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Energy Utilization Analysis of Buildings

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ENERGY UTILIZATION ANALYSIS OF BUILDINGS

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ABSTRACT

The accurate calculation of the energy requirements and heating and cooling equipment sizes for buildings is one of the most important, as well as one of the most difficult, problems facing the engineer. It is important because energy cost is an essential and significant element of the building's overall owning and operating cost and becoming a major determining factor in the selection of the heating, ventilating and air conditioning (HVAC) systems of buildings. The problem is difficult because of its complexity. It not only requires accurate determination of the heating and cooling loads, taking into account the continually varying outside weather and the frequently widely varying inside load conditions, but also determination of the performance of HVAC systems under varying conditions of partial load.

In this paper, the fundamental principles utilized in the procedures developed by American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) are explained and brief descriptions of the computer programs using these procedures are given. Such computer programs generally are capable of:

1) simulating the thermal response of a building to all sources of heat gains and losses,
2) accounting for all non-thermal energy requirements in the building or on the sites,
3) translating the building operating schedules into energy demand and consumption,
4) identifying the peak capacity requirements of heating and cooling equipment, and
5) performing an economic analysis that would select the most economical overall owning and operating cost equipment and energy source that minimize the building's life cycle cost.

INTRODUCTION

Energy Utilization Analysis of Buildings involves three principal steps. First is the calculation of heat loss and heat gain to the space.* Second is the determination of the heating and cooling loads imposed on the buildings HVAC systems. Third is the calculation of the energy input required by the HVAC systems to satisfy those loads. Each of these calculation steps can be carried out very simply to achieve approximate results, or with increasing degrees of complexity and sophistication as more accurate and more refined determinations of system performance are required.

The continually varying weather conditions outside a building and the frequently widely varying load conditions inside a building create a cooling/heating load that is always changing, not only in total amount, but in its distribution in the building. An exact calculation of this load is obviously impossible for many reasons. The dynamic response of the structure can only be approximated by calculations for small finite time intervals. Thus the objective of more refined calculation procedures is to obtain a closer and more realistic approximation of the heating and cooling load from which an energy calculation can be made.

Such refined calculation procedures, unfortunately, are both time-consuming and expensive. Without the use of advanced computer methods, they are literally impossible. The development of such calculation procedures has been justified on the basis that the more accurate calculation will result in overall savings in Life Cycle cost of a building due to more precise sizing of the equipment selected and more carefully controlled operation of the heating and cooling system.

Since the publication of the refined calculation procedures [1, 2]** by the ASHRAE Task Group on Energy Requirements for Heating and Cooling of Buildings, several computer programs, in both the public-domain and the private sector, have been developed based on these procedures [3, 4].

FUNDAMENTAL PRINCIPLES

Correct sizing of the HVAC systems and the accurate determination of energy requirements have always been difficult, not because the physics or economics is conceptually elusive, but because the task requires a huge amount of computation. For example, the correct prediction of heating and cooling loads on the heating and cooling equipment is important because the maximum loads determine the size, and hence the

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* Space is defined as a room or a group of rooms which would be treated as a single load.

**Numbers in the brackets refer to the bibliography and references.

initial cost, of the system components, while the summation over time of the energy required by the system to meet the heating and cooling load represents a large fraction of system operating costs and may, therefore, determine the choice of system. If the maximum loads are under-estimated, the equipment may be undersized and fail to do its job, and if the maximum loads are overestimated, the equipment will be larger, less efficient and more expensive than necessary.

Before the advanced computer methods, only a crude energy calculation was practical, and the engineer had to rely mainly on experience. The effect of outside weather was approximated by degree days, transient heat conduction was approximated by equivalent temperature differentials, and combined radiative/convective boundary conditions were approximated using the "sol-air" temperature. HVAC system equipment was, therefore, usually grossly oversized, as a hedge against a poor guess, and energy requirements could not be predicted with enough accuracy to make a rational choice between different HVAC systems.

Such inexactness in determining energy requirements of buildings is a problem for a number of reasons. Rising construction and energy costs are driving building owners to insist that equipment be no larger than absolutely necessary. Because of energy conservation drives, energy utilizing equipment and energy conserving methods are more varied now than at any time in the past. Architects and building owners would like the flexibility of knowing quickly and accurately the effects of changes in such factors as materials, building design, unconventional energy sources and HVAC systems on heating and cooling energy costs. Energy requirements, which entail the greatest amount of guesswork in conventional calculations, now have become the dominant concern in the selection of heating and cooling equipment and the design of buildings.

The accurate determination of the energy requirements for a building depends on many factors including the following:

1) position of the sun and hourly intensity of the solar radiation,

2) ambient weather conditions,

3) shading of the exterior surfaces,

4) heat transfer and storage characteristics of building elements,

5) secondary HVAC systems and their operation,

6) primary HVAC systems and their performance,

7) variable internal heat sources,

8) characteristics of conventional and unconventional energy sources.

Each of these factors will have various amounts of influence on the energy consumption of a building, reflecting the wide variations in architecture, use, materials, equipment, etc. available to an engineer.

The procedures developed by ASHRAE include the effects of all the factors mentioned above, and the computer programs using these procedures usually consist of four sub-programs executed in sequence, with the output of one becoming the input to the next. The function of each sub-program is summarized below.

1) Load Calculation Sub-program. Calculates sensible and latent components of hourly heating and cooling loads for each space in the building.

2) Secondary HVAC Systems Simulation Sub-program. Modifies sensible and latent components of hourly heating and cooling loads for each zone* in the building by taking into account the outside air requirements, building operating schedules, secondary HVAC systems control schedules and internal temperature and humidity variations.

3) Primary HVAC Systems Simulation Sub-program. Converts hourly heating and cooling loads into energy requirements based on partial load characteristics of the primary HVAC systems.

4) Economic Analysis Sub-program. Calculates the annual overall owning and operating cost of building by taking into account various combinations of HVAC systems and energy sources.

LOAD CALCULATION SUB-PROGRAM

The Load Calculation Sub-program, a complex of heat transfer, environment, and geometry subroutines, calculates sensible and latent components of hourly heating and cooling loads imposed upon the building HVAC system by each space at each hour. The input to the Load Calculation Sub-program reflects building architecture, building structure, the building surroundings, local weather, and the pertinent astronomy of the sun.

The ASHRAE load calculation procedure utilizes the Convolution Principle to account for the thermal storage (time delay) effect of the building structure.

When one time series $\{A\}$ is influenced by another time series $\{B\}$,

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*Zone is defined as group of spaces whose load profiles are closely correlated and which can, therefore, be lumped together as a single load on the HVAC system.

the relation between these two series may be expressed in a linear form as:

$$A_t = \sum_{i=0}^{n} X_i B_{t-i} \qquad \text{for } t = 0, 1, 2 \ldots \qquad (1)$$

In this example, the value of A at time t is expressed as a linear function of B at t = t, t-1, t-2, ... t-n with X_0, X_1 ... X_n being the time-independent coefficients. The relationship given in Eq. (1) is called the Convolution Principle and X_0, X_1 ... X_n are called the filter coefficients in the mathematics of time series analysis. They are called the response factors when referring to heat gain/loss calculations, and weighting factors when referring to the hourly load calculation in the ASHRAE load calculation procedure. In the above expression, the time series {A} is said to be calculated "by convolving" the time series {B} with the response factors {X}

In the ASHRAE load calculation procedure, the Convolution Principle is used:

1) to compute the transient heat gain (loss is considered as negative heat gain) through exterior walls/roofs;

2) to compute the time delay between heat gain to a space and resulting cooling (heating is considered a negative cooling load) load on the HVAC system;

3) to compute space temperature deviation.

The value of n in the Eq. (1) depends upon the degree that, at n hours previous to time t, B would influence the value of A_t. If the influence of B_{t-n} upon A_t is insignificant, X_n is nearly zero and the values of B beyond t-n hour is of no importance.* If the storage effect does not exist, the value of n will be zero. In this case, all response factors, except the first term X_0, will be equal to zero and calculations reduce to steady-state.

To understand the use of Convolution Principle, as an example, heat gain into the building through a wall/roof is explained as follows.

The value of heat gain Q into the building through a wall/roof depends on the present value, and the past history of the temperature difference ΔT between the inside air and the outside surface of the wall/roof.

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*Details of the technique which makes it possible to shorten the computation time and to lessen the computer memory requirement for carrying out the convolution calculations are described in Appendix A.

In other words, the graph of the schedule of Q vs. time t depends on the graph of the schedule of ΔT vs. t as shown in Fig. 1.

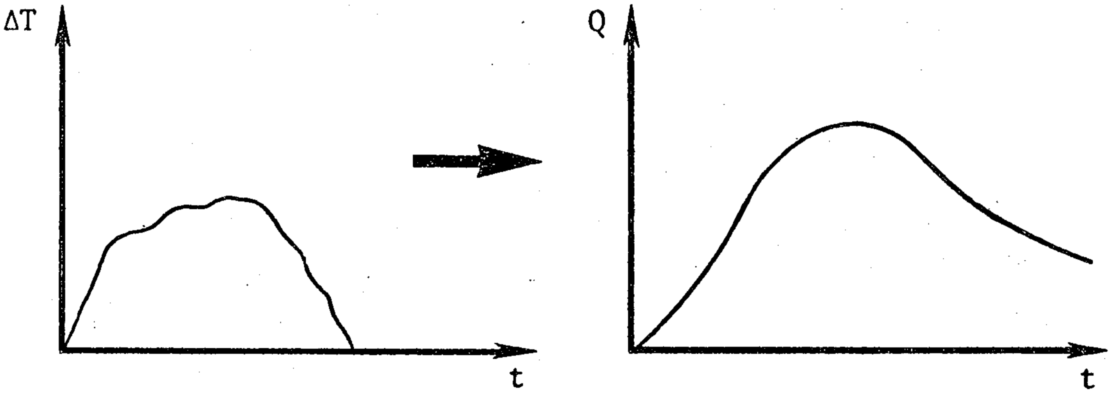

Fig. 1 Dependence of heat gain schedule on temperature difference schedule.

If it were necessary to compute Q for each hour, on the basis of the hourly history of ΔT, the differential equation of heat conduction would have to be repeatedly solved. This time-consuming operation, however, can be simplified so that Q need be determined as a function of t for only one temperature difference schedule. When a unit height isosceles triangle temperature pulse is used as one temperature schedule, the values of Q at successive equal-time intervals elicited by this unit height isosceles triangle, are the response factors r_0, r_1,... of the wall/roof construction as shown in Fig. 2.

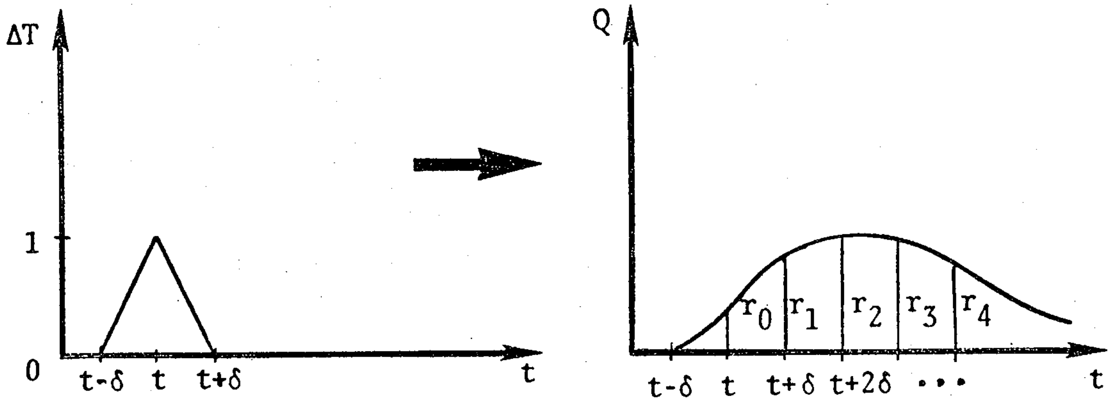

Fig. 2 Heat gain schedule for a unit height isosceles triangle temperature pulse showing response factors.

Any arbitrary schedule of ΔT (Fig. 3a) may be approximated by a schedule of $\Delta T'$, whose values agree with those of ΔT at integral multiples of the time interval δ. This schedule of approximate temperature differences $\Delta T'$, may be resolved into a series of isosceles triangle pulses (ΔT_1, ΔT_2, ΔT_3, ΔT_4, and ΔT_5 in Fig. 3c) which, when added together, give exactly $\Delta T'$. Each of these component pulses has a base width, or duration of 2δ, a peak occurring at each integral multiple of δ, and a height equal to the value of $\Delta T'$ at the time of the pulse's peak. Each pulse alone would elicit its own heat gain schedule as shown in Figs. 3d, 3e, 3f, 3g and 3h. The heat gain schedules elicited by the individual pulses are all the same except for two differences. Their heights are proportional to the heights of the pulses which elicit them, and each is moved to the right on the time axis as far as the pulse which elicited it.

The values of the individual responses, $Q_1 \ldots Q_5$, may be added at each value of time, to give the curve of sums (Fig. 3i). The superposition theorem asserts that the curve of sums is exactly the heat gain schedule which would be elicited by the approximate temperature difference schedule $\Delta T'$. Due to the smoothing effect of the heat transfer process, ΔT and $\Delta T'$ give nearly the same heat gain schedule. Therefore, the curve of sums is very nearly the heat gain schedule elicited by the original temperature difference schedule ΔT. Using Eq. (1), this method of resolution and recombination can be expressed mathematically as:

$$Q_t = \sum_{i=0}^{n} r_i \Delta T_{t-i}, \qquad (2)$$

whereas Q_t equals the heat gain at the time t; ΔT_{t-i} equals the temperature difference at t and in hours previous to t; r_i equals the i^{th} response factor for the wall/roof. It should be noted that the response factors are the only information about the wall/roof which appears in Eq. (2). Thus, the response factors characterize completely the thermal and physical properties of the wall/roof structure and, alone describe how the structure absorbs and releases heat over a prolonged period of time.

Combination of the instantaneous heat gains from lights, internal sources, infiltration, heat conduction through walls and roofs and solar radiation through windows, yields the cooling load. This combining procedure is not a simple addition because heat gains from different sources undergo different time delays before appearing as loads on the HVAC system. That is, instantaneous heat gains from different sources, which enter a space at the same instant, may appear as cooling loads at different times later. The time delays result from heat storage by solid objects. For example, instantaneous heat gain from solar radiation is immediately absorbed by walls and furnishings, and transfers later to the inside air, over an extended time period, through the

FIGURE 3a

ACTUAL ΔT SCHEDULE ACROSS WALL

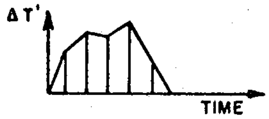

FIGURE 3b

APPROXIMATE ΔT SCHEDULE

FIGURE 3c

THE COMPONENT PULSES

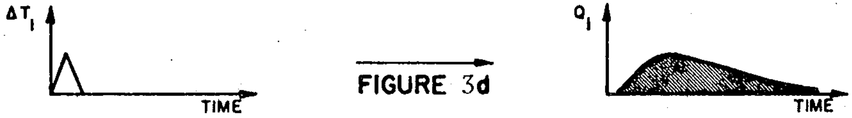

FIGURE 3d

FIRST COMPONENT PULSE AND HEAT GAIN IT ELICITS

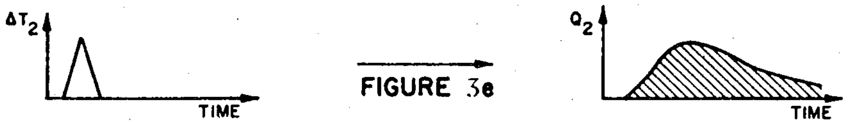

FIGURE 3e

SECOND COMPONENT PULSE AND HEAT GAIN IT ELICITS

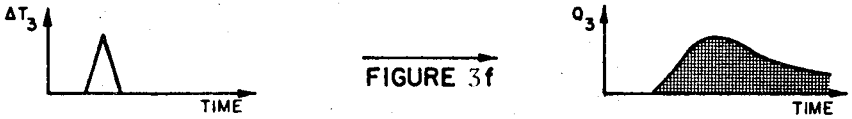

FIGURE 3f

THIRD COMPONENT PULSE AND HEAT GAIN IT ELICITS

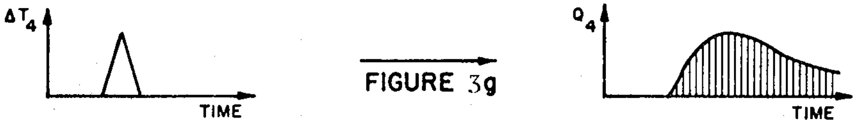

FIGURE 3g

FOURTH COMPONENT PULSE AND HEAT GAIN IT ELICITS

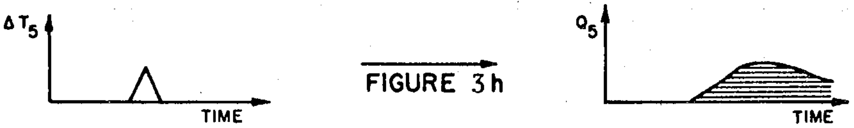

FIGURE 3h

FIFTH COMPONENT PULSE AND HEAT GAIN IT ELICITS

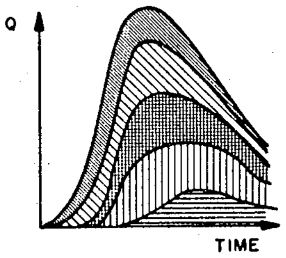

FIGURE 3i
CURVE OF SUMS

Fig. 3 Graphical Explanation of the Convolution Principle convective air film which surrounds these objects. The effect of time delays can be taken into account by using Eq. (1) with proper weighting factors. In this case, {A} and {B} correspond to cooling load and heat gain respectively.

Any deviation of space temperature from a constant design set-point has an important effect on the cooling load. Cooling load has usually been calculated and cooling equipment sized under the assumption that the space temperature remains constant at the design set point. In reality, space temperature does not remain constant and due to its deviation, the actual rate of heat extraction from the space (or zone) differs from the calculated cooling load. Therefore, using operation characteristics of the secondary HVAC systems, space temperature and heat extraction rate must be re-calculated as the final step before energy requirements are determined. In this phase of calculations, the Convolution Principle also becomes a very needed and useful tool.

The comparison of calculated and measured air temperatures of a room and calculated instantaneous heat gain and cooling load and measured heat extraction rate is shown in Fig. 4 [5] for an experimental building. This figure clearly indicates the fundamental difference between heat gain, cooling load and heat extraction rate.

The validity of the Convolution Principle was demonstrated by several researchers. Fig. 5 [6] shows the comparison of calculated and measured values related to thermal performance of an experimental building. As seen from this figure, calculated and measured values are in very good agreement with each other indicating validity of the Convolution Principle.

PRIMARY AND SECONDARY HVAC SYSTEMS SIMULATION SUB-PROGRAM

Due to outside air requirements, building operating schedules, HVAC systems control schedules and internal temperature and humidity variations, the hourly heating and cooling required from primary HVAC systems differ markedly from the hourly heating and cooling loads calculated in the Load Calculation Sub-program. The primary and secondary HVAC systems simulation sub-program performs three functions.

1) Sizes energy-consuming primary HVAC systems component (chillers, boilers, pumps, cooling towers, etc.) and secondary HVAC systems using peak zone and peak building heating and cooling loads determined by the Load Calculation Sub-program.

2) For each hour of the analysis, translates hourly loads and outside air loads by means of the individual performance characteristics of each fan system to obtain the hourly thermal requirements that must be provided by the primary HVAC systems.

3) Through the use of part load performance characteristics of the primary HVAC systems, converts the thermal requirements into energy requirements.

The fundamental principle of system simulation can be expressed as "predicting the operating quantities (pressures, temperatures, energy- and fluid-flow rates) within a system at the condition where all energy and material balances, all equations of state of working substances, and all performance characteristics of individual components are satisfied".

At this point, it is important to recognize that the term "system simulation" is frequently applied to the process of predicting the performance of a system undergoing changes of operating variables with respect to time. In such a simulation, the thermal capacities of equipment are crucial. Because of the present state of art, the simulation of primary and secondary HVAC systems is restricted to steady-state simulation with the assumption that during each hour of time interval the operation stays as steady-state but varies from one hour to another. It is essential that the dynamic characteristics of the building be considered in the hourly cooling and heating load calculation. However, since the dynamic response of most HVAC systems is much more rapid than that of the building, a steady-state simulation of the HVAC systems can be justified as adequate for most energy calculations. Because part-load performance characteristics are crucial to energy utilization analysis of buildings, the performance data of the HVAC system components must be available for the entire range of operating conditions that might be experienced during the operation of the system.

The data given by manufacturers are usually in the form of tables and/or graphs--a form which has been convenient for engineers. For computer programs that perform energy calculations, however, performance characteristics represented in equation form will be most convenient.

ECONOMIC ANALYSIS SUB-PROGRAM

The Economic Analysis Sub-program calculates the annual overall owning and operating cost of a building by taking into account all life cycle costs for each primary and secondary HVAC system analyzed. Life cycle costs are those expenditures which occur either once or periodically over the life of the building and include cost of energy, cost of equipment in terms of first costs and replacement costs which occur if the expected life of the equipment is less than that of the building, cost of maintenance (material and labor), cost of periodic overhaul (material and labor), salvage value of equipment at the end of building life, and costs of floor space which might result due to the unavailability of space occupied by equipment for other purposes. Once the life cycle costs have been accumulated and entered into the Economic Analysis Sub-program, the sub-program then proceeds to determine the present value of all life cycle costs over the expected life of the building and calculate the annual overall owning and operating cost which is construed to mean the uniform annual cost, considering all life cycle costs, to the owner during the lifetime of the building.

CONCLUSIONS

It is generally accepted that buildings can be designed to be energy conservative if their thermal insulation is increased; window size, air leakage, and lighting levels decreased; shading devices properly installed; HVAC systems adequately designed, installed, and maintained; and their thermal storage capability most fully utilized. These energy saving features, however, must be considered with reference to numerous constraints, such as added costs for material, construction and maintenance, conformance to local building codes, occupancy life styles, aesthetics, construction practices, and availability of equipment.

In spite of these constraints, there is sufficient engineering information and technical basis that exist today to warrant extensive studies on various design alternatives including unconventional energy sources for heating and cooling the building to minimize the wasteful use of conventional energy sources.

With the procedures developed by ASHRAE, most of the non-conventional or innovative ideas on structures, HVAC systems and controls could be studied. Some of the unique features that could be tried are:

1) Effect of the thickness, type and relative position in the construction of insulation of exterior wall/roof,

2) Effect of thermal storage of interior walls, floor-ceiling sandwich and furniture,

3) Effect of occupant, lighting and equipment schedules,

4) Evaluation of intentionally undersized primary HVAC systems by calculating the room temperature and humidity deviations from a design set point,

5) Evaluation of indoor thermal environment of various zones during the intermediate season, such as spring and autumn, when the heating or cooling requirements for these zones may not be in phase with that of the building as a whole,

6) Effect of intermittent operation, such as the shutdown of HVAC systems during the nighttime or on weekends,

7) Effective use of ventilation (outside air),

8) Effective use of internal and external shading,

9) Off-peak heating or cooling of buildings to shave the peak heating or cooling demand,

10) Use of solar energy for heating and cooling buildings.

In reality, a computer program using the procedures developed by ASHRAE becomes a very powerful toll for an engineer analyzing energy utilization of buildings. Such a computer program, where the size of the investment and energy conservation drive justifies the expense of the computations, may be used, in all stages of decision-making as follows.

1) Pre-design selection of the basic elements of building, primary and secondary HVAC systems and energy source.

2) Evaluation, during the design stage, of specific design concepts and modifications.

3) Evaluation, during construction, of contractor proposals for deviations from the construction plans and specifications.

4) Monitoring the operation and maintenance of the finished building so as to provide the greatest return on utility investment.

In such a computer program, both the effort required for input data preparation and computer running times depend on the complexity of the building under consideration and the degree of analysis desired. To reduce input data preparation and computer running times, a new state of the art computer program called Cal-ERDA has been developed and released to public use recently [7]. It is believed that this computer program has answers to the needs of the engineering community. It is an easy-to-use, fast running, completely documented, public-domain and economically operated computer program. In the program the emphasis has been given on input, efficiency of computation, flexibility of operation, and utility of output. A key factor in attaining these improvements has been achieved by the development of a free formated problem oriented "BUILDING DESIGN LANGUAGE" which greatly facilitates the user's task in defining the building and its HVAC systems.

BIBLIOGRAPHY AND REFERENCES

1. Procedure for Determining Heating and Cooling Loads for Computerized Energy Calculations - Algorithms for Building Heat Transfer Subroutines, Edited by Metin Lokmanhekim, ASHRAE 1971.

2. Procedures for Simulating the Performance of Components and Systems for Energy Calculations, Edited by Wilbert F. Stoecker, ASHRAE 1971.

3. Metin Lokmanhekim, Calculation of Energy Requirements with the Proposed ASHRAE Algorithms for U.S. Postal Buildings, presented at First International Symposium on Use of Computers for Environmental Engineering Related to Buildings, Gaithersburg, Maryland, U.S.A., Nov. 30-Dec. 2, 1970.

4. Metin Lokmanhekim, Calculation of Cooling and Heating Loads with Proposed ASHRAE Procedure and Energy Utilization Analysis of Buildings, University of Hawaii, Department of Mechanical Engineering, 1971-72 Academic Year National Science Foundation Distinguished Visiting Scholar Seminar, Honolulu, Hawaii, U.S.A., Dec. 10-13, 1971.

5. Gintas P. Mitalas, An Experimental Check on the Weighting Factor Method of Calculating Room Cooling Load, Presented at ASHRAE Annual Meeting, Denver, Colorado, U.S.A., June 30-July 2, 1969.

6. Tamami Kusuda, NBSLD, The Computer Program for Heating and Cooling Loads in Buildings, U.S. Department of Commerce, National Bureau of Standards, 1976.

7. Metin Lokmanhekim, et. al, Cal-ERDA, A New State of the Art Computer Program for the Energy Utilization Analysis of Buildings, Presented at Third International Symposium on the Use of Computers for Environmental Engineering Related to Buildings, Banff, Alberta, Canada, May 10-12, 1978.

APPENDIX A

FUNDAMENTALS OF EFFICIENT USE OF CONVOLUTION PRINCIPLE*

The Convolution Principle can be expressed mathematically as

$$Q_t = \sum_{i=0}^{n} r_i \Delta T_{t-i} \qquad (1)$$

Utilization of Eq. (1), as is, requires extensive computation time at each simulated hour. However, use of a subtle modification of it which allows n to equal infinity, if it is necessary, saves tremendous amounts of computation time. The explanation of the fundamentals of this efficient use technique is given as follows.

At time t-1, Eq. (1) can be written explicitly as

$$Q_{t-1} = r_0 \Delta T_{t-1} + r_1 \Delta T_{t-2} + \cdots + r_m \Delta T_{t-m-1} + r_{m+1} \Delta T_{t-m-2} + r_{m+2} \Delta T_{t-m-3} + \cdots \qquad (2)$$

If response factors reach a Common Ratio at the m^{th} hour after a unit height isosceles triangle pulse is applied, then the Common Ratio R can be expressed as

$$R = \frac{r_{m+2}}{r_{m+1}} = \frac{r_{m+3}}{r_{m+2}} = \ldots \qquad (3)$$

Using Eq. (3), Eq. (2) can be written as

$$Q_{t-1} = r_0 \Delta T_{t-1} + r_1 \Delta T_{t-2} + \cdots + r_m \Delta T_{t-m-1} + r_{m+1} (\Delta T_{t-m-2} + R \Delta T_{t-m-3} + \ldots) \qquad (4)$$

---

*The technique described here was developed by M. Lokmanhekim and C.C. Groth and presented at the Conference on Heat Transfer in Buildings of International Center for Heat and Mass Transfer, Dubrovnik, Yugoslavia, Aug. 29-Sept. 2, 1977.

Similarly, at time t, Eq. (1) can be written as

$$Q_t = r_0 \, \Delta T_t + r_1 \, \Delta T_{t-1} + \dots + r_m \, \Delta T_{t-m} + r_{m+1} \, (\Delta T_{t-m-1} + R \, \Delta T_{t-m-2} + \dots) \tag{5}$$

Multiplying both sides of Eq. (4) by R, subtracting the result from Eq. (5), rearranging the terms and solving for Q_t gives

$$\begin{aligned} Q_t - RQ_{t-1} = {} & r_0 \, \Delta T_t + r_1 \, \Delta T_{t-1} + \dots + r_m \, \Delta T_{t-m} \\ & + r_{m+1} \, (\Delta T_{t-m-1} + R \, \Delta T_{t-m-2} + \dots) \\ & - R \, r_0 \, \Delta T_{t-1} - R \, r_1 \, \Delta T_{t-2} - \dots - R \, r_m \, \Delta T_{t-m-1} \\ & - r_{m+1} \, (R \, \Delta t_{t-m-2} + R^2 \, \Delta T_{t-m-3} + \dots) \\ = {} & r_0 \, \Delta T_t + (r_1 - R \, r_0) \, \Delta T_{t-1} + \dots + (r_{m+1} - R \, r_m) \, \Delta T_{t-m-1} \end{aligned}$$

Let $r'_0 = r_0$

$r'_1 = r_1 - R \, r_0$

$\cdot$

$\cdot$

$\cdot$

$r'_{m+1} = r_{m+1} - R \, r_m$

Then

$$Q_t - R \, Q_{t-1} = r'_0 \, \Delta T_t + r'_1 \, \Delta T_{t-1} + \dots + r'_{m+1} \, \Delta T_{t-m-1}$$

and

$$Q_t = R \, Q_{t-1} + \sum_{i=0}^{m} r'_m \, \Delta T_{t-i} \tag{6}$$

Careful examination of Eq. (6) shows that once Q_{t-1} is calculated, it can be stored and later used in the calculation of Q_t along with a few multiplications and additions saving tremendous amounts of computation time compared to repetitive use of Eq. (1) for each simulated hour.

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