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Publication Date
2013-03-11

University of California, Berkeley
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Resource Economics

CUDARE Working Papers

Year 2013 *Paper 1108R2*
(Year 2010, 1st revision *Paper 1108R)*

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Tracking the Libor Rate

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Abstract

With an eye to providing a methodology for tracking the dynamic integrity of prices for important market indicators, in this paper we use Benford second digit reference distribution to track the daily London Interbank Offered Rate (Libor) over the period 2005-2008. This reference, known as Benford's law, is present in many naturally occurring numerical data sets as well as in several financial data sets. We find that in two recent periods Libor rates depart significantly from the expected Benford reference distribution. This raises potential concerns relative to the unbiased nature of the signals coming from the sixteen banks from which the Libor is computed and the usefulness of the Libor as a major economic indicator.

Keywords: market rate data, Libor, Benford's law, second digit distributions, JEL classification: C10, C24.

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[1] We thank the editor Mark Taylor and two anonymous referees as well as comments by Albert Metz, Leo Simon, Gordon Rausser, Laura Schechter, John Cochrane, the British Bankers Association, assistance in data collection by Susan Press, and support from the Giannini Foundation of Agricultural Economics. Abrantes-Metz: LECG and Leonard N. Stern School of Business, New York University; Villas-Boas and Judge: Department of Agricultural and Resource Economics, University of California, Berkeley. Corresponding author: rabrantes-metz@lecg.com. The views expressed in this study belong solely to the authors and should not be attributed to LECG or its clients.

"The Libor, or the London interbank offered rate, is a daily average of what banks charge other banks to lend money in London. (…) Libor (is) "the dial on the engine of the car," showing how much power the economy has. "And right now it is indicating that the car is severely overheating."

in "Lending remains squeezed as investors look for guidance as to what Washington will do next"

CNN.com, September 30, 2008.

1. Introduction

Economic data serve as a necessary basis for choices and decisions relative to the functioning of an economic system. Since the economic choice-decision-agent system runs on information, if the information on which choice is based is flawed, serious allocation and distributive economic implications follow. The economic outcomes that fill our newspapers and television screens presently, remind us how fragile these data really are. Consequently, there is a pressing need for some objective way to track and screen the performance and integrity of important economic indicators such as interest rates. It is also important for the functioning of economic systems that this happens in real time and not ex-post. In this context, in this paper we track the daily London Interbank Offered Rate (Libor) over the last three years and suggest a method that may be used to identify if the Libor, through signals from sixteen banks, is performing its intended price signaling function.

The method we propose is to contrast the distribution of the empirical second digits (SD) of the Libor interest rate, to a reference distribution known as Benford's SD law. According to this law, in many naturally occurring numerical data sets and in several financial data sets, the digits follow a logarithmic weakly monotonic distribution. Given this empirical phenomenon, Benford's first significant digit (FSD) law has been used in many settings to check data integrity.[2] In these data checks, not only the FSD but also higher digits have been used to detect potential irregularities. In the case of the Libor rate,

[2] Among many who have used Benford's law to check the validity of purported scientific data in the social sciences see Varian (1972), Giles (2007), Cho and Gaines (2007), and Judge and Schechter (2009). See Abrantes-Metz and Bajari (2009) how statistical methods have started to be used in antitrust and finance to detect a variety of conspiracies and manipulations.

and interest rates in general, these do not often vary much over limited periods of time. Prices do tend to present some stickiness over nominal values which are represented by the first significant digit, and simply for this reason it is likely that the Benford's law will be violated for the FSD, as it will not naturally span the one to nine digits space. In this case, it is possible to use Benford second digit (SD) law as a reference distribution to track interest rates empirical second digit (0 to 9) frequencies.

The Libor acts as a benchmark for the US Dollar, Sterling, Euro, and Yen and represents the rate at which each of the banks in the sixteen member panel perceive they could raise unsecured funds. Libor rates are quoted daily on ten major currencies. In this study we focus on the US Libor rate. The US Libor rate emerges as follows: 16 banks submit daily rate quotes to the British Bankers Association (BBA), the middle 8 rate quotes submitted are converted into Libor via a simple arithmetic mean, and the process is overseen by an independent committee of market participants and by the Foreign Exchange and Money Market Committee. Banks individual quotes are submitted anonymously. Each member bank (player) of the group of 16 receives a private signal and then submits a rate quote to the center which then makes a decision based on the middle 8 quotes. It is assumed the signals received are passed on based on market data and are thus without player bias.

Since the Libor is used as a benchmark in setting interest rate contracts worldwide, its relevance extends beyond interbank lending, to international conglomerates, to small borrowers, and to subprime mortgages. It is a central rate in interest rate swaps and the majority of floating rate securities and loans refer to it. Several hundreds of trillions of dollars of swaps and contracts are indexed to the Libor. Given its importance, issues arising with a Libor rate over- or understatement can have implications in many other markets and thereby have broad economic policy considerations and impacts.

2. Benford's Law

In this paper, in order to identify tampering and human influence on market processes we use a method that is based on the empirical phenomenon known as Benford's law (1938). According to this law, in many naturally occurring numerical data sets, the leading digits are not uniformly distributed but instead follow a logarithmic weakly monotonic distribution. Under Benford's law the digit distributions for the first significant digit (FSD) and second digit (SD) are given by

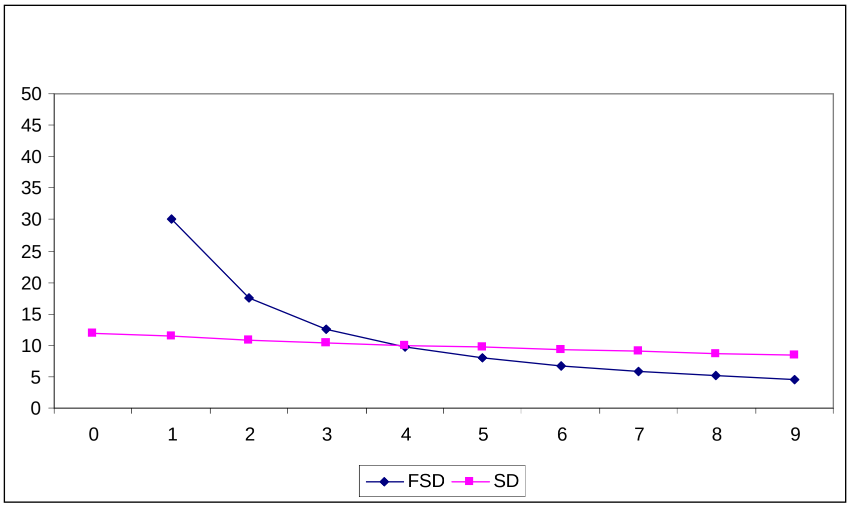
$$\text{Prob(FSD}= d) = \log_{10}(1+d^{-1}),\ d = 1, 2, \ldots, 9 \qquad (2.1.)$$

and

$$\text{Prob(SD}= d) = \sum_{k=1}^{9} \log_{10}\left(1+(10k+d)^{-1}\right),\ d = 0,..,9, \qquad (2.2)$$

respectively. A graph of the first two digits' distributions is presented in Figure I.

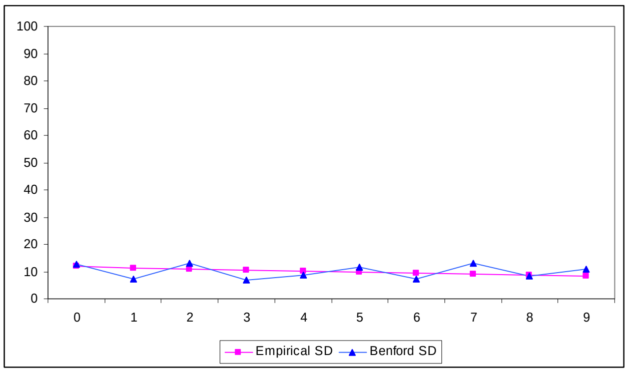
Figure I. Benford's Law First and Second Digits Frequencies



Since Benford (1938), others have published studies showing that Benford's FSD law applies to a surprisingly large number of data sets, including populations of cities, electricity usage, and the daily returns to the Dow Jones. Market data reflect nominal values that often do not vary much over limited periods of time. Interest rates are a case in point and can, for example, take the value around 4 percent for a considerable time. Although the FSD's of the Libor interest rates for the time period under study do not span the nine digit space, the second (see equation 2.2) and following digits data may be expected to naturally do so.

Consider the empirical SD distribution of the Federal Funds Rate for the period 1987-2005.[3] To evaluate the divergence between Benford and the data second digit (SD) distribution we use the traditional Chi-Square goodness of-fit tests.[4] In Figure II we note the visual agreement between the empirical SD and the Benford SD distribution. In terms of a goodness of fit between the two distributions a χ^2 of 6.47 results, meaning that we cannot reject the null hypothesis of distribution equality at the ten percent level of statistical significance.

Figure II. 2nd Digit Fed. Funds Rate Empirical (1987-2005) & Benford Distributions



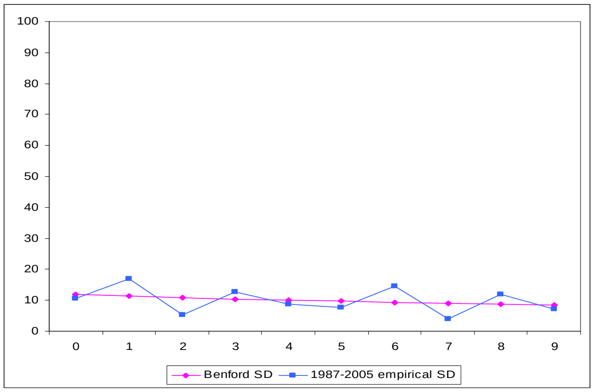
3. Empirical Results – Tracking the Libor

Our objective is to use scale invariant data that may be expected to track Benford SD distribution, and in the event of behavioral departures evaluate their basis. To indicate the incidence of Benford like digit distributions as it relates to Libor we use the daily one-month Libor rates over the period 1987 to 2005.

[3] The Federal Funds Effective (FFE) rate is the interest rate at which banks (and other depository institutions) lend balances through the Federal Reserve Bank to other depository institutions. Because this rate is usually applied to overnight loans, it represents a short term rate of borrowing between banks, making it a suitable benchmark for our study.

[4] $\chi^2 = \sum_{i=0}^{9} (e_i - b_i)^2 / b_i$, where e_i is the observed frequency in each bin in the empirical data, b_i is the frequency expected by Benford. This statistic has *9* degrees of freedom with the *10*, *5*, and *1* percent critical significance values of 14.98, 16.92 and 21.97, respectively.

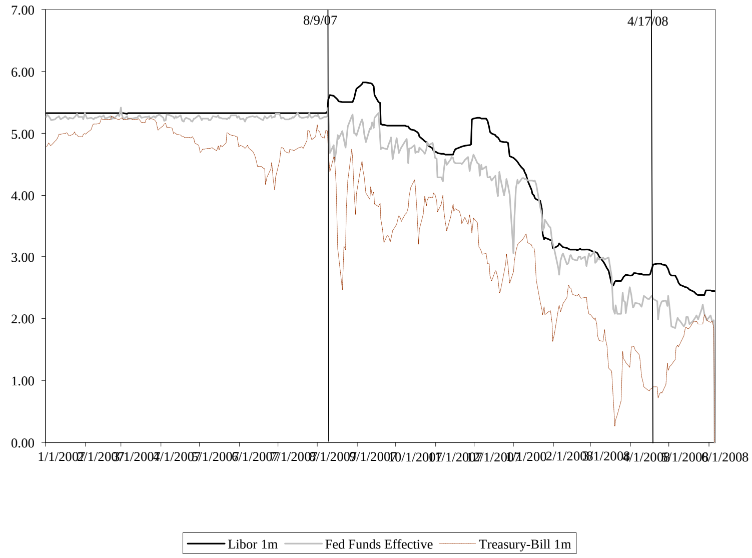
Figure III. Second Digit Libor Empirical (1987-2005) and Benford Distributions



In Figure III the empirical frequencies of each second digit of the rates oscillate around the Benford distribution with a corresponding chi square of 13.89. This means we cannot reject the null hypothesis of equality at the ten percent level of statistical significance. Although statistically we cannot reject the equality of distributions hypothesis, the departure of the Libor from the Benford SD reference distribution is much more pronounced than for the Federal Funds rate displayed in Figure II. Furthermore, when looking at the one month Libor for the period January 2007 to the end of May 2008, as well as the Fed Funds Effective Rate and the one month Treasury-Bill (T-Bill) in figure IV[5] two striking features emerge: i) the nearly constant Libor for at least seven and a half months and ii) the different volatility pattern between the Libor rate and the benchmark rates T-bill and Fed Funds Rate, when no reason seems to justify such pattern differences.

[5] A short-term debt obligation backed by the U.S. government with a maturity of less than one year. T-bills are sold in denominations of $1,000 up to a maximum purchase of $5 million and commonly have maturities of one month (four weeks), three months (13 weeks) or six months (26 weeks). The T-bill is also an important benchmark for short-term contracts with a price that is determined by the market.

Figure IV. **Libor 1m, Fed Funds Effective Rate and Treasury-Bill 1m**



In order to identify periods in which the Libor empirical distribution deviates from Benford we now proceed to test the closeness of these two distributions for rolling six months periods, starting in August 2005. Specifically, our major concern is with the Libor performance in recent periods. We break up our analysis into before and after August 2007, a time pointed by many as the “official start” of the financial crisis, and therefore representing a natural structural break in our study of the Libor rate.

3.1 Tracking the Libor rate until August 2007

To track the Libor we analyze the empirical frequencies of the second digits of the daily 1-month-Libor rates for six month periods, and contrast these frequencies over time with the Benford reference SD distribution.[6] The first column of Table 1 contains the Benford SD frequencies that are monotonically decreasing, starting at 0.12 for digit zero and decreasing to 0.085 for the digit nine. In column 2 of Table 1 we report the empirical SD frequencies for the Libor for the year of 2005 until January 2006. We find that given the χ^2 value for the empirical SD frequencies these are not statistically significantly different from the Benford SD reference.

[6] We also note that all of the analyses presented in this paper were run for the 3 month Libor, with the same qualitative findings and in Abrantes-Metz and Villas-Boas (2010) using the individual banks quote data we perform similar second digit frequency divergence analyses yielding consistent results.

Table 1. Benford's SD Frequencies and Empirical SD Frequencies over Time

	1	2	3	4	5	6	7	8
	Benford	jan 05 to	aug 05 to	sept 05 to	oct 05 to	nov 05 to	dec 05 to	jan 06 to
SD		dec 05	jan 06	feb 06	mar 06	apr 06	may 06	jun 06
0	11.97	16.54	12.12	12.40	12.31	6.98	17.69	17.69
1	11.39	8.46	9.09	9.30	9.23	9.30	2.31	9.23
2	10.88	4.62	3.03	3.10	3.08	3.10	18.46	3.85
3	10.43	15.00	18.18	18.60	18.46	18.60	8.46	7.69
4	10.03	7.69	8.33	8.53	8.46	8.53	20.00	8.46
5	9.67	15.38	16.67	20.16	20.00	20.16	6.92	20.00
6	9.34	6.15	8.33	3.88	6.92	6.98	6.15	6.92
7	9.04	7.31	8.33	7.75	6.15	6.20	12.31	6.15
8	8.76	12.69	9.85	10.08	9.23	12.40	7.69	12.31
9	8.50	6.15	6.06	6.20	6.15	7.75	0.00	7.69
Chi-square SD Freq = SD Benford		15.86 **	18.24 *	28.16	25.70	29.10	37.22	22.76
Chi-square SD Freq=SD Uniform Freq		18.92 *	18.92 *	29.02	26.75	27.09	42.72	24.02

Source for Libor data: Historial 1 month Libor rates, British Bankers Association. (**) cannot reject similarity in frequencies at 1 percent; (*) cannot reject at 5 percent significance level.

Starting in February 2006, and continuing for eighteen six month periods in Tables 2 and 3 (which due to their size are presented at the end of the paper), the theoretical and empirical frequencies diverge and the chi-square distance measures escalate to χ^2 values over *800* and thus indicating significant statistical difference and major departures from the Benford SD distribution.

3.2 Tracking the Libor post August 2007

Starting for the period of August to December 2007, in the columns labeled 27 to 31 of Table 3, the SD frequencies return to expected Benford outcomes and chi-square values that lead to not rejecting the equality null hypothesis at acceptable statistical significance levels. Looking at more recent available data for 2008, the columns labeled 32 to 36 of Table 3, we find that the SD frequencies for the three periods, consisting of last trimester months of 2007 and the first trimester months in 2008, depart from the base period empirical frequencies and from Benford, and yield inflated χ^2 test values of 48.96, 29.18 and 36.90. The empirical frequencies for the period December 2007 to April 2008 return to Benford levels but diverge again, in line with the possibility of Libor manipulation by the panel banks,[7] in an increasing fashion from 77 to 380, from May 2008 to October 2008.

4. Summary and Implications

As we write this paper the US-global economy is still in a very difficult situation. Given the unfortunate economic outcome we currently live in, our focus is on the prices that guide the economic engine and the possibility that they may have been manipulated . To investigate this non market possibility, we have tracked the Libor, using as a methodology, the second digit distribution variant of Benford's law. As a result we have found that over an extended period of time there have been significant departures between the Benford and empirical second digit distributions. The behavioral departures of the Libor from the expected path, in particular a path that the Libor had followed for at least the prior twenty years, raise questions regarding the integrity and quality of its rate signals coming from individual banks and cry out for an answer. Based on our evidence, biased signals coming from the individual banks (agent aggregation bias), rate manipulation or collusion appear as one likely answer. We consider the analysis in this paper exploratory. However, we were not prepared for the large departures between the empirical Libor and the Benford reference distribution. In ongoing research we hope to develop an objective method for identifying possible human tampering and fraud in important market indicators.

Given its extensive use the economic consequences of a misbehaving Libor can be various and severe. From a distributive standpoint, if the level of Libor deviates from its market level, it will affect an artificial and inefficient redistribution of wealth from one group of people to another. If, for example, the level is too low, borrowers, such as homeowners, gain at the expense of lenders. A more subtle consequence is to distort other prices in the economy. A lower Libor induces a lower mortgage rate, makes it easier to buy homes, substituting homes away for other goods. This artificially inflates

[7] On April 17, 2008, the Wall Street Journal (WSJ) published the news that the BBA intended to investigate the composition of these rates and this initiated a series of articles (see e.g. Abrantes-Metz et al, 2008).

the prices of homes and related goods such as furniture, for example, while deflating the prices of other goods. The immediate implications of a non market determined Libor, over a prolonged period of time, have the potential to lead to bubbles and meltdowns of the type we are currently experiencing. This brings us back to our earlier point which is that we need objective-predictive ways to track the behavior of important economic indicators such as the Libor in real time instead of ex-post- historical-descriptive analyses.

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Table 2. Benford's and Empirical SD Frequencies over time

	1	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	Benford	feb 06 to	mar 06 to	apr 06 to	may 06 tc	jun 06 til	jul 06 to	aug 06 to	set 06 to	out 06 to	nov 06 to	dec 06 to	jan 07 to	feb 07 to	mar 07 to	apr 07 to	may 07 tc	jun 07 to
SD		jul 06	aug 06	sept 06	oct 06	nov 06	dec 06	jan 07	feb 07	mar 07	apr 07	may 07	jun 07	jul 07	aug 07	sept 07	oct 07	nov 07
0	11.97	13.29	17.42	17.69	15.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.79	3.82
1	11.39	6.94	9.09	9.23	9.09	6.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.15	12.12	12.21
2	10.88	2.89	3.79	3.85	3.79	3.82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.53
3	10.43	38.73	34.85	51.54	67.42	84.73	95.38	98.48	100.00	100.00	100.00	100.00	100.00	100.00	87.12	71.54	54.55	37.40
4	10.03	3.47	4.55	4.62	4.55	4.58	4.62	1.52	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.77	0.76	0.76
5	9.67	9.83	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	9.85	10.77	10.61	10.69
6	9.34	5.20	4.55	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.27	3.08	3.03	11.45
7	9.04	4.62	6.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.08	6.06	10.69
8	8.76	9.25	12.12	5.38	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.62	7.58	9.92
9	8.50	5.78	7.58	7.69	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.52	1.53
Chi-square																		
SD Freq = SD Benford		93.68	82.20	201.99	365.50	595.70	774.11	829.81	858.50	858.50	858.50	858.50	858.50	858.50	639.60	410.42	222.86	98.68
Chi-square																		
SD Freq=SD Uniform		100.92	89.73	217.87	389.27	626.19	811.87	870.06	900.00	900.00	900.00	900.00	900.00	900.00	670.73	431.25	235.54	102.58

Source for Libor data: Historial 1 month Libor rates, British Bankers Association. (**) cannot reject similarity in frequencies at 1 percent; (*) cannot reject at 5 percent significance level.

Table 3. Benford's and Empirical SD Frequencies over time

SD	1 Benford	26 jul 07 to dec 07	27 aug 07 to dec 07	28 sept 07 t jan 08	29 oct 07 to feb 08	30 nov 07 to mar 08	31 dec 07 to apr 08	32 jan 08 to jun 08	33 feb 08 to jul 08	34 mar 08 to ag 08	35 abr 08 to sept 08	36 may 08 to oct 08
0	11.97	4.58	6.06	6.15	9.92	6.15	6.15	4.58	3.85	3.88	0.77	3.25
1	11.39	12.98	13.64	29.23	22.90	16.92	16.92	13.73	15.38	0.00	2.31	4.88
2	10.88	6.87	10.61	11.54	11.45	11.54	10.00	3.92	0.77	0.00	0.77	2.44
3	10.43	21.37	7.58	3.08	3.05	3.08	6.92	5.88	3.85	3.88	3.85	4.88
4	10.03	0.76	2.27	2.31	1.53	1.54	6.92	34.64	39.23	55.04	63.85	68.29
5	9.67	11.45	14.39	4.62	6.11	6.15	10.00	7.84	6.15	6.20	3.85	7.32
6	9.34	12.21	12.12	10.00	12.98	13.08	7.69	5.88	6.92	6.98	3.08	3.25
7	9.04	10.69	11.36	10.77	11.45	17.69	13.85	11.76	13.08	13.18	13.08	4.88
8	8.76	14.50	14.39	14.62	12.21	16.92	14.62	8.50	10.00	10.08	7.69	0.81
9	8.50	4.58	7.58	7.69	8.40	6.92	6.92	3.27	0.77	0.78	0.77	0.00
Chi-square												
SD Freq = SD Benford		33.39	17.61 **	48.96	29.18	36.90	14.82**	77.53	116.37	244.76	336.68	380.10
Chi-square												
SD Freq=SD Uniform		31.98	14.55**	55.02	32.21	32.54	13.27**	77.66	116.32	242.22	335.29	381.63

Source for Libor data: Historial 1 month Libor rates, British Bankers Association. (**) cannot reject similarity in frequencies at 1 percent; (*) cannot reject at 5 percent significance level.