometry: UCSMP.

The University of Chicago School Mathematics Program (UCSMP) provides a comprehensive course of geometry aligned with the NCTM standards titled *Geometry: UCSMP* (Benson, 2009). The book provides

problem solving, everyday application, algebra review, and the use of calculators and computers through TI-Nspired calculators. The book also provides teacher support and implementation ideas, even a section called 'accommodating the learner' for teachers to adjust the lesson based on student needs. Overall, this book provides a remarkable geometry curriculum but like the other reviewed curricula, the variability of teaching strategies of the teacher play a key role in students learning the content presented in this book.

Compared to *Discovering Geometry, Geometry: UCSMP* offers a more traditional method as this text has complete conjectures and is designed to provide students will all the information they need to be successful geometry students. *Geometry: UCSMP* presents problem solving skill practices through the assignment questions. Each section of the *Geometry: UCSMP* text has two sets of questions titled "Covering the ideas" and "Applying the mathematics." The latter section gives students the opportunity to practice problem-solving skills assuming the students have already internalized these methods and are able to successfully solve problems. The only support provided for the teacher is the 'accommodating the learner' section that appears to reinforce a concept instead of problem-solving skills.

Geometry: UCSMP offers very little support for discourse. It is at the discretion of the teacher to create discourse opportunities if using any of the three textbooks reviewed in this chapter. As previously discussed, discourse provides students with an opportunity to improve their understanding as well as learn from their peers.

Motivation and metacognition are minor components of *Geometry: UCSMP.* This text provides various projects, integrated ideas, and real-world connections to entice student interest and increase their motivation to learn the content. *Geometry: UCSMP* also makes the best attempt at incorporating miscognizant activities for the student. These strategies include self-tests, 'quiz yourself' quizzes, and writing opportunities for students to clarify their thinking.

Geometry: Concepts and skills.

Larson (2010) offers a traditional way of learning geometry. Each topic or section consists of conjectures and/or theorems followed by related exercise problems. Each conjecture and theorem is provided for students, which does not provide the student with time for wonder or inquiry in a collaborative learning environment (Bayer, 1990). This book covers the basic Geometry standards and offers sections of standardized practice tests. However, there are few practice problems at the heart of each section. There are online resources for student support such as practice quizzes, test, a parent guide at www.classzone.com, and projects after every other chapter. In general, this book offers basic geometry content with little support for teaching problem solving skills, discourse skills, and promoting motivation and metacognition.

Similar to the other two curricula *Geometry: Concepts and Skills* has difficult problems for students to solve, yet lacks explicit methods for problem solving. Again, it is up to the individual teachers to teach problem-solving

strategies as well as teach the geometry content. Even though problem solving is not explicitly discussed in any of the teacher editions of these textbooks, they all have exercise problems that offer students time to master the content if they understand how to apply the new concept to these problems. All three curricula reviewed provide enough practice problems for students to master content knowledge but not enough to master problem-solving skills. The practice to master these skills is necessary to ensure that students can apply new conjectures or properties to new problems. I have witnessed this lack of problem-solving skills with my geometry students. They often need constant reinforcement and scaffolding to make this application happen as well as the time to master the new content (Bransford, 2000). Just because the opportunity for mastery is present does not necessarily mean that students will be able to have this time for practice. It is up to the teacher to support students in the process whether it is during class or during homework assignments.

Compared to *Discovering Geometry* and *Geometry: UCSMP*, *Geometry: Concepts and Skills* has even fewer opportunities for students to engage in classroom discussions and does not offer explicit discourse practices. As well, it does allow students to be self-reflective of their learning, take ownership of what they are learning, and be motivated to learn. Students who are intrinsically motivated and interested in learning will find all three of these texts adequate for learning geometry. For the students who are not

intrinsically motivated, it is up to the teacher to provide motivational opportunities to help get students interested in geometry.

Conclusion

According to NCTM guidelines, there are three criteria for high quality curriculum. A mathematics curriculum should be "coherent, should focus on important mathematics, should be well articulated across the grades" (NCTM, 2000, p. 15). Using these guidelines, the above textbooks are of high quality solely based on math content. However, they do not provide a teacher with quality material to promote student motivation, students' problem-solving skill development, and discourse opportunities. Teachers must supplement their own instructional material to reach best practices because the curricula do not provide this information.

There is an opportunity to develop curriculum that will allow students to master problem-solving skills and increase motivation through discourse opportunities. This type of curriculum will provide teachers strategies for adjusting a traditional geometry textbook to better prepare students to be motivated problem solvers.

Proposal for New Curriculum

Current curriculum offers the necessary content required by state standards but provides teachers with little support in developing problem solving skills and meaningful discourse in the classroom. In order for students to improve these skills, teachers must design lessons that incorporate meaningful discourse but allows students to practice new skills.

The proposed curriculum will involve a shift from in teacher-centered discussions to student-centered discussions. Students will engage in meaningful discussions as well as practice and mastery skills. What effects will discourse have on problem solving, motivation, and metacognition? Will creating opportunities for discourse in the classroom improve these constructs? This study will attempt to provide some insights into these important questions.

Chapter V: Solving Out Loud

This curriculum was developed to improve students' problem solving and discourse skills. As a secondary mathematics teacher for the past seven years, I have observed that students in this course often lack confidence in their mathematical ability and struggle to improve as math students. In addition, I noticed that these struggling students are also reluctant to talk about math. I designed this curriculum to build students' confidence as math students so that they can speak about math and problem solving with awareness and thoughtfulness. The curriculum has one overarching goal and three sub-goals. The overarching goal is to improve students' confidence and ability in problem solving and the three sub-goals are to improve students' mathematical discourse, motivation, and metacognition of mathematical learning.

In order to improve problem solving strategies, students need to be aware of these strategies themselves and their own problem solving abilities. Problem solving strategies include understanding the problem, determining a way to solve the problem, and actually solving the problem. Such strategies may even include a 'guess and check' method but students need to be aware of the strategies they have used and speak of their methods and explain what they did to someone else. In explaining their problem solving strategies, students engage in discourse and can learn from each other. Explaining a concept is another way for students to demonstrate their understanding of the concept (Wiggins & McTighe, 2005).

Another attribute for students to become better problem solvers is being motivated to improve and be aware of their learning. Because these students are not intrinsically motivated by the fun and joy of learning math, the motivation may need to be an intrinsic feeling of accomplishment fueled by confidence. Metacognition plays a role in teaching students to be aware of their own learning and the progress they can make when improving their confidence and abilities to problem solve.

Features and Activities

Solving Out Loud provides an opportunity for students to talk about math while engaged in mathematical problem solving. The activities also allow students to reflect on their learning and progress. The core activity in *Solving Out Loud* is a fast-paced mathematics activity that incorporates communication, discourse, problem solving, and reflection. The second activity allows students to openly discuss and debate a variety of topics that include math, motivation, and learning strategies. Both of these activities incorporate at least three of the features: problem solving, discourse, motivation/persistence, and metacognition as seen in Figure 1.

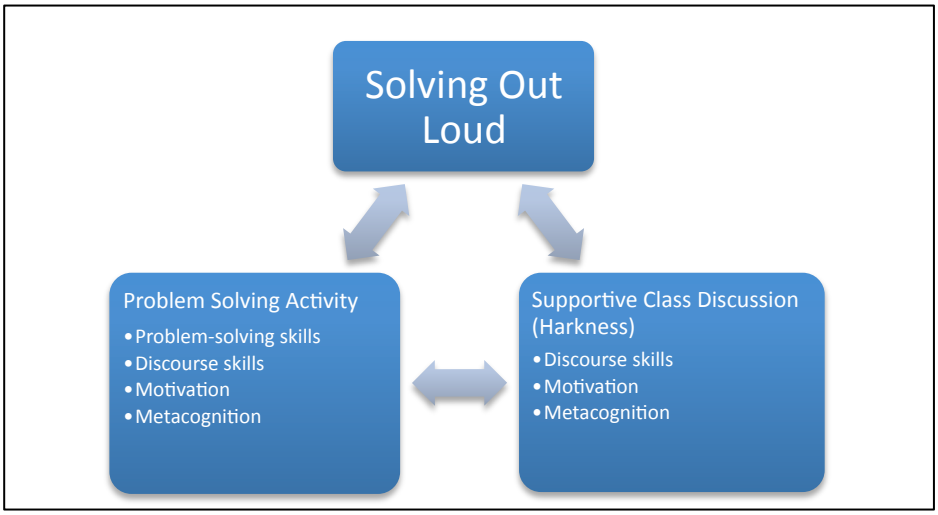

Figure 1: Components of *Solving Out Loud*

Problem solving

Problem solving strategies are discussed, practiced, and reinforced with *Solving Out Loud*. Students practice solving difficult problems supported by group conversations. The problem solving activity usually occurs at the end of the class period as a review of material learned in class. Students receive one to three challenging problems one at a time. In small groups, students have 60-90 seconds to come up with a plan for solving the problem. As a whole class, students discuss the plans and may come to an agreement on a single strategy. The students then solve the problem either individually or in their small groups. Lastly, the class discusses the solution and whether or not their method of solving the problem was reasonable. This process can be cycled through one to three problems per class depending on the amount of time and/or variety of problems that are applicable to the day's lesson.

In choosing problems that will be used for these activities, it is very important that the problem is challenging but within the Zone of Proximal Development (ZPD) of the students (Vygotsky, 1978) meaning that the problem is not one they have yet mastered but one that is possible for them to learn with support. If the problem is too easy, students do not want to talk about the problem but rather jump straight to solving it. If the problem is too difficult, the students do not yet have confidence to attempt it. I began selecting problems that were similar to problems students tended to generally skip on homework assignments. These problems are often the challenging ones that require more reading and thought. Through these problem solving activities, students are able to develop confidence in their mathematical problem-solving abilities and learn how to approach challenging problems.

Discourse

There are two forms of discourse activities incorporated into *Solving Out Loud*. The first activity is the fast-paced problem solving activity. Students practice and perfect discourse skills during this activity as they share their problem solving strategies. The other activity is a supportive class discussion that provides students a time to converse in class about any topic the teacher proposes. The teacher may ask students to discuss the reasons for learning a certain concept or the connections a concept has to a current event. These class discussions are similar to a Socratic Seminar, which is often based on a text, or passage students have read, or a Harkness discussion, which begins with a debatable and intriguing question. These

discussions happen mostly once a week and are a time for students to reflect, ponder, question, and debate various ideas or concepts.

Both of these discourse activities require little to no participation from the teacher. Students begin to listen and speak to each other without looking to the teacher for reassurance. The teacher may need to steer the conversation or ask further probing questions but for the most part the teacher's role is that of a bystander, listener and observer. From my experience, it is often necessary to remind students "pretend I am not here, and speak honestly" or "remember that I will not confirm your statements, you need to listen to each other."

In order to lead a good class discussion, it is important for the questions to be contentious and stimulating. It must spark interest, wonder, and allow students to really dissect the topic. Examples of class discussion questions that I used include: "What motivates you?", "what makes a good math student?", followed by "is being a good math student a natural talent or something you can get better at?", and "in what real world situations might you use <insert topic>?".

The kind of discussions used in *Solving Out Loud* work best when students are facing each other and sitting around a table together. Arranging desks into a circle, U-shape, or rectangular formation will allow students to face each other or it may be best to have students form a circle either standing or sitting. The key factor is for students to face each other so they can listen, respond, and see each other during the conversation.

Figure 2: Classroom formation for *Solving Out Loud*

Motivation

Motivation is a key component to student learning success and plays an important role in student growth and improvement in math. Students often have a self-defeating mindset about their mathematical ability. Motivation can help improve this mindset by allowing students to understand their own ability to improve. A student's motivation to do well can come from a variety of intrinsic and extrinsic factors. Because of this, it is difficult to motivate students through the sheer fun of learning math. Instead, it is important to

motivate students through the idea that it is possible for them to be successful and understand math (Dweck, 2006).

Solving Out Loud explicitly incorporates motivation. Students are often asked to discuss motivating features during the *Solving Out Loud* discussions or they individually reflect on their motivation after the problem solving activities. I found it necessary to discuss motivation with my students and I wanted them to be reassured that despite the different levels of motivation students may have, together they can encourage each other to improve problem solving strategies.

Metacognition

Similar to motivation, metacognition is an important component of *Solving Out Loud*. In order for students to improve their problem solving skills, they need to be aware of what they have done in the past and what skills they need to improve (Zemelman, 2005). Many students fail to reflect and think about their learning and cognitive patterns. *Solving Out Loud* gives students an opportunity to have internal reflection as well as group reflections to know if their learning and problem solving strategies are on track and to become aware of areas for improvement. Through the metacognitive process students can also become aware of their successes, thus supporting motivation as well.

Settings Where *Solving Out Loud* Can Be Used

This curriculum was designed for a high school geometry class. Although I designed the curriculum for geometry content, it can be used for other math courses as well as other grade levels. This curriculum could

possibly be useful with science curriculum or any course that wishes to improve students' problem solving and discourse skills.

Overall Goals of the Study

Solving Out Loud is designed to improve discourse skills, problem-solving skills, motivation, and metacognition in students. An evaluation of data from the students' work, recorded class conversations, teacher field notes, homework, and pre and post surveys was designed to determine the effectiveness of *Solving Out Loud*. An account of the implementation of *Solving Out Loud* is provided in the next chapter.

Chapter VI. Implementation of *Solving Out Loud*

I implemented *Solving Out Loud* in a high school geometry class. The class met three times a week with one 45-minute period and two 90-minute periods. I employed the curriculum activities two to three times a week. The topics of the class during implementation were the study of right triangles, the Pythagorean theorem, and right triangle trigonometry all of which was supported by *Discovering Geometry* (Serra, 2008), our class textbook.

This chapter provides a detailed description of the setting and the teacher of *Solving Out Loud* as well as a narrative of its implementation. Because *Solving Out Loud* has two types of class discussion activities, description of the implementation of these activities are separate and not necessarily in chorological order.

The Setting of *Solving Out Loud*

I implemented my curriculum at a small independent school, Mountain View Academy (as stated previously, all names of people and places used in this paper are pseudonyms). This school opened in 2007, with the current enrollment being approximately 350 students in grades seven through twelve, of which forty-eight percent are males and fifty-two percent are females. Enrollment is based on a completed application, interviews with the parents and students, Independent School Entrance Examination (ISEE) results, transcripts, and recommendations. The school charges tuition; however approximately forty percent of the students receive tuition assistance.

The mission of the school has three foci: academic excellence, ethical responsibility, and global engagement. Students learn about each aspect through academics, clubs, arts, athletics, service learning, and global travel programs. Two key philosophical cornerstones of the school are interdisciplinary integration and the use of Harkness discussions as an instructional method. Each teacher sits on a grade-level team to develop and implement integrated projects and topics throughout the school year that often tie into the mission of the school.

Solving Out Loud can be implemented with any general class discussion, however Harkness was the method used during this implementation. The Harkness discussion is named after Edward S. Harkness who in 1930 donated a large table to Philips Exeter Academy (Philips Exeter Academy, 2010). This method of instruction allows students and the teacher to sit at the table together to share and discuss ideas. In a typical discussion of this type, the teacher poses a Harkness question for the students and they converse, debate, and/or share ideas. An example of a Harkness question for a film class may be, assuming the students had viewed the movie PYSCHO, “What roles does lighting play in Hitchcock's PSYCHO?” (G. Cooper, personal communication, February 9, 2011). All students are involved, from asking probing questions to summarizing. Ideally, they are motivated by each other to carry the conversation to intellectual and interesting points.

Figure 3: Traditional Harkness table

The Teacher

I have been teaching mathematics for seven years; each year I have taught least one Geometry class. My first two years of teaching were at a large comprehensive high school in New Jersey. I taught two different levels of Geometry with at most thirty students in each class. Some of my classes were co-taught with a special education teacher. Many of the students had special needs such as IEP or 504 plans. The co-teaching environment was unique and productive. Students received individual assistance and we could differentiate the instruction so all the students were learning in the best way for

them. This experience taught me to design my lessons for all learning types. I also learned the investigative approach to learning. I began to design my lessons so the students would discover the geometric properties followed by application.

My next two years teaching was at a startup charter school in Southern California. When I was hired, I was the only math teacher. I designed the curriculum and picked all the textbooks for the three math courses we offered. For the two years I taught there, the school was an independent learning charter school. This structure challenged my teaching methods and altered my approach to curriculum design.

I am currently at Mountain View Academy for my third year. I was drawn to this school by their mission, which ends with the words "…and prepare students for a purposeful life." Can we really do that while teaching students how to solve a system of equations? For the past two years at PRS, I have been able to reflect, revamp, and reinforce my teaching style. I have also created advisory curriculum and held the position of Activities Coordinator.

In my classroom, I also try to provide meaningful contextual connections for my students. It is important for me to see my students in interacting venues outside the classroom. I try to watch the soccer games and play performances. My most valued take away from these observations is that my class is not the most important thing in their life. I know that math will not

be every student's favorite class but my hope is that all my students understand it, have confidence in their ability to learn it, and appreciate it.

This has led me to develop *Solving Out Loud*, which incorporates classroom discussions to improve problem solving, discourse, motivation, and metacognition. As a teacher, I struggled with the large class discussions and had a difficult time taking myself out of the conversation and allowing the students to converse in Harkness style. I wanted an opportunity to hone my own teaching skills as well as improve the skills of my students.

Prior to implementing *Solving Out Loud*

First, students completed a survey designed to assess their perception of their problem solving skills, motivation, and even the use of (Harkness) discussions in class. I made the survey with guidance from *The General Self-Efficacy Scale* which was originally developed by Matthias Jerusalem and Ralf Schwarzer in 1981 (The General Self-Efficacy Scale, 2011).

Next, with SmartBoard software, I made a set of slides I originally called "discourse practice." The discourse process was divided into parts highlighted on the accompanying slides. Slide #1 stated, "Discuss the steps needed to solve this problem." Students were then given 60-90 seconds to discuss the problem in small groups. As a class, the students discussed their ideas for solving the problem and usually came to a consensus. The next slide declared, "Now solve it" and students carried out the discussed plan to solve the problem. The third slide asked, "Did we determine a reasonable way to solve the problem?" As a class, we reviewed their problem solving plan and

the answer to the problem. Following this format, students answered one to three questions that incorporated concepts learned in class that day.

As a review of what they learned that day, I tried this method of discourse practice at the end of class for three class periods. Each time there were two or three problems for the students to complete. I picked challenging problems that were similar to homework problems.

After these first three trial runs, I noticed some important factors about the questions the students were solving. The most successful discussions were about challenging problems with multiple steps needed to reach the solution; if the problems were too easy, the students did not want to discuss the problem. I also realized the students needed a designated space to write out their plans and a space to solve the problem. A few students were not writing anything down and had no way of illustrating their understanding of the problem beyond the discussion. Finally, I realized that this activity alone was not enough for students to practice discourse skills. I needed another activity to scaffold the practice of discourse skills for a student-centered discussion. My curriculum needed another opportunity for large class discussions as well as practice in discourse and problem solving discussions.

Students discussed discourse and problem solving

I continued my pre-implementation with two class discussions. For each discussion, students brainstormed ideas in small groups of three students before they shared their ideas in the large class discussion. One

student compiled a list on the board from all the ideas that were shared during the class discussion.

The first discussion was around the question “What does it mean to participate in a class (Harkness) discussion?” The students did not hesitate to generate a list of ideas. Once the groups were finished and they shared their ideas, one student wrote all the ideas on the whiteboard. A few students debated about the semantics of some words such as the difference between not interrupting and being respectful. They all finally agreed on this list. From this list, I was able to create a rubric that I used to tally the students’ participation during each supportive class (Harkness) discussion (see Appendix). Figure 4 shows the list they generated:

Class Discussion (Harkness) Participation:
Record/take notes
Listening/paying attention
No interruptions/be respectful
Use useful info and details
Debate (both sides of the issue/topic)
Summarize/confirm key points
Responding/building off ideas of others
Ask clarifying questions

Figure 4: Student generate list of ways to participate in a class discussion

The second supportive class (Harkness) discussion was about problem solving. I asked the class to generate what they thought were the steps to solving a problem. This discussion followed the same format at the previous one except that each group wrote their steps on the large whiteboard and we compared the different answers side by side. We synthesized their words and

ideas. Their steps were all very similar. However, the students did not come up with the word 'plan' and although they used a variety of other words that all meant 'make a plan', I had to present that word to them and they all agreed to use it. Here were the results:

Steps to problem solving:
(1) Read and understand the problem
a. Know what the question is asking
b. Reread the problem
c. Break the problem down
(2) Make a plan
a. Write down the steps needed to solve the problem
b. Organize information
c. See what you can do with what you have
d. Draw a picture
(3) Solve (executing the plan)
a. Attempting the problem
b. Show your work
c. Draw a picture
d. Trial and error (guess and check)
(4) Check your answer

Figure 5: Student generate list of steps to problem solving

What struck me during this conversation was the similarity of these steps to Polya's (1945) heuristic method of problem solving and the students' blatant disregard for the steps. One student said, "you have to read and understand the problem then solve it, but I never read the problem." Many students admitted to not reading problems or checking their answers. Although students were aware of the problem solving strategies, they did not put them to use. I know that *Solving Out Loud* must guide students through these steps and help students internalize the procedures by giving them the

opportunity to think about their problem solving skills and reflect on their effort and knowledge of solving problems.

Implementation of *Solving Out Loud*

Solving Out Loud has two main components: supportive class discussion and the problem solving activity. The problem solving activity is the core of *Solving Out Loud* and is best suited for the end of a lesson while the supportive class discussion (Harkness) activity is best at the beginning of a lesson spread sporadically throughout a unit. During implementation, each day consisted of a lesson to introduce the new topics with the support of *Solving Out Loud*. *Solving Out Loud* provided the necessary support for students to practice their problem solving skills as well as practice the new content they were learning. Table 2 shows the timeline of the implementation.

Table 1: Implementation schedule of *Solving Out Loud*

Day	Topic of Study	Activities
1	The Pythagorean Theorem and it's converse	Supportive Class Discussion: Who makes use of right triangles and why?
2	The Pythagorean Theorem and its converse	Supportive Class Discussion: How has math made your life easier?
		Problem Solving Activity: Finding the longest diagonal in a rectangular prism
3	Two Special Right Triangles (45-45-90 and 30-60-90) Chapter 9 Quiz	Problem Solving Activity: Find the perimeter of a rectangular field given the diagonal and one side.
4	Group work	Problem Solving Activity: (1) Find the coordinates of a point on a unit circle if the angle measures 45º. (2) Find the amount of paint needed to paint the eaves of a house.
5	Story Problems	Supportive Class Discussion: What motivates you to do what you do?
		No Problem Solving Activity: due to in class story problems activity.
6	Distance in Coordinate Geometry	Problem Solving Activity: A ramp connects a platform with a sidewalk. Given the length of a platform and the distance it is from the ground. Find the distance from the base of the platform to the sidewalk.
7	Test	
8	Right Triangle Trigonometry	Supportive Class Discussion: What makes a student a good math student?
		Problem Solving Activity: A boy is flying a kite. Given the angle of elevation and the length of string the boy let out, find the height of the kite from the ground.
9	Trigonometry Application	Problem Solving Activity – determine the height of the flagpole using trigonometry and a clinometer.
10	Law of Sines and Cosines	Problem Solving Activity: The CCTV tower in Beijing, China is not perpendicular to the ground. Using the law of sines, find the angle at which the tower leans.
11	Story Problems with Trigonometry	Problem Solving Activity: No PSA due to in class story problem activity.
12	Trigonometry Quiz	

Implementation of the Supportive Class Discussions

I decided to use the supportive class (Harkness) discussions and discourse in my geometry class because, although the students have Harkness discussions in English and History, they are unfamiliar with this type of discourse in a mathematics classroom. Most discussions in a math class are teacher driven and teacher-centered. Students look to the teacher as the main, and sometimes only, voice of knowledge. It was through my attempts to get students to openly discuss their mathematical problem solving strategies that I decided to focus on improving their discourse skills and problem solving skills through communication. Ten out of the thirteen students in this particular class were lower performing math students who often do not complete homework assignments, perform poorly on assessments, and lack confidence in their ability to do math and speak about it. Accordingly, the students' low achievement influenced my decision to use supportive class discussion in math as a means to promote motivation and problem solving.

Day one began with the discussion (Harkness) question, "Who uses right triangles and why do they use them?" I used this question because I wanted to tap into students' prior knowledge of right triangles and to introduce the new unit of study. The conversation started with some excitement but after just a few minutes the conversation stopped. One student said, "Ms. King, you really need to work on writing better Harkness questions." So began my process of writing discussion (Harkness) questions. From the perspective of my colleagues at my school, discussion (Harkness) questions cannot be too

narrow or steer students to a particular set of answers. Some even suggested having students generate the questions to be used in the next class.

Two weeks later, after a mid-winter break, I revisited the supportive class discussion and asked the class to discuss this question "How has math made your life easier?" The conversation, which was recorded with a digital video camera, went like this:

> Student A said, "It hasn't!"
> Student B tells a story of how she corrected someone who gave her incorrect change.
> Student C said, "Cooking," which prompted this response from student A "oh, that's a good one".
> Student D: "I don't know if you guys saw The Social Network but they used a lot of math equations to make that website, I mean a lot and they were really complicated"
> Student E: "Yeah, I took a class last year where we made a website and it took all year to make the website and it involved a lot of math"

My goal for this conversation was to remind students of the everyday math they encounter and that they often use math without always being aware that they are using math. A few minutes into the conversation, I realized that the students had nothing to debate. They all just listed information and everyone agreed with each other. There was nothing captivating or debatable about this first Harkness question. This conversation was just like the first. There was no debate, no questions, and very little excitement. Although these two supportive class discussions did not spur a wondrous conversation, they did give students the opportunity to engage in dialogue. Almost all students were involved, listening and offering ideas.

Stepping away from the topic of math or geometry, I decided to use the supportive class discussions as a means to get at issues that are rarely discussed in the classroom. I asked the students this question "What motivates you to do the things that you do?" The students openly debated and shared ideas, concerns, and situations that resulted from different motivating factors. The conversation lasted for roughly twenty minutes. Here are some examples of what they said are motivating factors in their lives:

My mom
Rewards
My parents
Competition
The urge to be successful
Because I enjoy them
Sports – competition
How it is going to help you in the future…will motivate what you want to do
You can always be motivated to study so you can be successful
Something that really motivates me is the reward
I could really care less about the reward
Being proud of yourself….like getting 100% on a math test

Figure 6: Motivating factors expressed by individual students

These statements show that each student had their own individualized motivating factors. Some were motivated by grades or to please parents; no student mentioned that they were motivated to learn for the sake of learning math. It appears that learning math was just the means to achieve something else. At this point, it was unclear to me if *Solving Out Loud* would be able to increase students' motivation in learning math.

I followed up the first question with this question: “at your age and in your life what are things that you are motivated to do, that could be academic/nonacademic?” Students provided very honest and candid answers. One student brought the conversation to academics and suggests that the teacher can help motivate students to learn math besides just knowing the material for a quiz or test. Here is a sample of this conversation:

> Cindy: Even though we are the youngest we are all motivated to go to college
> George: I am the exact opposite; I am not motivated by academics. I am in the ‘now’ kind of guy…I am motivated by having fun and living life
> Jordan: Yes you want to have certain goals but as George said you want to live your life in the now you are only going to be this age once.
> Luke: You should have a balance between having fun when you can or when it’s appropriate and do your school work when necessary.
> Brooke: I often question, why am I doing this but the idea then becomes….’why not study for the test, why not cheer on the soccer team’

The class discussion above provided insight not only into the students’ motivation but also into their discourse skills. The students did not hold back their opinion when appropriate, they corrected each other when someone interrupted, they rebutted someone’s statement, and they spoke with confidence. The students’ comments helped me revise the reflection questions that I asked during the problem solving activity portion of *Solving Out Loud*. I needed to ask questions that pertained to students’ motivation and whether or not they were interested in learning the concepts and in improving their problem solving skills.

Four days later we had our next supportive class discussion. The discussion question was, “what makes a student a good math student?” I asked this question because I wanted to see if the lack of motivation was coming from an internal believe about math students and their perception of what it takes to be a math student. A majority of the students stated that it was up to the each individual student to try their best and study. Some students expressed the idea that the teacher plays a part to make sure students are getting the material before they move to other topics.

These supportive class discussions provided me with an opportunity to hear my students talk in class in a format that we had never used before. It also gave them a venue to talk about topics that are rarely discussed in a math class as well as the chance to practice their discourse skills. They were beginning to develop discourse skills through these conversations and a culture of conversation in the class. *Solving Out Loud* allows students the opportunity to build discourse skills in a variety of ways and to build a cohesive class community through these conversations.

Implementation of the Problem Solving Activities

The other portion of the implementation of *Solving Out Loud* was of the problem solving activities. These activities were fast-paced. They were designed to give students enough time to think through the problem but the time is limited so if they happen to be solving it incorrectly they would not feel like they had wasted time. Heinz (2010) suggests that “capable and confident problem solvers and legitimate contributors to mathematical discourse” (Heinz,

2010, p. 318) can be developed through a particular discourse setup. Heinz (2010) suggest a class setting that allows students to work on a problem in groups for a short duration in time followed by a student lead discussion in which students were able explain and rephrase strategies that other students may not have thought of.

Solving Out Loud incorporates this design. The conversation that takes place during the problem solving activities gives students this necessary feedback and the chance to hear other methods for solving the same problem. Figure 7 shows the scaffolded steps of problem solving activities from *Solving Out Loud* and the goals associated with each step.

Steps of the problem solving activities	**Goal**
Reading the problem and determining a plan	Students improve problem-solving skills by reading and understanding the problem.
Discussing the plan for solving the problem	Students improve discourse skills and develop confidence in their problem solving abilities.
Carrying out the plan and solving the problem	Students improve problem-solving skills and develop confidence in their problem solving abilities.
Reflecting on the answer and the steps taken to reach the answer	Students determine if they have a reasonable answer and reflect on the problem solving process.
Reflection questions	Students individually reflect on the learning process.

Figure 7: The steps of the problem solving activities from *Solving Out Loud*

The first problem students faced was to find the length of a diagonal in a prism. This problem met the right level of difficulty because no student was able to answer it immediately. The conversation that followed the small group discussions showed that many students did not trust their initial ideas on how to solve this problem. They eventually came to a compromise but I needed to help steer the conversation in the right direction by asking clarifying questions of some student comments. Such as “can you use the name of the line segment you are referring to, such as $\overline{AC}$ or $\overline{AG}$?” or “can you explain why finding the area may help us answer this question?” I could tell that some of the students were getting frustrated with this problem. A few students understood how to solve it but had a hard time articulating their method where as other students were just staring at the board not sure where to start or who to listen to. The problem is illustrated in Figure 8.

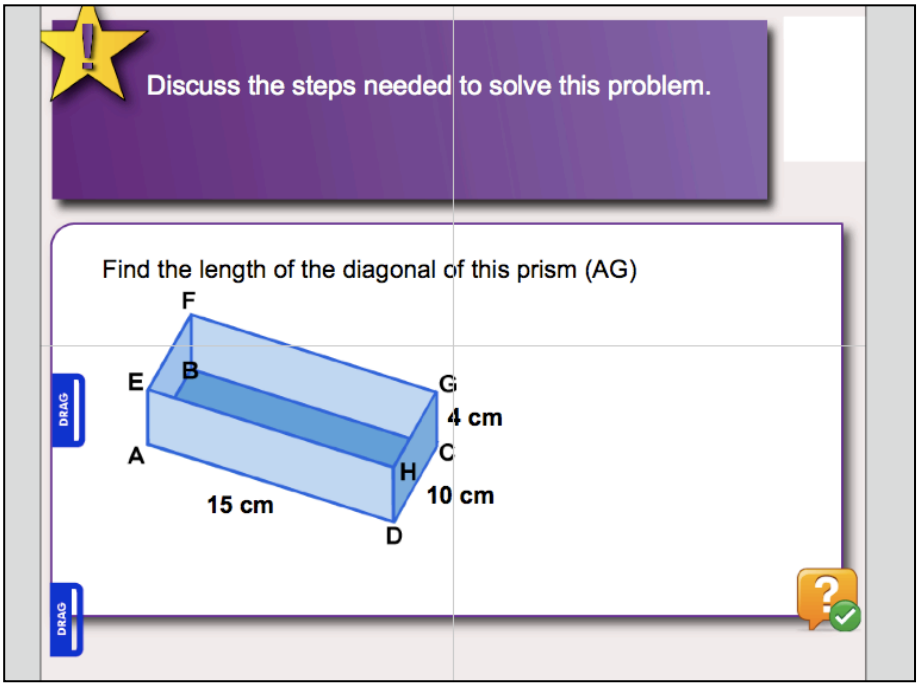

Figure 8: Sample problem from *Solving Out Loud's* problem solving activity

Problem from http://exchange.smarttech.com (2011)

After the conversation, I realized that many students were not writing. Instead, they were relying on one group member to do all the writing for the group. I needed to provide students with a worksheet that would allow them to fill in their ideas and even reflect on the process afterwards. I created a worksheet with four squares providing students the space to write work or ideas that require multiple stages. Using the four square worksheet allowed students a place to write their work as well as a place to answer the reflection questions at the end of each activity. The four square worksheet is provided in Figure 9.

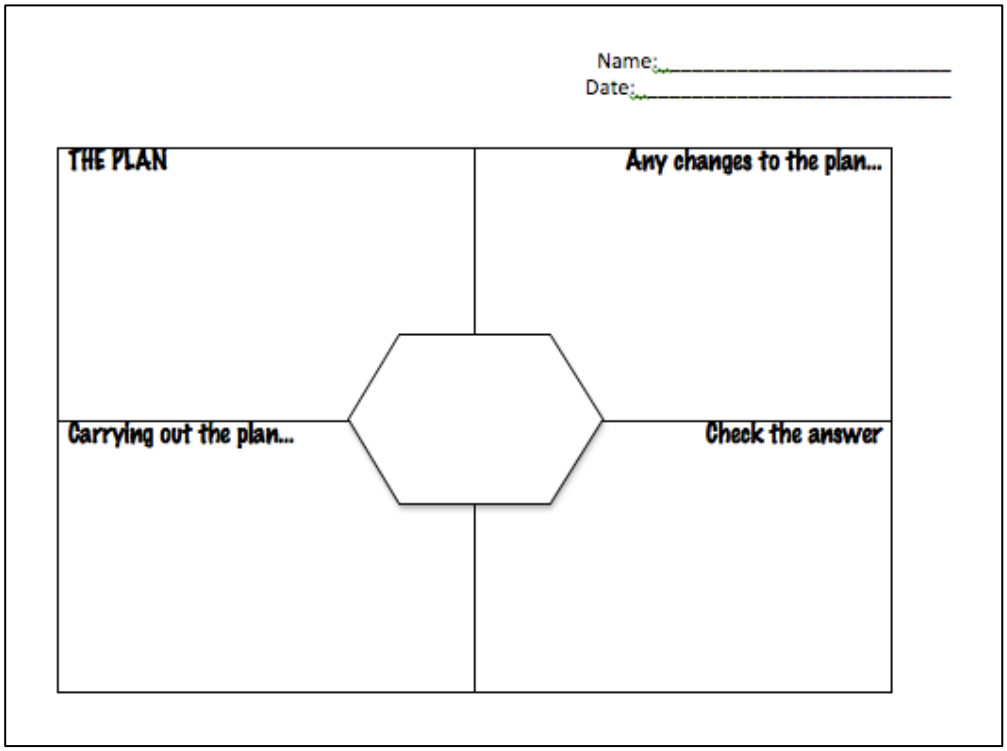

Figure 9: Four Square Worksheet used for *Solving Out Loud*

I conducted the problem solving activity in the next class meeting. Some students cheered with delight, “oh yes this activity again”; others remained indifferent to the process. One student in particular, who usually passively sat in class, changed his whole body language. He sat up straight, got his pencil in hand, and was eagerly waiting to see the problem. Once I revealed the problem he got right to work with his group. These actions were the most I had seen from this student all year. He was communicating, smiling, and working on math.

This problem solving activity followed its normal course. The students had just sixty seconds to make a plan to answer the question and all students were engaged in small group discussions during this minute. The problem was to find the amount of fencing need for the perimeter of a rectangular field given the length of the diagonal and the length of one side. The conversation that followed confirmed that students were becoming more confident in their problem solving skills but that many were still making mistakes and not using the discussion to its full advantage. A few mentioned the need to use Pythagorean Theorem and some students suggested drawing a picture.

After the conversation, all students were eager to solve the problem. When checking the answer, many students made the mistake of not finding the perimeter. They simply found the missing side length. Going back to 'check your answer' proved to be a productive step for many students because they failed to answer the actual question. In finding just the unknown side of the field, they failed to find the perimeter. Since checking answers was not a common practice for these students, it was important for *Solving Out Loud* to have this process for students to learn its value.

That same student mentioned above who was eager to get to work did not answer the question correctly. His group treated the triangle as a $30° – 60° – 90°$ triangle. This was most likely because students learned about special triangles that day. Because of this, I stressed the importance of drawing a picture and making sure they had all the necessary information and are not assuming anything to be true that is not present in the problem.

Moments like this show the importance of the teacher as listener and supporter of the student-centered class discussions in *Solving Out Loud.* Even when the conversations are student-centered a teacher may need to interject when inaccurate information is taking the conversation on a tangent. This was more evident on the next day of implementation.

On the fourth day of implementation, the students had to find the coordinates of a point on a unit circle where the angle measured 45º. The students had previously studied circles and although they had not seen a unit circle before in this manner, the problem required them to make a right triangle and use their new knowledge of $45° – 45° – 90°$ in order to find the solution. The problem proved to be a struggle for the students and I had to probe their conversation a little more than I had on previous discussions. Once students realized they needed to create the right triangle they were able to solve the problem. Again, the class conversation helped students to solve the problem by hearing many of the possible methods.

The following day students practiced their problem solving skills with right triangle story problems. Although the students had experienced the problem solving activities a few times, they were more motivated to work on these story problems than they were in the previous units. Almost all students got to work immediately and each group conversed about the problems instead of silently working on the problems. Implementing the problem solving activities followed by class time to practice more problems in groups was a

valuable addition to *Solving Out Loud*. This practice time gave students time to apply their understanding and master the content (Bransford, 2000).

During the implementation, I asked students to provide feedback regarding this activity and asked for their opinions to make it better. They all wanted to change the four square worksheet. Their suggestions included: needing more room, do not want to rewrite the work if we are making changes to it, and do not check the answer. Using this information, I adapted the four-square sheet to the model seen in Figure 10. Allowing students the chance to provide feedback and suggestions can give students a sense of ownership of their learning as well as motivation to continue learning (Deci, 1996).

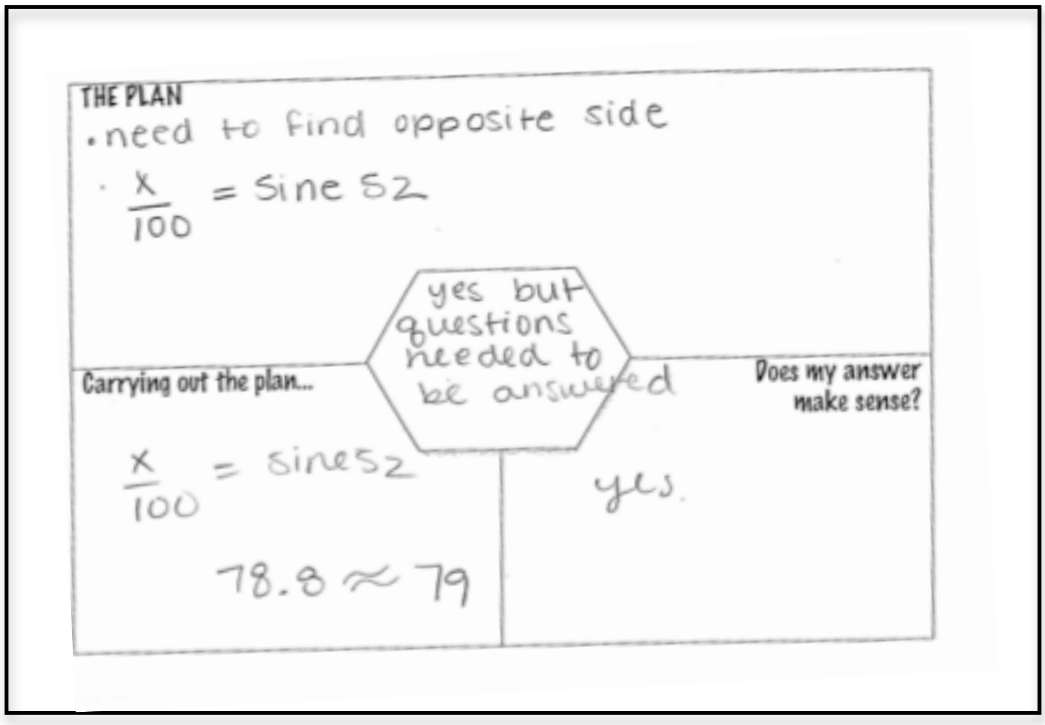

Figure 10: Adapted four square worksheet from student feedback

I continued to implement the problem solving activities in three more class meetings. I tried selecting problems in the ZPD for the students but at times that proved to be a challenging task. For each implementation, students were able to debate various problem-solving strategies and most were able to succeed in answering each question. When the first implementation was over, I continued to use *Solving Out Loud* in this class and I introduced it to my two Advanced Algebra classes with much success.

Incorporating Motivation and Metacognition into *Solving Out Loud*

Solving Out Loud incorporates motivation and metacognition throughout all the activities. Motivation and metacognition were explicitly discussed with students and they became aware of their own motivation and metacognitive abilities. Students were asked at the end of each problem solving activity to reflect on the activity itself or on their own learning. Some reflection questions include: how did the discussion help you answer this problem?, what do you like and/or dislike about this activity?, and did this problem need a discussion?

During pre-implementation and post-implementation, students completed a survey intended to measure their perception of problem-solving and discourse skills. The activities in *Solving Out Loud* incorporated the development and awareness of metacognitive thinking and reflection.

Throughout the implementation, I found it important and useful to explicitly talk about these ideas with my students. As teachers, we may expect our students to be self-reflective but from my experiences, students do not know how to do this. I think that through *Solving Out Loud*, students were

able to learn about their own motivation and bring awareness to their learning in a way they had never done before in a math class.

Conclusion and Modifications of *Solving Out Loud*

Overall, the activities in of *Solving Out Loud* are designed to support student problem solving skills by allowing them to practice challenging problems in class and learn from their peers through class discussion. The activities were also designed to improve discourse skills, motivation, and metacognition. *Solving Out Loud* allows students to make mistakes in a supportive environment where they can quickly learn from these mistakes and adapt their problem solving.

The goals of *Solving Out Loud* were to improve students' confidence and ability in problem solving as well as to improve students' mathematical discourse, motivation, and metacognition of mathematical learning. Chapter VII will address the findings that show that the goals of *Solving Out Loud* were met.

Chapter VII. Evaluation of *Solving Out Loud*

The goals for *Solving Out Loud* were to improve students' confidence and ability in problem solving, discourse skills, motivation, and metacognition in math. I focused on activities to improve both discourse skills and problem-solving skills. These focused activities allowed students to openly discuss mathematical ideas as well as practice their problem solving strategies. I also incorporated student motivation and metacognition into those activities.

Data Collection Strategies

To evaluate *Solving Out Loud*, I used a variety of data collection strategies. I made observations and wrote field notes during the supportive class discussions and the problem solving activities. The field notes took between five to fifteen minutes per implementation. I also digitally recorded these conversations and activities. At the end of the implementation, I watched the recordings and added additional notes to my field notes. Students also completed a survey before and after implementation. I collected and evaluated the students' work throughout implementation.

Problem solving

I evaluated students' problem solving skills with student work completed during the problem solving activities. Student work during the problem solving activities was in the form of a written four square worksheet created for *Solving Out Loud*. This work was analyzed for any changes in problem solving ability and use of problem solving steps by the students. I also evaluated students' pre and post survey. I also evaluated students' work done

on homework and assessments before and during the implementation. The completion of challenging homework questions was reviewed for any changes in their attempt and ability to solve challenging problems.

Discourse

Discourse skills were evaluated throughout the implementation using digital recordings, field notes, rubrics, and student work. For the problem solving activities, I evaluated students' contributions to the conversations using field notes and the digital recordings and compared them to the work they completed during the activities. In particular, I looked at the amount and type of contributions students made to the class discussions. I compared my field notes and rubrics from activity to activity as well as from beginning to end of the implementation. A rubric was used for the supportive class discussion. This rubric was generated from a class discussion on participation; the students created and approved the rubric shown in Figure 11.

Discussion ⇓ Students ⇒													
Listening/paying attention													
No interruptions/be respectful													
Use useful info and details													
Debate (both sides of the issue/topic)													
Summarize/confirm key points													
Responding/building off ideas of others													
Ask clarifying questions													
Record/take notes													

Figure 11: Rubric for supportive class discussion

Motivation

Motivation was evaluated throughout the implementation of *Solving Out Loud*. I took field notes indicating students' involvement and interaction in the class discussions, students' body language demonstrating engagement in the activities, and overall attitudes. I paid particular attention to engagement and attitude towards the class discussion and activities. I compared observations from activity to activity as well as from beginning to end of implementation.

Metacognition

I evaluated the students' awareness of metacognition and their ability to judge their own learning and improvement using a pre and post survey

instrument. I also evaluated students' answers to reflection questions from the problem solving activity, which were asked after each implementation. Student responses were compared from activity to activity and over the entire implementation process.

Pre-Implementation Survey Sample Questions:
Response Format
1 = Not at all true 2 = Hardly true 3 = Moderately true 4 = Exactly true
I can always manage to solve difficult problems if I try hard enough. 1 2 3 4
I can solve most problems if I invest the necessary effort. 1 2 3 4
When I am confronted with a problem, I can usually find the solution. 1 2 3 4

Post-Implementation Survey Sample Questions:
When you arrived at a math problem you did not know how to solve, what do you do and why?

On a scale from 1-5 (1 being poor and 5 being excellent), how would you describe your problem solving skills in math and why?

Figure 12: Example of survey questions for pre-implementation and post- implementation surveys

Findings

Goal: Improve students' confidence and ability in problem solving

Problem solving finding #1

My first finding indicates that persistent problem solving increased confidence over the course of implementation.

Before implementation, students were often skipping challenging homework problems and not receiving full credit on their homework assignments. When we discussed these problems in class, students did not engage in conversation on strategies to solve the problem. Instead they sat, often avoiding eye contact until we moved on to another problem. From these

homework assignments and lack of class discussion, it was clear that students were not confident in their ability to problem solve. They made little to no effort to attempt these problems or talk about them in class. Only twelve out of thirty-seven homework assignments averaged the top score of 4 out of 4 points before the implementation of *Solving Out Loud*. Four points is earned for full completion of homework, meaning that every problem was attempted, and for accuracy. The homework average before implementation was a score of 3.6 out of 4 points and during implementation the homework average rose to 3.78 out of 4 points. Although this is not a sizeable increase, this data suggests that students were gaining confidence in their ability to solve these challenging problems. Students gain confidence through persistence. The increase in homework averages suggests that students were persistent in attempting the challenging problems they once skipped which supports the development of confidence.

I also analyzed the students' assessment scores during the pre and post implementation of *Solving Out Loud*. Similar to the homework scores, the assessment scores rose only a small amount. The quiz and test averages before implementation was 82.7% and the average during implementation was 83.7%. Again, this suggests that confidence in problem solving abilities was on the rise during the implementation of *Solving Out Loud*.

While reviewing the data for the *Solving Out Loud*, it was clear that not all students reached their full level of confidence in their problem solving ability. While problem solving is a skill that may take a student years to fine-

tune and master, *Solving Out Loud* gave students the opportunity to begin the process and to be completely aware that this is something they need to focus on and work on in order to improve their learning. *Solving Out Loud* was just a starting point for further problem solving development because it was clear that problem-solving skills improved but the study duration was not long enough to show if these skills were in fact mastered by the students.

Problem solving finding #2

My findings indicate that students' problem solving skills improved over the course of implementation.

At the end of implementation, I reviewed the answers from the post-implementation survey. The first question that pertained to problem solving was for students to describe how the problem solving activity improved their ability to solve problems. I coded their responses into three categories: positive, negative, and indifferent. Positive responses indicated that the student felt their problem solving skills improved. A negative response indicated that the student believed that the problem solving activity did not improve their problem solving skills. Finally, an indifferent response meant that the student felt that their problem solving skills were neither improved or were undeterminable. Nine out of the thirteen students had a positive experience. It is also interesting to note the actual reasons why students had a positive experience with *Solving Out Loud*. A breakdown of student responses and comments from all students in the class are in Figure 13.

Negative:	**Positive:**
not really, maybe a bit	They helped by understanding how to break down problems
	making a pre-plan & checking plus speed
	It opened up the possibilities of doing problems in different ways, which helped me to use a trial and error process.
	It gave me more practice and hands on approach
	it helped me learn to take my time
Indifferent:	If I was stuck, the whole class helped
certain problems were helped but others were not	It helped me think about my plan first, not just dive right into the problem
I don't know	By setting up my own plan of action before attempting to tackle the problem head on, it helps my problem solving b/c I start on hard problems differently now
	It helped me plan how to solve the problems

Figure 13: Students' written responses to the effectiveness of *Solving Out Loud*

I compared the responses above to my field notes taken during the problem solving activity and found some correlating factors among the students' answers and my own observations. The two students who participated the least in the class discussions prior to the implementation had responded as having a belief that the curriculum had a positive effect on their problem solving skills, as indicated by responses in Figure 13. The single student who had a negative response to the problem solving question was often off task and had to be reminded engage in class activities. The student who answered indifferently with the statement that "certain problems were

helped but others were not" rarely spoke up in the discussions; sometimes he answered problems correctly and sometimes he did not. I could agree with both students that *Solving Out Loud* did not effectively improve their individual problem solving skills. This may not be the case if the implementation of Solving Out Loud was longer and these students were given more time to hone their problem solving skills.

Next, I reviewed and coded the work that students completed on the four square sheets during the problem solving activity. I categorized and coded the student work as correct (3), accurate but not complete (2), incorrect (1), or blank(-). Here is a sample of student work from the problem solving activity. Figure 14 illustrates student work that had a correct plan and reached the correct answer, this was coded as 2. Figure 15 shows the student had no understanding of the problem and did not solve it even after the class discussion; his work was coded as blank.

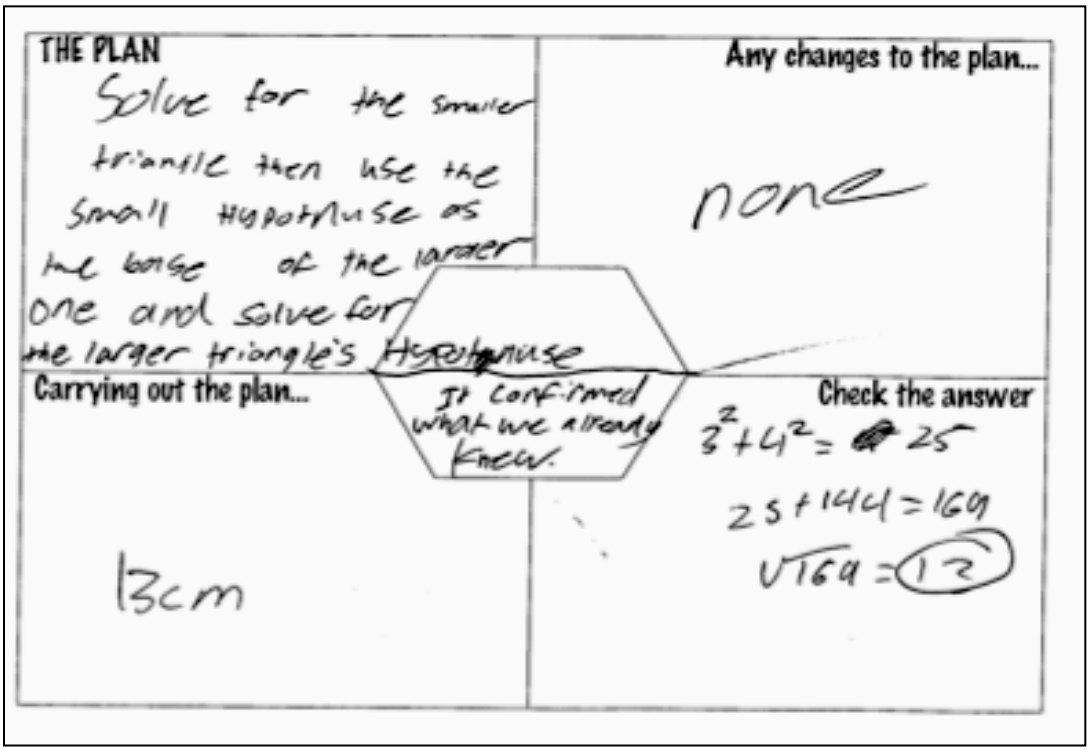

Figure 14: Example of student work with correct plan and correct answer

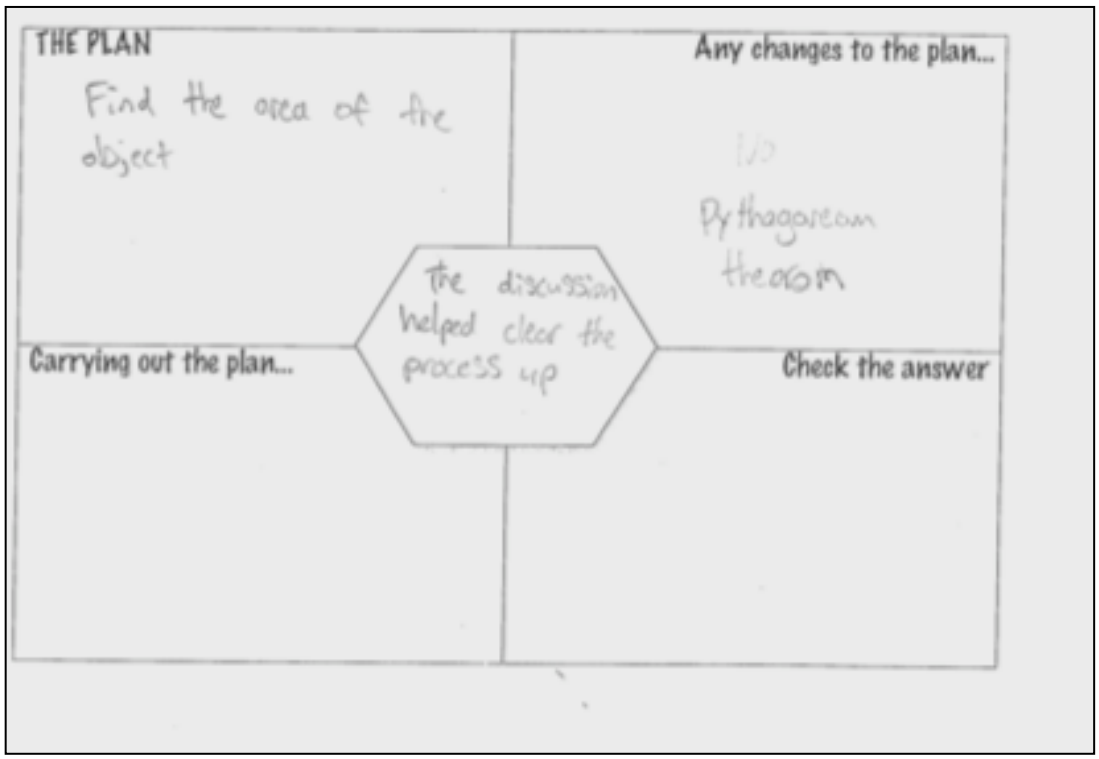

Figure 15: Example of student work of incomplete plan and no answer

I reviewed the work of all the students such as those shown in Figure 14 and Figure 15. At first, I reviewed the type of work done in the "Plan" section of the four square sheet and compared it to the whether or not the students reached the correct answer. For the first three sessions of the problem solving activity, many students did not write their complete plan. The number of students who wrote a correct plan for solving the problem increased from two students on the first implementation to ten students on the last implementation. Students were more efficient in writing out their steps for problem solving versus just stating "Pythagorean's theorem." This shows that

Solving Out Loud provided a scaffold for strengthening problem solving skills because students were able to completely answer challenging questions as the implementation continued.

Table 2: Breakdown of work done for the "PLAN" in the problem solving activity

	Question 1 (From Day 2)		Question 2 (From Day 3)		Question 3 (From Day 4)		Question 4 (From Day 6)		Question 5 (From Day 8)	
Type of work done for the "PLAN"	Number of student responses	Number of students with the correct answer	Number of student responses	Number of students with the correct answer	Number of student responses	Number of students with the correct answer	Number of student responses	Number of students with the correct answer	Number of student responses	Number of students with the correct answer
Correct	2	1	2	2	4	4	6	5	10	8
Accurate but not complete	6	6	9	8	8	8	0	-	0	-
Incorrect	2	0	0	-	0	-	5	4	2	2
Blank	1	0	2	0	1	1	0	-	1	0
Absent	1	-	0	-	0	-	0	-	0	-

Comparing the responses across the implementation period shows that there was an increase in correct plans and answers. I also looked at individual student responses and noted that the same three students were the ones who left their "plans" blank. One of these students was the same one mentioned in the previous finding who had a negative response to *Solving Out Loud* and

rarely participated in the discussions. This student did not improve his problem solving strategies but was honest in his personal evaluation.

The last two questions, shown in the Table 2, were from day six and day eight of implementation. For the implementation of these days the four square sheet had been adapted based on student feedback so just looking at the students' four square was not enough to determine if they has a complete plan. I had to compare the work they did with my field notes and the digital recording of the activity to conclude that a student had a correct plan or if they altered the plan after the class conversation. I was able to determine this through cross-referencing my data by the contributions the students made to the conversation.

Goal: Improve students' discourse skills

Discourse finding

Students' academic discourse improved over the course of implementation. In general, I also found that students became more willing to converse in class. Students increased their willingness to communicate and learned to listen to each other and not rely on the teacher as the only expert.

Students were able to improve communication about problem-solving strategies and learn from each other during conversations based on problem solving. Using field notes and digital recordings for the problem solving activities, I was able to determine that students did improve their discourse skills over the course of implementation. In my review, I looked for active listening, the sharing of ideas, and thoughtful responses. Finally, I reviewed

the change that occurred in student work during the problem solving activities after the conversations occurred. I looked to see if the conversation helped students solve the problem. I noted if students had an incomplete plan or an inaccurate plan but they had the right work and the right answer this suggested that the conversation helped them understand how to solve the problem.

The first implementation of the problem solving activity involved an informative discussion because many students did not understand how to solve the problem. Most knew that the solution would involve the Pythagorean theorem however they were uncertain as to why they needed it. During this conversation, the students debated and questioned each other and one student who rarely spoke up in the class provided insightful reasoning and explanation. My field notes show a row of exclamation points because this was the first time all year this one student spoke in class without being prompted. There were only three students not involved in this conversation and two of them did not find the correct solution to the problem.

The analysis of the student work from the problem solving activity, field notes, and digital recordings showed that at most 85% students were able to explain their ideas in the class discussion and that other students were able to listen and learn from these students. In total, there were 23 instances where a student had an accurate but incomplete plan yet still arrived the correct solution to the problem. There were nine instances where students had an incorrect plan yet six of those nine ended up with the correct answer. This

illustrated that the large group discussion helped these students learn the process needed to solve this problem and reach the correct answer.

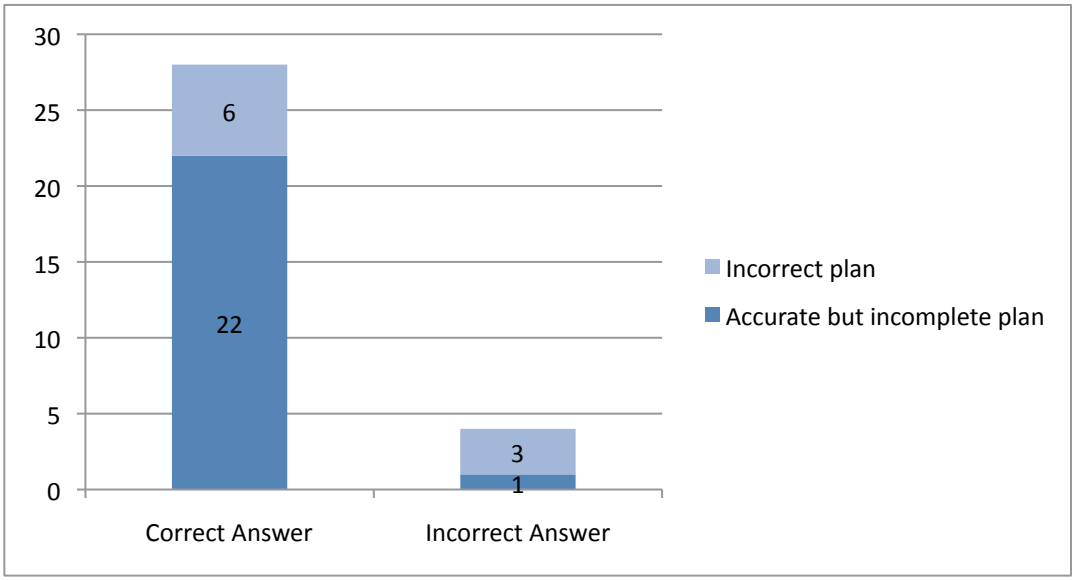

Figure 16: Breakdown of students' "plan" and number of correct or incorrect answers per "plan"

Next, I analyzed the rubrics from the Harkness discussions that were conducted during the implementation of *Solving Out Loud*. On the first day of implementation, one student said "you cannot do Harkness in a math class" and at the end of the implementation, this same student wrote, "I feel like they [Harkness discussion] are helpful and useful."

During implementation, I noticed that more students were engaged in the supportive class discussions. Instead of the same two to three students participating, there would be eleven or twelve students participating. Only one student did not participate in any of the Harkness discussions. This student was the student mentioned in the problem solving finding #2 who was indifferent to the activities. He would quietly speak with his small group but

never once participated in the supportive class discussion. This student also wrote “not sure if they (class discussions) are really helping.” With the majority of the students participating in the supportive class discussions as well as problem solving activity discussions, the students’ skills improved within these discussions. Although the students may not realize the immediate skills learned in a supportive class discussion, the improvement is clear in the conversation and the value the conversation had when students are listening, paying attention, and participating.

Goal: Improve students’ motivation

Motivation Finding

Solving Out Loud showed an increase in student motivation and persistence to participate in class and work through challenging problems. From the start of the school year, I noticed that this class of students would often skip the most difficult homework problems. In conducting the problem solving activities I used problems that were similar to the problems students tended to skip on their homework. I observed that students were motivated and persistent to complete these problems during the implementation. All students conversed with their group and almost all students engaged in the class discussions and attempted each problem.

There was also an increase of students who completed the problem solving activities with a correct answer. The first implementation had 54% of students reach the correct answer with accompanying work and the final

implementation had 77% of students reaching this same goal. This illustrates that students were persistent and motivated to complete the task.

Part of *Solving Out Loud* was to explicitly talk about motivation and give students the opportunity to discuss motivation factors for them in their learning. In addition to talking about motivation, *Solving Out Loud* provided opportunities for student input and feedback about the activities. The rubric was created by students and the four square worksheet was adapted from their suggestions and comments. Providing opportunity for input and ownership can help raise motivation (Deci, 1996).

Students also discussed what it means to be a good math student and they debated whether that was a natural talent or an achievable skill. The following sample dialogue illustrates the students' feelings about motivation regarding being a good math student.

> Teacher: Is being good at math a natural talent or something you can work at?
> George and Matt: Both
> Cindy: I like this class but in elementary school, I was bad at math but I always have to work hard at it, I'm not good at quick multiplication
> Luke: I think anyone can be good you just have to try
> Hannah: You aren't born with it
> Luke: You need to want to do it; you cannot have a negative attitude
> Henry: Like setting yourself up to fail
> Matt: Someone who is good at math naturally they can quickly get their homework done
> Cindy: As long as you are willing to put forth effort

The students' argument regarding the nature of one's talent lying in the hands of the student matches Dweck's (2006) idea of growth mind-set versus

fixed mind-set. A growth mind-set student accepts challenges and learns from mistakes and failures whereas a fixed mind-set student will pass on the blame for failure to others or circumstances outside of their control. Fix mind-set students would rather not study for a test since they 'fail anyways' but a growth mind-set student believes in their effort to give it their best and succeed. The students' comments indicated that there are naturally gifted math students but everyone else needs to give it a concerted effort and take on the responsibility of learning the content. I hope that the students remember to put forth the concerted effort and take responsibility as they strive to improve their problem solving skills in geometry. This leads to the final goal of metacognition.

Goal: Improve students' metacognition

Metacognition finding

Students' metacognition of their own learning improved during the implementation of *Solving Out Loud*. To analyze a change in the metacognition of students towards their learning, I took field notes during the activities, reviewed students' answers to reflection questions, and their answers from the pre-implementation and post-implementation survey.

I incorporated the use of reflection questions into *Solving Out Loud* to develop metacognitive and self-reflective behaviors. I would ask students a reflective question at the end of each problem solving activity. The students would write their responses in the center hexagon on the four square worksheet. These reflection questions offered students a chance to think about their learning and what they were doing to show they understood the

content. Sample questions include “why did you like or dislike the activity?” and “how did the discussion help you answer the problem?” Here are some answers to the latter question.

How did the discussion help you answer this problem?
To reassure that my plan would come to a correct answer
Confirmed what we already know
To confirm that our plan was correct
Helped double check our way of solving the problem and comparing answers
Helped our class figure out a plan towards solving the problem
The discussion helped me confirm what I believed we had to do, so therefore it gave me the confidence to complete the problem
We could talk with everyone and confirm what we had already known
It made it so people could input ideas and give scenarios to solve
b/c we talked about the ways to go about solving the problem and which one is the best

Figure 17: Sample of student responses to reflection question

To further analyze the development of behaviors that suggest metacognitive thinking, I looked at the pre-implementation and post-implementation surveys. In the first pre-implementation surveys every student in the class either answered “moderately true” or “exactly true” to the statement: *When I am confronted with a problem, I can usually find the solution.* At the early stages of this curriculum, the answers to this statement showed that the students had very little awareness of their actual abilities because they were not correctly solving problems and skipping the more challenging homework problem.

After the implementation of *Solving Out Loud*, students gave much more honest and accurate answers to their problem solving abilities. They became more insightful and aware of their abilities. I reviewed two specific questions from the post implementation survey. Both of these questions asked students to rank their problem solving skills. The first question was**:** on a scale from 1-5 (1 being poor and 5 being excellent), how would you describe your problem solving skills in math and why? Eight out of thirteen students ranked their problem solving skills as a 3 or higher. Students' ranking of math problem solving skills can be seen in Figure 18. It is important to note one student who scored himself as not being a good problem solver, a level one on the survey, stated, "I have not fully learned the skills yet" where as in the pre-implementation he felt as if he could solve most problems. This change in his scoring highlights that *Solving Out Loud* gave students the opportunity to develop metacognitive skills and learn how to be self-reflective and self-aware about their learning and skills.

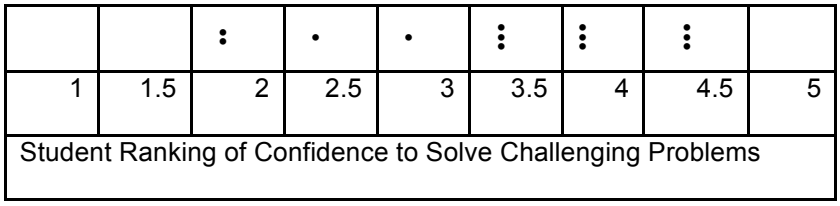

Figure 18: Student ranking of problem solving skills in math

The second question I reviewed was: on a scale from 1-5 (1 being not all and 5 being VERY) How confident are you to attempt to solve challenging

problems either on your homework or in class and why? Even more students, ten out of thirteen, ranked their confidence at a 3 or higher.

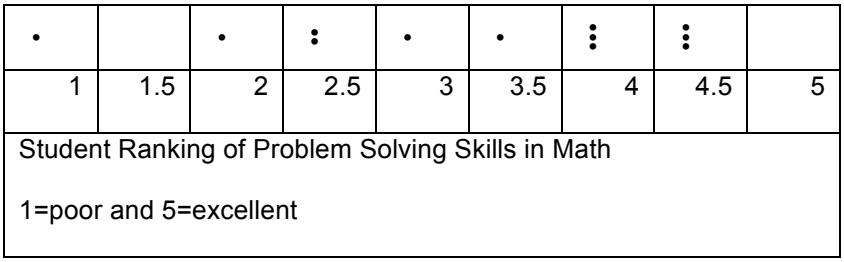

Figure 19: Student ranking of confidence to solve challenging problems

Summary of Findings

Overall, students felt that the activities in *Solving Out Loud* provided them the opportunity to practice problem-solving skills, talk about math, and learn to reflect on their learning. *Solving Out Loud* sparked an interest with a few students who otherwise did not participate in class discussions. I believe *Solving Out Loud* is a sincere approach at supporting and scaffolding problem solving and discourse skills in a mathematics class. Most students felt that the experiences of the problem solving activities had a positive impact on their learning and improvement of problem-solving skills.

Solving Out Loud provided the students in my Geometry class the scaffolding and support to engage in structured problem solving activities. These problem solving activities were supported and enhanced by the student-centered supportive class (Harkness) discussions that were adapted as part of the curriculum. As shown in the data, students increased their

mathematical persistence in problem solving, following a similar problem solving protocol as described by Polya (1945). They reported both increases in their own mathematical confidence, engaged more readily in mathematical discourse, and became active listeners to their peers. Their own metacognitive awareness showed growth as students were able to evaluate their own mathematical skills more realistically, being able to point out both their strengths and areas for further growth.

The *Solving Out Loud* curriculum did not have the same impact with all students, as evidenced by the one or two students who continued to struggle with engaging in the problem solving and mathematical discussions. The curriculum does show promise as a scaffolding to help students develop their own problem solving skills, discourse abilities, and metacognition.

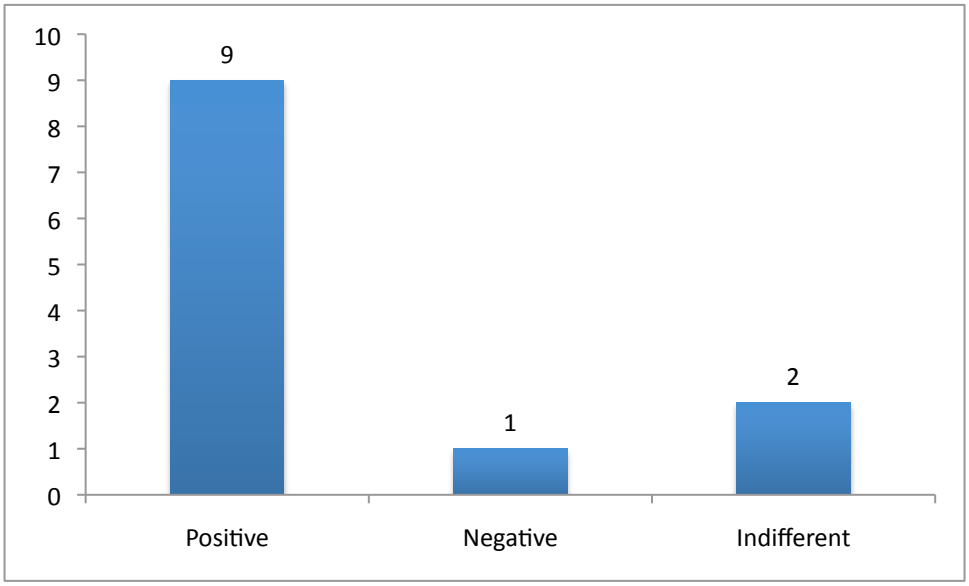

Figure 20: Student response to the effectiveness of the problem solving activities in *Solving Out Loud*

Chapter VIII: Conclusion

The goals of *Solving Out Loud* were to improve students' problem solving skills, discourse skills, motivation, and metacognition. Small group and large class discussions were the method to improve these skills. Students honed their problem solving skills as well as actively reflecting on their motivation and learning styles.

Solving Out Loud requires the teacher to step out of the spotlight and let student voices become the center of the conversation. Students will begin to trust each other's mathematical input instead of turning to the teacher for final approval. The teacher's role is one of mediator, summarizer, and/or questioner. The teacher does not control the conversation instead poses a question and lets the students debate and discuss it. In my previous experience as a math teacher, I was usually the center of most discourse in the classroom. Even when students were up at the board explaining a problem their peers usually looked to me as the final voice of authority, as if to say 'I'll agree, if the teacher agrees'. *Solving Out Loud* allows the students to trust themselves and each other in the problem solving process and to begin to actively listen to what is said in class discussion.

In using *Solving Out Loud* it is important to note that it is designed as supplemental curriculum to enhance the current curriculum in any math or even a science course. It is important to pick problems for students to discuss and solve that are similar to problems the students tend to skip on homework. These problems will vary depending on the ability and motivation of one's

students. It is also important to use supportive class discussion questions that are debatable, intriguing, and thought provoking. I also found it important to explicitly discuss learning and motivation with my students. I had never done this before and I realized that they are very interesting in talking about themselves and what motivates them and what learning/studying styles work best for them.

Depending on the level of students' discourse skills, it may be important to start slow and allow them to discuss questions like "what makes a good math student?" and "is excelling at math a natural talent or can a person become good at math?". Letting the students debate these questions gave me great insight into their feelings towards learning math and their discourse skills. Using this information, I was able to prepare questions for further discussions that scaffold their communication and listening skills.

It is also very important to be flexible when implementing *Solving Out Loud*. Some conversations may take twenty minutes and others may only last two minutes. I learned to read the class to determine an appropriate place to stop a conversation or interject another probing question. This requires the teacher to actively listen to the conversation as well. Active teacher monitoring is necessary to ask additional questions as it keeps students interested and motivates their involvement if the conversation is dynamic.

Flexibility is also key for the problem solving activities in *Solving Out Loud*. I originally imagined students solving up to 3 problems every time they did this activity. However, there were times when just one problem was

sufficient; the conversation was worthwhile and students were all motivated to problem solve. At these moments, I did not want to push forward too fast just to get through more problems; I let the conversations take their natural course and stopped them when appropriate. There were some problems the students never saw but I found the depth of the learning was more important then the need to cover all the problems I had preselected.

Scaffolding the discourse skills and problem solving skills is also a very important component when working with *Solving Out Loud*. After I implemented this curriculum, my class had to stop all normal instruction for a Geometry wide balsa wood bridge building competition. This competition took three weeks and provided little time for any other instruction or curriculum learning. I was surprised to see my students, who had been improving their problem-solving skills, fall short in their groups when they came across problems with their bridge. I would have thought this opportunity would be ideal for showing off their problem solving skills. Unfortunately, I took away *Solving Out Loud* and left the students with no opportunity to share their problems and discuss solutions with the class. Future implementation may require a gradual release of the scaffolding for *Solving Out Loud* and give students the opportunity to practice the problem solving skills on their own.

Future modifications for *Solving Out Loud* may include the chance for students to work independently on creating a plan first, then discussing it with their smaller group, and finally the supportive class discussion. A few weeks after the implementation of *Solving Out Loud*, I tried this strategy and found

that students were just as willing to ‘make a plan’ on their own as they were when working in small groups. Another modification may be to allow students to pick problems and to write questions for *Solving Out Loud* for the Harkness discussions.

In planning for further curriculum development on the topic of discourse and problem solving skills, I think teachers and researchers should do a more extensive study on student centered discourse and its use and value in mathematics classrooms. From my experience, students in an honors course are much more confident in discussing math while less proficient students in other classes are very reluctant to communicate in the classroom. My school has the unique ability to focus on Harkness discussion and has discourse as a cornerstone of the school’s educational philosophy. Students are accustomed to talking openly about a variety of topics in the humanities but are not as comfortable with this model in a math class.

Appendix

SOLVING OUT LOUD

Using discourse as means to promote problem solving, motivation, metacognition in a mathematics classroom.

By: Megan E. King

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Section 1 - Introduction

Letter to the Teacher

Tips for Implementation

Outline of Implementation

LETTER TO THE TEACHER

Dear fellow educator,

I once asked my geometry class “Am I speaking German?” and the students continued to stare at me blankly. They had no interest in answering any of my questions not even this bad joke. I asked this question because I felt like my students did not understand a word I was saying. Was it me? Are they unmotivated? Is the investigation/activity/interaction not engaging enough?

Of course, I do have classes that openly engage in conversations about the math content they are learning but this only seems to happen in my honors classes. In the non-honors class, the students who are not ‘good at math’ as they see it hardly speak up and engage in any dialogue.

I was determined to change this learning environment. I wanted to hear my students talking about math, exciting to learn, motivated, and even reflecting on their learning. I created *Solving Out Loud* to promote the improvement of problem solving skills, discourse skills, motivation, and metacognition through discourse. I gave students the opportunity to talk about learning as well as strategies to problem solve. We discussed what it meant to be a good math student as well as what motivates them as teenagers and why they may not be excited to learn math.

Most importantly, *Solving Out Loud* created a student-centered discussion environment. As the teacher, I was part of the conversation from asking questions to occasionally probing for more information. However, I took myself out of the conversation on purpose so students could listen to each other and learn from each other. This proved to be very valuable when we discussed strategies for solving particular problems. Students began to listen to each other and not look to me as the only knowledgeable person in the room. They began to share ideas and strategies; they were building

confidence in their ability and openly trying to problem solving with the help of the class conversation.

Solving Out Loud is a supplement to your current curriculum. Teach the way you usually teach. *Solving Out Loud* only requires five to fifteen minutes of class time for students to converse and practice problem solving. This can be done only a few times a week to give your students time to talk in class and to let them work through problems and ideas.

I quickly realized I learn more about my students and what they need to learn when I listen to them. Take the time to listen.

Sincerely,

Megan E. King

TIPS FOR IMPLEMENTING *SOLVING OUT LOUD*

SUPPRORTIVE CLASS DISCUSSIONS

-These can occur sporadically throughout a unit when applicable.

-These discussions start with an open ended and debatable question.

-The students share ideas and converse about the question presented.

-The teacher is often out of the conversation, letting the students tackle the topic from their own point of view and ideas.

TALK ABOUT LEARNING

- These discussions are a great time to discuss motivation and learning.
- Be open with your students about what you want them to improve
- In order to be aware of their learning, students may need to learn how to be metacognitive math students

PROBLEM SOLVING ACTIVITY

-This process can be repeated 1-3 times per class period. Used best at the end of class as a review of new concepts.

-Picking the homework problems students tend to skip is a great place to start!

-Students are presented with a challenging yet solvable problem.

-In small groups, they have 60-90 seconds to come up with a plan for solving the problem

-Next, the students discuss the plans as a class and see if they can come up with an agreement on how to solve the problem and/or clarify any confusion

-Finally, the students solve the problem and the class discusses the answers

SETUP CLASSROOM

-*Solving out Loud* works best when students sit in a circular u-shaped formation.

-Rows of desks do not allow for ideal class conversation.

GENERAL TIPS:

Take yourself out of the equation....Solving Out Loud is about student discourse and how they can learn from each other. As the teacher, you many need to guide the conversations but for the most part, you must allow the students to see each other as a local expert and trust what their peers are saying.

OUTLINE OF IMPLEMENTATION

Day	Topic of Study	Activities
1	The Pythagorean Theorem and it's converse	Supportive Class Discussion: Who makes use of right triangles and why?
2	The Pythagorean Theorem and its converse	Supportive Class Discussion: How has math made your life easier?
		Problem Solving Activity: Finding the longest diagonal in a rectangular prim
3	Two Special Right Triangles (45-45-90 and 30-60-90) Chapter 9 Quiz	Problem Solving Activity: Find the perimeter of a rectangular field given the diagonal and one side.
4	Group work	Problem Solving Activity: (1) Find the coordinates of a point on a unit circle if the angle measures 45º. . (2) Find the amount of paint needed to paint the eaves of a house.
5	Story Problems	Supportive Class Discussion: What motivates you to do what you do?
		No Problem Solving Activity: due to in class story problems activity.
6	Distance in Coordinate Geometry	Problem Solving Activity: A ramp connects a platform with a sidewalk. Given the length of a platform and the distance it is from the ground. Find the distance from the base of the platform to the sidewalk.
7	Test	
8	Right Triangle Trigonometry	Supportive Class Discussion: What makes a student a good math student?
		Problem Solving Activity: A boy is flying a kite, given the angle of elevation and the length of string the boy let out. Find the height of the kite from the ground.
9	Trigonometry Application	Problem Solving Activity – determine the height of the flagpole using trigonometry and a clinometer
10	Law of Sines and Cosines	Problem Solving Activity: The CCTV tower in Beijing China is not perpendicular to the ground. Using the law of sines, find the angle at which the tower leans.
11	Story Problems with Trigonometry	Problem Solving Activity: No PSA due to in class story problem activity.
12	Trigonometry Quiz	

Section 2 - Surveys

Pre-implementation Survey

Post-implementation Survey

PRE-IMPLEMENTATION SURVEY

Response Format

1 = Not at all true 2 = Hardly true 3 = Moderately true 4 = Exactly true

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1. It is easy for me to stick to my aims and accomplish my goals. 1 2 3 4

   Comment:

---

2. I can always manage to solve difficult problems if I try hard enough. 1 2 3 4

   Comment:

---

3. I am confident that I could deal efficiently with unexpected challenges. 1 2 3 4

   Comment:

---

4. Thanks to my resourcefulness, know how to handle unforeseen situations. 1 2 3 4

   Comment:

---

5. I am good at figuring out tricky situations. 1 2 3 4

   Comment:

---

6. I can solve most problems if I invest the necessary effort. 1 2 3 4

   Comment:

7. When I am confronted with a problem, I can usually find the solution. 1 2 3 4

Comment:

8. When in class, I enjoy and depend on the contribution of peers to help my own understanding of a topic. 1 2 3 4

Comment:

9. Class conversations and Harkness discussions are a valuable way for me to learn information. 1 2 3 4

Comment:

10. I value the opinion of my peers in this class. 1 2 3 4

Comment:

11. I prefer to work alone on my geometry work. 1 2 3 4

Comment:

12. I prefer to work in small groups on my geometry work. 1 2 3 4

Comment:

13. Geometry is _____ for me because…

POST IMPLEMENTAITON SURVEY

Please thoughtfully answer the following questions. Your comments will not effect your grade or my opinion of you!

(1) How do you feel about Harkness discussions in math?

(2) How do Harkness discussions in math compare to Harkness in other classes?

(3) How did you feel about the review activity (with the four box sheet) we did during chapters 9 and 10?

(4) Describe how (if at all) the review activity improved your ability to solve problems?

(5) If you could change anything about the activity, what would you change and why?

(6) When you arrived at a math problem you did not know how to solve, what do you do and why?

(7) On a scale from 1-5 (1 being poor and 5 being excellent), how would you describe your problem solving skills in math and why?

(8) On a scale from 1-5 (1 being not all and 5 being VERY) How confident are you to attempt to solve challenging problems either on your homework or in class and why?

Section 3 - Supportive Class Discussion Materials

Description

“Having a class discussion on class discussions”

Writing Discussion Questions

Brief History on Harkness

Discussion Rubric

DESCRIPTION

There are two forms of discourse activities incorporated into *Solving Out Loud*. The first activity is the fast-paced problem solving activity. Students practice and perfect discourse skills during this activity as they share their problem solving strategies. The other activity is a supportive class discussion that provides students a time to converse in class about any topic the teacher proposes. The teacher may ask students to discuss the reasons for learning a certain concept or the connections a concept has to a current event. These class discussions are similar to a Socratic Seminar, which is often based on a text, or passage students have read, or a Harkness discussion, which begins with a debatable and intriguing question. These discussions happen mostly once a week and are a time for students to reflect, ponder, question, and debate various ideas or concepts.

Both of these discourse activities require little to no participation from the teacher. Students begin to listen and speak to each other without looking to the teacher for reassurance. The teacher may need to steer the conversation or ask further probing questions but for the most part the teacher's role is that of a bystander, listening and watching. From my experience, it is often necessary to remind students "pretend I am not here, and speak honestly" or "remember that I will not confirm your statements, you need to listen to each other."

In order to lead a good class discussion, it is important for the questions to be contentious and stimulating. It must spark interest, wonder, and allow

students to really dissect the topic. Examples of class discussion questions that I used include: “What motivates you?”, “what makes a good math student?”, followed by “is being a good math student a natural talent or something you can get better at?”, and “in what real world situations might you use <insert topic>?”.

The kind of discussions used in *Solving Out Loud* work best when students are facing each other and sitting around a table together. Arranging desks into a circle, U-shape, or rectangular formation will allow students to face each other or it may be best to have students form a circle either standing or sitting. The key factor is for students to face each other so they can listen, respond, and see each other during the conversation.

"HAVING A CLASS DISCUSSION ON CLASS DISCUSSIONS"

Solving Out Loud began with two conversations

(1) What does it mean to participate in a supportive class (Harkness) discussion?"

Students need to discuss what it means to be involved in a class conversation. This can start with a small group conversation and end with a whole class discussion. The class can come to a general agreement and create a list used to make a tally rubric for future Harkness discussions.

Here is a sample list:

Class Discussion Participation:

Record/take notes

Listening/paying attention

No interruptions/be respectful

Use useful info and details

Debate (both sides of the issue/topic)

Summarize/confirm key points

Responding/building off ideas of others

Ask clarifying questions

(2) Supportive Class (Harkness) Discussion on Problem Solving.

Students discuss what they think are the steps to proper and effective problem solving.. This can start with a small group conversation and end with a whole class discussion. The class can come to a general agreement and create a list used to as a guide for the improvement of problem solving skills.

Here is a sample list:

Steps to problem solving:

(1) Read and understand the problem
   a. Know what the question is ask
   b. Reread the problem
   c. Break the problem down

(2) Make a plan
   a. Write down the steps needed to solve the problem
   b. Organize information
   c. See what you can do with what you have
   d. Draw a picture

(3) Solve (executing the plan)
   a. Attempting the problem
   b. Show your work
   c. Draw a picture
   d. Trial and error (guess and check)

(4) Check your answer

WRITING DISCUSSION QUESTIONS

- Discussion question writing is not easy. In order to lead a proper purposeful discussion, the leading question must be debatable and intriguing. It must spark interest, wonder, and allow students to really dissect the topic.

- *Solving Out Loud* discussion questions
  - "What motivates you?"
  - "What makes a good math student?", followed by "is being a good math student a natural talent or something you can get better at?"
  - "In what real world situations might you use <insert topic>?"

  Please note that questions can relate to the current topic of study or even tie the current content to current events or integration points with other subjects.

BRIEF HISTORY ON HARKNESS (A TYPE OF DISCUSSION)

Harkness discussions are a type of discussions similar to a Socratic Seminar. Any class can have a large discussion but Harkness is unique in it's style and questions. There is even a Harkness table that is a large oval dining table that seats twenty!

Brief History of Harkness:

The Harkness discussion is named after Edward S. Harkness who in 1930 donated a large table to Philips Exeter Academy (Philips Exeter Academy, 2010). This method of instruction allows students and the teacher to sit at the table together to share and discuss ideas. A typical class size is twelve to sixteen students with students sitting around an oval Harkness table or octagonal arrangement of tables. Students are often learning through these Harkness discussion, a type of discourse. In a typical discussion of this type, the teacher poses a Harkness question for the students and they converse, debate, and/or share ideas. All students are involved, from asking probing questions to summarizing. Ideally, they are motivated by each other to carry the conversation to intellectual and interesting points.

DISCUSSION RUBRIC

This was created from what students decided are the ways to participate in a class discussion. They all approved this rubric since they technically made it.

Discussion ⇓ Students ⇒													
Listening/paying attention													
No interruptions/be respectful													
Use useful info and details													
Debate (both sides of the issue/topic)													
Summarize/confirm key points													
Responding/building off ideas of others													
Ask clarifying questions													
Record/take notes													

Section 4 - PROBLEM SOLVING ACTIVITY

Description

Sample Questions

Reflection Questions

Original four square worksheet

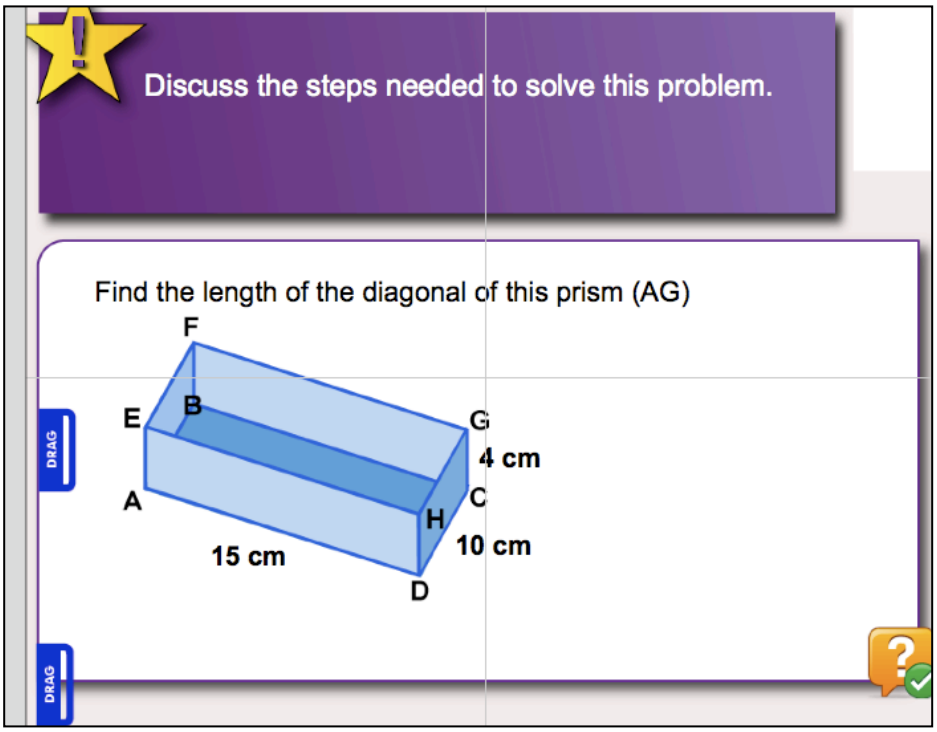
Adapted four square worksheet

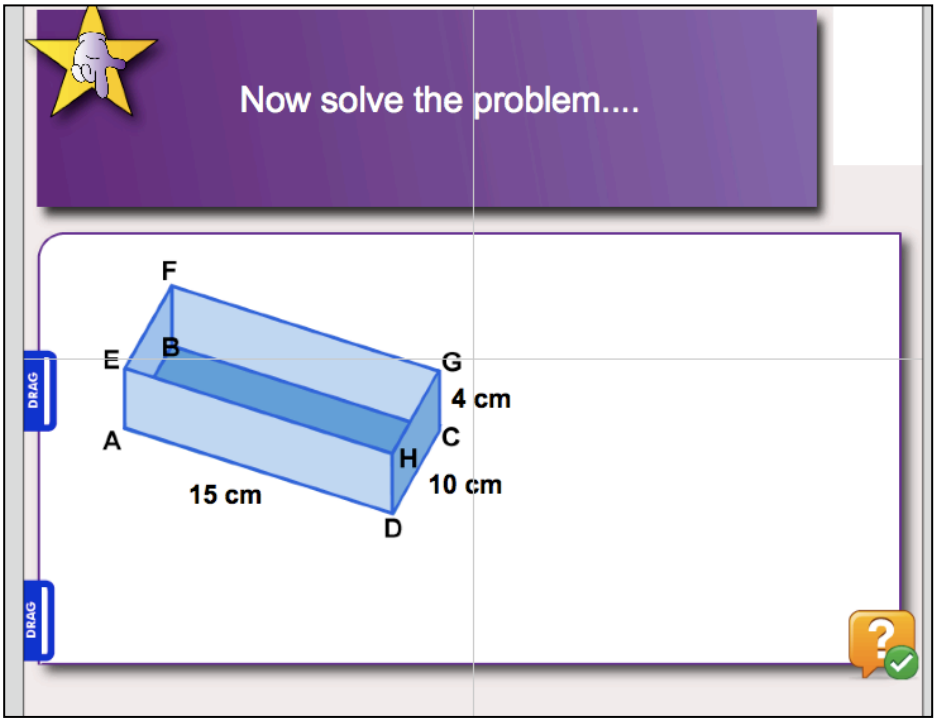
DESCRIPTION OF PROBLEM SOLVING ACTIVITY

Problem solving strategies are discussed, practiced, and reinforced with *Solving Out Loud*. Students practice solving difficult problems supported by group conversations. The problem solving activity usually occurs at the end of the class period as a review of material learned in class. Students receive one to three challenging problems one at a time. In small groups, students have 60-90 seconds to come up with a plan for solving the problem. As a whole class, students discuss the plans and may come to an agreement. The students then solve the problem either individually or in their small group. Lastly, the class then discusses the answer and whether or not their method of solving the problem was reasonable. This process can be cycled through one to three problems per class depending on the amount of time and/or variety of problems that are applicable to the day's lesson.

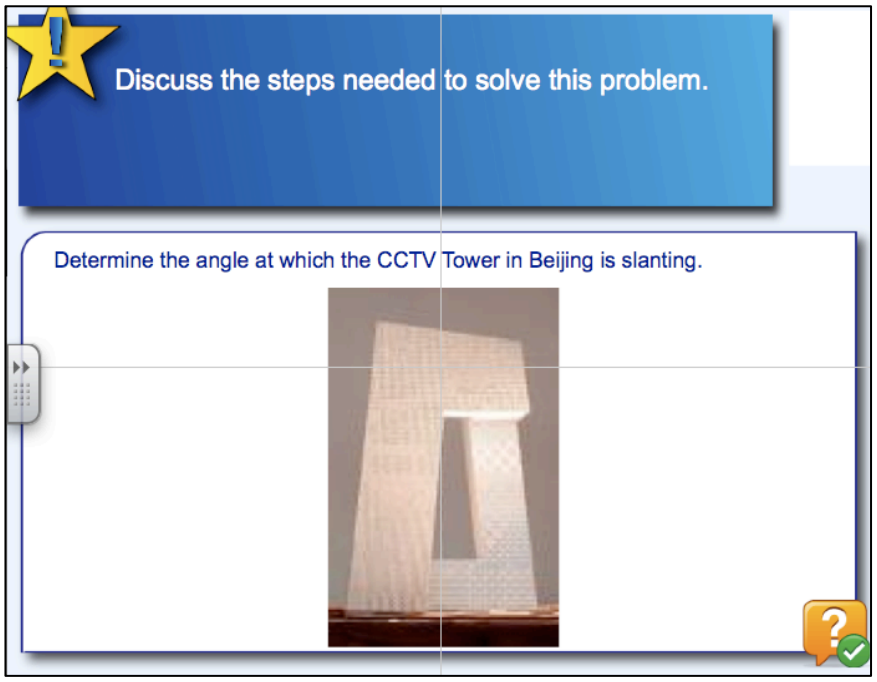
SAMPLE PROBLEMS FROM SOLVING OUT LOUD

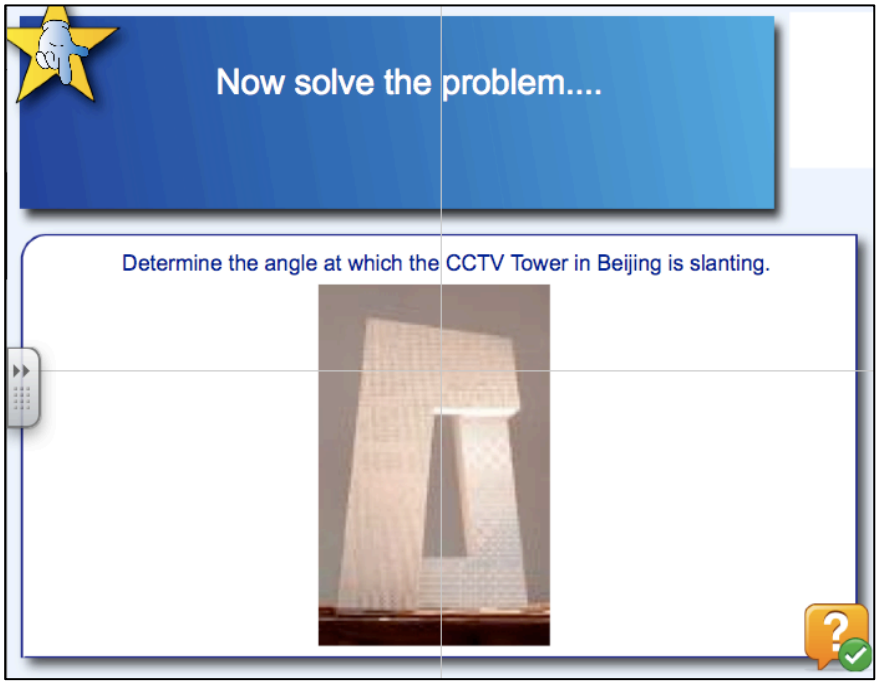
Problem retrieved from http://exchange.smarttech.com (2011)





I created this problem to integrate with the study of China in 9th grade history.





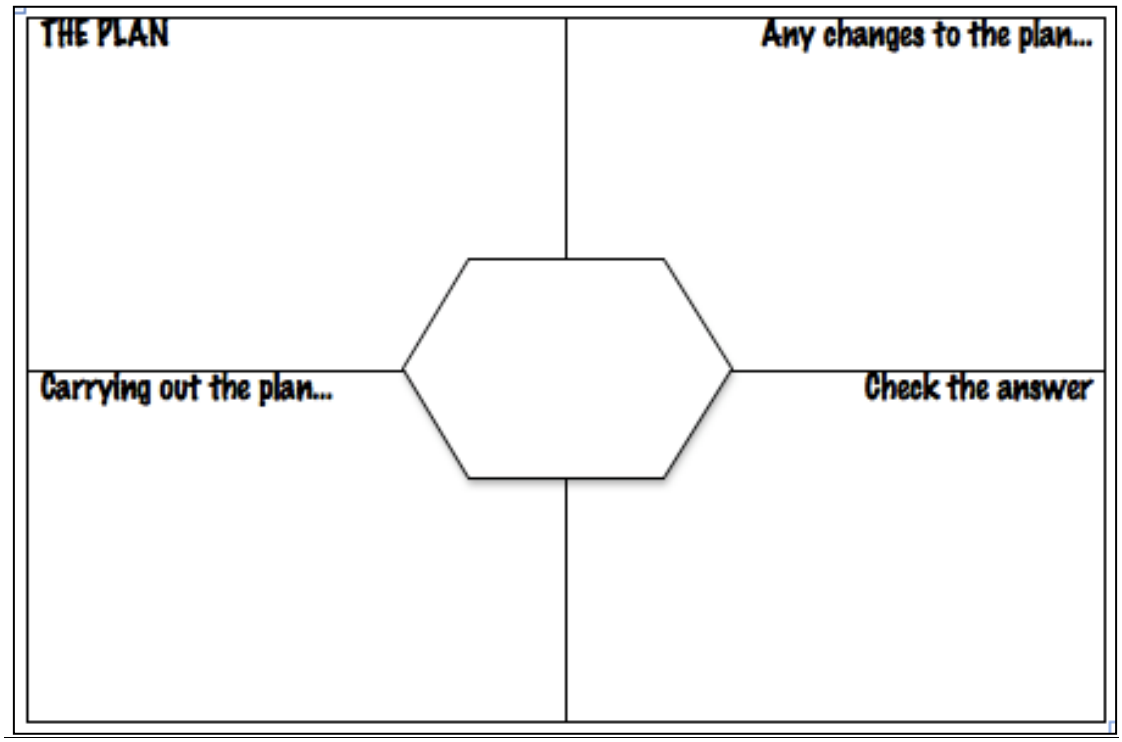
REFLECTION QUESTIONS

Here are questions to ask students for after the activity, such as:

- How did the discussion help you answer this problem?
- What do you like and/or dislike about this activity?
- How can we change the 4-square sheet?
- Did this problem need a discussion?
- What other ways are there to solve this problem?

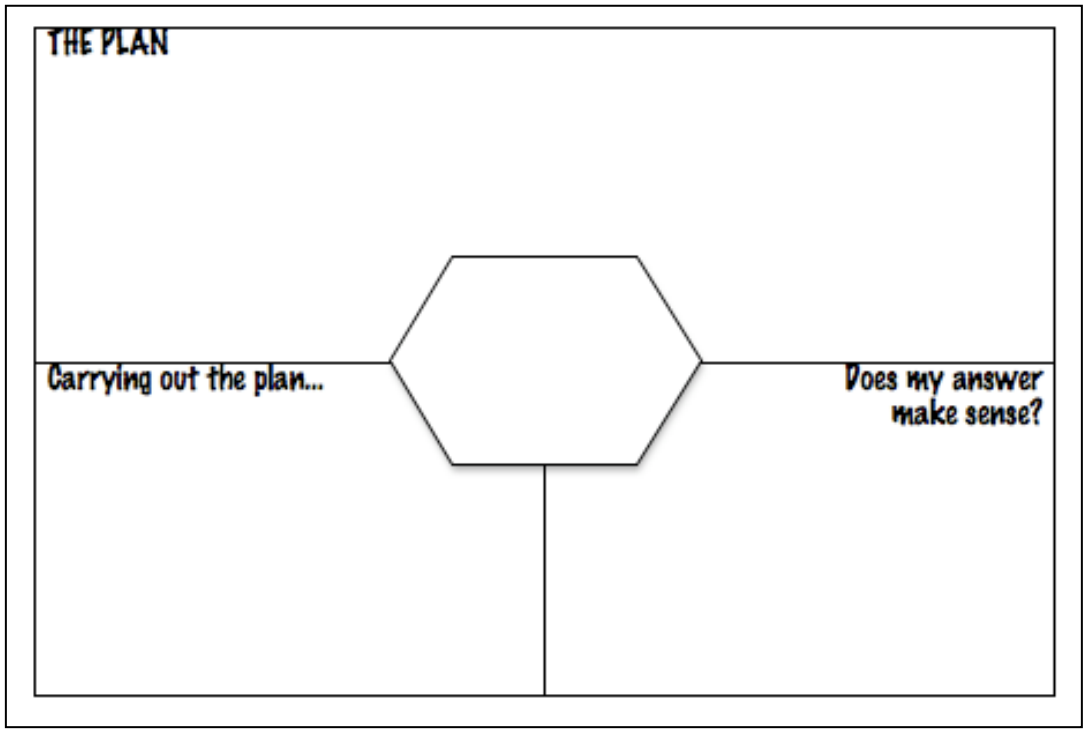
FOUR SQUARE WORKSHEET

Use to support problem solving activity



STUDENT IMPROVED FOUR SQUARE WORKSHEET

Students' input altered the four square to make it more user friendly



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