Entrepreneurial Innovation*

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Abstract

This paper presents an equilibrium model in which the process of firm formation and technology adoption is endogenous. Individuals decide whether to work in an existing firm for a posted wage, or to establish a new firm. Entrepreneurs hire a single worker and choose a production technology from a fixed set. The stochastic properties of different technologies are known with different, and exogenously specified, degrees of precision. We use Dempster's (1967) *lower probabilities* to characterize these differences in objective precision of risk information.

Individuals in the model are heterogeneous with respect to their tolerance of imprecise risk. This heterogeneity determines which technologies are adopted in equilibrium, the number of firms adopting each active technology, firm structure (risk attitudes of owner and worker), and the wage differentials across firms adopting different technologies. We can also parametrically alter the risk precision associated with a given technology to examine the effect on equilibrium. This comparative static exercise suggests an explanation for the commonly observed S-shaped diffusion profile for successful innovations.

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1 Introduction

In this paper we build a model of an *entrepreneurial* economy, in the sense that all agents are alert to the possibility of establishing new markets, as well as participating in existing ones. This alertness to innovative opportunities is introduced using a refinement of the usual logic of competitive equilibrium, which we call "innovation-proofness". Equilibria must exclude the possibility of establishing a firm in a new market and Pareto improving upon the welfare of the firm's participants.[1] We show that innovation-proof equilibria exist under very mild conditions. Thus, entrepreneurial alertness is fully compatible with the smooth functioning of competitive markets (as one might hope!).

Importantly, and in contrast to Kirzner's (1997) theory of entrepreneurship, all agents in the economy are equally alert to innovative opportunities. Instead, they differ in their willingness to *act* in response to these opportunities, which is determined by their *preferences*. Thus, in our model, as in most neoclassical competitive analysis, all features of equilibrium flow from the standard primitives: preferences and technologies.

However, one of the chief novelties of our framework lies in the specification of these primitives. In order to distinguish well-established technologies from new innovations, we employ Dempster's (1967) *lower probabilities* to parametrize the degree of precision of information about the stochastic properties of different technologies. These lower probabilities allow risk information to be more or less precise. The degree of precision is therefore exogenous and objective, and commonly understood by all agents.

Agents' evaluations of innovative opportunities depend on their attitudes to "imprecise risks". Drawing on the decision-theoretic models of Jaffray (1989, 1991) and others, we distinguish two types: *Bulls*, who interpret imprecise risk information in a maximally optimistic fashion; and their pessimistic counterparts, the *bears*. This, we believe, is a very fruitful framework to study the impact of entrepreneurial activity on competitive equilibria.

A key feature of such entrepreneurial activity is firm formation. The archetypal entrepreneur is a visionary individual – a Bill Gates or a Richard Branson – who leaves a secure job in order to build a new enterprise. Hence, in our model, individuals, rather than firms, are the decision-making units, and firm formation is endogenous. Each individual decides whether to be an employee or to establish a firm, and in the latter case, whether to set up in an existing market or forge a new one using a novel technology. We may therefore determine the equilibrium number of firms in each industry, and also the "character" of individual firms. As Drèze (1985, p.5) points out, firms, unlike human beings, have no "visceral reaction to uncertainty". In the face of uncertainty and incomplete markets, the objectives of the firm need to be induced from the objectives of owners and workers.

We distinguish three firm types. One is "entrepreneurial", with Bull workers sharing the optimistic vision of their Bullish employer regarding a technology of highly uncertain value. Another type of firm, evident in more staid industries, is "traditional" in character: Bulls hire bears, with the former receiving a surplus for bearing the residual risks of the enterprise. If risks are very precisely known, firms may even be "bureaucratic", with bears at all levels of the organization.

[1]For similar refinements in somewhat different contexts, see Hart (1980) and Makowski (1980).

Because Dempster's lower probabilities allow the precision of risk information to be parametrically specified, one may study the effects of precision on the properties of equilibria. A comparative static exercise of this sort is carried out in Section 5. In addition to tracking changes in industry size and wage differentials, we also observe transitions in the character of firms. Even for a simple two-industry economy, a surprisingly rich picture emerges.

These comparative statics convey some useful lessons for understanding the S-shaped diffusion profile for successful innovations. Unlike the diffusion models of Jensen (1982) and Vettas (1998), in which firms are the units of decision-making, our analysis reveals the important role of changing firm structure over the diffusion path. Initially, innovative firms are of the entrepreneurial type, and diffusion is retarded by the limited pool of suitably optimistic workers, and the high wages necessary to prevent them from starting rival firms. However, once information about the new technology becomes sufficiently precise, bears are willing to work in the new industry. During this phase, equilibrium firm structure in the new industry becomes traditional. This enlarges the pool of potential workers, and lowers equilibrium wages, as bears derive utility directly from the "insurance" value of wage income. Diffusion now proceeds apace.

The remainder of the paper is organized as follows. The next section describes the economy, the use of lower probabilities to characterize imprecise risk, and the decision-making rules employed by Bulls and bears. Section 3 introduces the equilibrium concept, and discusses issues of existence. Some general observations on the occupational choices made in equilibrium are offered in Section 4. Section 5 examines equilibrium comparative statics in uncertainty using a simple example, and discusses possible lessons about diffusion. Section 6 concludes. Three appendices contains various formal arguments omitted from the body of the text.

2 Imprecise risk, decision-making, and the economy

The economy consists of several industries. Each industry's revenue is subject to risk, but risk may be imprecisely specified. Individuals differ in how they evaluate these revenues. In this setting, heterogeneity in the evaluation of stochastic processes with vague information is central to the model. We first explain how to formalize imprecise risk using *belief functions* (or *lower probabilities*), and then describe individual decision-making. Finally, we introduce an economy in which production risks are described by belief functions.

2.1 Belief functions and imprecise risk

Imagine a situation in which there is randomness that is partially, but not fully, probabilized. Ellsberg's famous 3-color experiment provides a convenient example.[2] An urn contains 90 balls; 30 balls are known to be red, while each of the remaining 60 balls is either yellow or black. Hence, according to the objective information, the number of yellow balls in the urn lies between 0 and 60. Likewise for the number of black balls. If a ball is drawn at random from the urn, there is partial information about event probabilities on the state space $\Theta = \{r, y, b\}$. For example, if $\pi(E)$ denotes the probability of $E \subseteq \Theta$, then $\pi(\{r\}) = 1/3$ and $\pi(\{y, b\}) = 2/3$,

[2] Our discussion of this example is adapted from Hendon *et al.* (1994).

but all we know about $\pi(\{y\})$ and $\pi(\{b\})$ is that they lie in the interval $\left[0, \frac{2}{3}\right]$. Similarly, the probabilities $\pi(\{r,y\})$ and $\pi(\{r,b\})$ lie in $\left[\frac{1}{3}, 1\right]$.

There are several ways to describe this information about event probabilities. One is simply to assign the appropriate intervals to each event, as we did above. Another is to specify the set of probabilities on Θ consistent with the information. For the present example, this set is

$$\left\{ \pi \in \Delta(\Theta) \;\middle|\; \pi(\{r\}) = \frac{1}{3} \right\} \tag{1}$$

(where $\Delta(\Theta)$ denotes the set of all probabilities on Θ). Yet another method is to describe the *lower envelope* of the set (1). Let $\underline{v}$ denote this lower envelope. Thus, $\underline{v}(\{r\}) = \underline{v}(\{r,y\}) = \underline{v}(\{r,b\}) = 1/3$, $\underline{v}(\{y\}) = \underline{v}(\{b\}) = 0$, and $\underline{v}(\{y,b\}) = 2/3$. Observe that this lower envelope does not omit any information, since the upper bound on the probability of any event E is precisely $1 - \underline{v}(E^c)$. In other words, the interval containing the "true" probability of event E is $[\underline{v}(E), 1 - \underline{v}(E^c)]$.

The function $\underline{v}$ is called a *belief function* (Shafer (1976)), and was introduced as a description of imprecise risk by Dempster (1967).[3] Not all situations of imprecise risk can be completely characterized by a belief function, since not all probability sets can be fully characterized by their lower envelopes. The situations that do admit a belief function characterization may be described as follows. Let there be a set S of fundamental states, endowed with a σ–algebra Σ and a commonly known probability measure p, and a measurable *information correspondence* $\Gamma : S \twoheadrightarrow \Theta$, mapping elements of S into subsets of Θ. The correspondence Γ generates imprecise probabilistic information about events in Θ. If fundamental state $s \in S$ is realized, the available information implies that some state $\theta \in \Gamma(s)$ also occurs, but nothing else is known about the relative likelihoods of states in $\Gamma(s)$. If Γ is singleton-valued (a *function*), then p and Γ together induce a probability on Θ. Otherwise, the available information about the stochastic process by which states in Θ are realized may be too vague to yield a well-defined probability.

In Ellsberg's example, one may choose $S = \{r, yb\}$. Then $p(\{r\}) = 1/3$, $p(\{yb\}) = 2/3$, $\Gamma(r) = \{r\}$, and $\Gamma(yb) = \{y, b\}$. More generally, S may describe a partition of Θ for which objective probabilities are known. However, other interpretations are also possible.

The *upper probability* of event $E \subseteq \Theta$ is

$$\overline{v}(E) := p(\{s \in S \mid \Gamma(s) \cap E \neq \emptyset\})$$

and the *lower probability* is

$$\underline{v}(E) := p(\{s \in S \mid \emptyset \neq \Gamma(s) \subseteq E\}).$$

The probability interval for event E is therefore $[\underline{v}(E), \overline{v}(E)]$. When Γ is singleton-valued, the upper and lower probabilities coincide, and their common value is an ordinary probability measure. This model therefore provides a natural vehicle for studying the comparative static effects of changes to the nature or degree of uncertainty. If $\Gamma(s) \subseteq \hat{\Gamma}(s)$ for all $s \in S$, then it is natural to say that Γ describes a situation involving less uncertainty than $\hat{\Gamma}$: the induced probability intervals on each event "shrink".

[3] See also Mukerji (1997) for a useful discussion of belief functions in the context of economic decision-making.

Dempster (1967) showed that $\overline{v}(E) = 1 - \underline{v}(E^c)$, so we may dispense with the upper probability (which is simply the "dual" of the lower probability). The lower probability constructed in this manner is a belief function. Furthermore, the set of probabilities on Θ compatible with the given information is

$$\{\pi \in \Delta(\Theta) \mid \pi(E) \geq \underline{v}(E) \quad \forall E \subseteq \Theta\} \tag{2}$$

The set (2) is called the *core* of the belief function $\underline{v}$, denoted core $(\underline{v})$. It is closed and convex.[4]

2.2 Bulls and bears

Objects of choice are imprecise monetary lotteries represented by random variables $f : \Theta \to \mathcal{R}_+$. Given a specification for (S, Σ, p) and Γ, each such lottery induces a belief function on $\mathcal{R}_+$. Models of decision-making in this framework have been axiomatized by Jaffray (1989, 1991), Hendon *et al.* (1994) and Jaffray and Wakker (1994).[5] We adopt two important special cases of the models in Jaffray (1989) and Hendon *et al.* (1994). However, following Jaffray and Wakker (1994), the state space Θ is kept explicit in the description of the objects of choice, rather than representing random variables directly as belief functions on $\mathcal{R}_+$.

We assume that all agents are risk neutral: preferences order purely risky prospects by their expected return. However we distinguish two types of agents – *Bulls* and *bears* – according to the decision weights they assign to outcomes when probabilities are imprecisely known. Bulls choose the most "optimistic" element of the set (2) of information-consistent probabilities; while bears choose the most "pessimistic". Therefore, Bulls evaluate f using

$$\mathbb{E}_{\overline{v}} f := \overline{\mathbb{E}} f = \max_{\pi \in \text{core}(\underline{v})} \mathbb{E}_\pi f$$

while bears employ the evaluation

$$\mathbb{E}_{\underline{v}} f := \underline{\mathbb{E}} f = \min_{\pi \in \text{core}(\underline{v})} \mathbb{E}_\pi f$$

where

$$\mathbb{E}_\pi f = \sum_{\theta \in \Theta} \pi(\theta) f(\theta).$$

Thus $\underline{\mathbb{E}} f \leq \overline{\mathbb{E}} f$. We call $\overline{\mathbb{E}} f$ the upper expected value of f; and $\underline{\mathbb{E}} f$ the lower expected value of f.

The operators defined above are not additive in general. In fact:

$$\mathbb{E}_{\underline{v}}(f + g) \geq \mathbb{E}_{\underline{v}} f + \mathbb{E}_{\underline{v}} g$$

and

$$\mathbb{E}_{\overline{v}}(f + g) \leq \mathbb{E}_{\overline{v}} f + \mathbb{E}_{\overline{v}} g.$$

A sufficient condition for additivity is comonotonicity of f and g (see Schmeidler (1986, 1989)). The variables f and g are comonotonic if there do not exist states $\theta, \theta' \in \Theta$ such that $f(\theta) > f(\theta')$ and $g(\theta) < g(\theta')$.

[4] We shall assume Θ to be a finite set throughout the paper, so $\Delta(\Theta)$ is a subset of Euclidean space.

[5] There is also a literature on belief functions as *subjective* constructs in the characterization of choice under uncertainty. See, for example, Jaffray and Phillipe (1997), Mukerji (1997) and Ghirardato (2001).

2.3 The Economy

The economy consists of a continuum of agents, indexed by the unit interval, and I technologies. Agents are heterogeneous in their attitudes to imprecise risk: some are bears, and the rest are Bulls. Each technology requires the human capital of two agents – an owner and a worker – in order to be productive. A firm is therefore a coalition of two agents, employing one of the I technologies. Many firms may operate the same technology. Choices of non-labor inputs are suppressed in the analysis. As Baumol (1968, 1993) points out, the entrepreneurial function is distinct from the managerial one; the latter being concerned with matters such as fine-tuning the input mix in production.

We shall describe the set of firms employing technology i as *industry* i. A firm's revenue depends on the technology it employs, the density of firms in each industry, plus some stochastic factors. Formally, the revenue function of a firm in industry i is a bounded function $R_i(\theta, \delta)$, where $\theta \in \Theta$ is the payoff-relevant state, and $\delta \in \mathcal{R}^I$ is a vector of densities, δ_i being the total density of firms in industry i. Since each firm hires exactly one worker, the density δ_i of firms in industry i is an element of $\left[0, \frac{1}{2}\right]$. The objectively known stochastic properties of the random variable $R_i(\cdot, \delta)$ are described using a fundamental measure space (S, Σ, p) and an information correspondence $\Gamma : S \twoheadrightarrow \Theta$. We take S to be the unit interval $[0, 1]$, Σ the usual Borel σ–algebra on S, and p the Lebesgue measure on (S, Σ), unless otherwise specified. The state space Θ is finite.

The focus of our analysis is on the labor markets, so output markets are not explicit in the model. The dependence of revenues on the industry density vector δ is intended to capture, in reduced form, the effects of output market competition. The nature of this competition is not specified. The model is therefore compatible with price-taking or strategic behavior in these markets. It should be noted, however, that we assume exogeneity of demand-side conditions in the output markets: firm revenues are independent of the equilibrium incomes of agents in the model.

We shall make the following technical assumption to guarantee existence of equilibria:

Assumption 1 *For each i, R_i is continuous in δ.*

Note that although each random variable $R_i(\cdot, \delta)$ is defined on the same state space Θ, this does not imply that each industry is subject to the same source of revenue uncertainty. For example, one may have $\theta \in \mathcal{R}^I$, and for each i, the value of R_i is independent of θ_j for every $j \neq i$. In this case, the uncertainty in industry i depends on the precision of the available information about the ith component of θ, and this may be quite different to the uncertainty surrounding the jth component.[6] For example, one might wish to distinguish established industries, whose stochastic revenues are known precisely, from currently inactive industries (potential innovations) for which there is great uncertainty about profitability. Our framework allows this distinction to be modelled in a natural way.

[6] Any correlation in these random shocks is irrelevant to individual decision-making in any case, since agents can work in only one firm. Correlation would affect their occupation choice only if they could divide their time among several jobs.

Summarizing the discussion so far:[7]

Definition 1 *An* economy *is an object*

$$\mathcal{E} = \left\{ (S, \Sigma, p), \Theta, \Gamma, \left\{ R_i : \Theta \times \left[0, \frac{1}{2}\right]^I \to \mathcal{R}_{++} \right\}_{i=1}^{I}, \alpha \right\}$$

where (i) Γ *is a measurable correspondence from* S *to* Θ*; (ii)* R_i *gives the revenue of a typical firm in industry* i*; and (iii)* $\alpha \in (0,1)$ *divides the unit interval into Bulls, agents with indices in the sub-interval* $[0, \alpha)$*, and bears, agents with indices in the sub-interval* $[\alpha, 1]$*.*

3 Equilibrium and innovation-proofness

The agents, who are all aware of the form and stochastic properties of the I revenue functions, simultaneously choose an occupation (owner or worker) and an industry. Firm formation is therefore endogenous in the model. Occupational choice is based on knowledge of the industry wage rates, and common expectations about the densities of firms that will form in each industry. In equilibrium, labor markets clear, and the density of firms in each industry is required to match agents' common expectations. The latter is analogous to the more familiar assumption that agents correctly anticipate prices in the various output markets.

Each agent in economy $\mathcal{E}$ has $2I$ occupational options: wage-earning or firm ownership in one of the I industries. Exactly one occupation is chosen: agents cannot divide their time among a portfolio of jobs.

Wage contracts are subject to limited liability. That is, we make the following institutional assumption:

Assumption 2 *An employment contract in industry* i *specifies a wage level* $w_i \geq 0$ *and contains the following limited liability clause: if state* $\theta \in \Theta$ *is realized, the employer pays the worker*

$$\min \{w_i, R_i(\theta, \delta)\}$$

The limited liability clause implies that wage earning need not generate a sure income.[8] Assumption 2 also imposes restrictions on the form of risk sharing that is feasible through labor contracts.[9] Note that all firms in the same industry must offer the same wage, but different

[7]The definition of $\mathcal{E}$ actually goes slightly beyond our discussion: revenue lotteries are required to be strictly positive with probability one.

[8]Although we have not done so, it would be straightforward to elaborate the model so that agents also have heterogeneous endowments of initial wealth, and must draw on personal wealth to pay wages if necessary (as in Kihlstrom and Laffont (1979)). Wealthier agents may then be able to pay lower wages in equilibrium because of the lower likelihood of default. Formally, this effect is similar to a credit constraint in a model where owners must invest in capital in order to start their businesses, and credit markets are imperfect. In each case, personal wealth has a positive effect on the profitability of business ownership. Hence, wealthier agents are more likely to become owners, as some empirical evidence – such as that in Evans and Jovanovic (1989) – suggests.

[9]Rigotti and Ryan (2000) consider fully state-contingent wage-contracting in a single technology model. See also Kelsey and Spanjers (1997).

wages may be posted in different industries. This reflects the fact that default contingencies are identical for two firms in the same industry, but may differ across industries. Hence, the same posted wage may induce different state-contingent incomes in different industries.

To evaluate returns from the various occupational options, agents need to know the vector $w = (w_1, w_2, \ldots, w_I)$ of wage rates for each industry, and the vector of industry densities $\delta = (\delta_1, \delta_2, \ldots, \delta_I)$. Given (w, δ), a Bull obtains utility

$$\overline{\mathbb{E}}\left[R_i(\theta, \delta) - \min\{w_i, R_i(\theta, \delta)\}\right]$$

from owning a firm in industry i, and utility

$$\overline{\mathbb{E}}\left[\min\{w_i, R_i(\theta, \delta)\}\right]$$

from being a worker in the same industry. The utility obtained by bears from these occupations may be described similarly. Let $\mathcal{O} = \{1, 2, \ldots, 2I\}$ be a set of indices for occupations in $\mathcal{E}$. We shall index the occupation of being a firm owner in industry i by $(2i-1) \in \mathcal{O}$, and the occupation of being a wage earner in industry i by $2i \in \mathcal{O}$. Given an economy $\mathcal{E}$ and vectors (w, δ), we define the optimal occupation choices as follows.

Definition 2 *The Bulls'* optimal occupation set $BR^{\mathcal{E},B}(w, \delta) \subseteq \mathcal{O}$ *is the set of occupations that maximize the upper expected value of their income.*[10] *Let* $y^{\mathcal{E},B}(w, \delta)$ *denote the upper expected value of occupations in* $BR^{\mathcal{E},B}(w, \delta)$ *with respect to* $\overline{v}$.

Definition 3 *The bears'* optimal occupation set $BR^{\mathcal{E},b}(w, \delta) \subseteq \mathcal{O}$ *is the set of occupations that maximize the lower expected value of their income.*[11] *Let* $y^{\mathcal{E},b}(w, \delta)$ *denote the lower expected value of occupations in* $BR^{\mathcal{E},b}(w, \delta)$ *with respect to* $\underline{v}$.

Hence, $y^{\mathcal{E},B}(w, \delta)$ (respectively, $y^{\mathcal{E},b}(w, \delta)$) is the maximum expected remuneration available, given (w, δ) and Bullish (respectively, bearish) evaluation of imprecise risks.

In addition to the vectors (w, δ), an equilibrium must also specify the occupation of each agent. This is described using an *allocation function.*

Definition 4 *An* allocation function *for the economy* $\mathcal{E}$ *is a Lebesgue measurable function* $\phi : [0, 1] \to \mathcal{O}$.

We may now state the definition of an equilibrium for $\mathcal{E}$.

Definition 5 *The triplet* (w, δ, ϕ) *is an* equilibrium *of* $\mathcal{E}$ *if*

(i) $\phi(j) \in BR^{\mathcal{E},B}(w, \delta) \qquad \forall j \in [0, \alpha)$;

(ii) $\phi(j) \in BR^{\mathcal{E},b}(w, \delta) \qquad \forall j \in [\alpha, 1]$; *and*

[10] Obviously, this income is given by $\overline{\mathbb{E}}[R_i(\theta, \delta) - \min\{w_i, R_i(\theta, \delta)\}]$ if a Bull owns a firm in industry i and by $\overline{\mathbb{E}}[\min\{w_i, R_i(\theta, \delta)\}]$ if a Bull is a worker in that same industry.

[11] Again, this income is given by $\underline{\mathbb{E}}[R_i(\theta, \delta) - \min\{w_i, R_i(\theta, \delta)\}]$ if a bear owns a firm in industry i and by $\underline{\mathbb{E}}[\min\{w_i, R_i(\theta, \delta)\}]$ if a bear is a worker in that same industry.

(iii) *$Leb\left[\phi^{-1}(2i)\right] = Leb\left[\phi^{-1}(2i-1)\right] = \delta_i$ for each $i \in \{1, 2, \ldots, I\}$, where Leb denotes Lebesgue measure.*

An equilibrium specifies wages, industry densities and individual occupations such that, when all agents anticipate (w, δ) and each chooses his or her occupation optimally, all labor markets clear and the common expectations of δ are confirmed. It is quite possible that some industries fail to operate in equilibrium: that is, the vector δ may have some zero components. Hence, the equilibrium endogenously determines which industries operate (for example, which innovations are implemented), as well as the wage rates and firm densities in each operating industry.

Theorem 3.1 *Under Assumptions 1 and 2, every economy $\mathcal{E}$ has an equilibrium (w, δ, ϕ).*

Proof: Let $\Phi(w, \delta)$ denote the set of allocation functions for $\mathcal{E}$ consistent with (w, δ). Denote the associated set of occupational density vectors as

$$Z(w, \delta) = \left\{ \left(Leb\left[\phi^{-1}(i)\right]\right)_{i=1}^{2I} \middle| \, \phi \in \Phi(w, \delta) \right\}$$

Notice that elements of $Z(w, \delta)$ belong to the unit simplex $\Delta^{2I-1} \subseteq \mathcal{R}_+^{2I}$. Now let us define the $2I \times 2I$ matrix $C = [c_{ij}]$ as follows:

$$c_{ij} = \begin{cases} 1 & if \quad j = 2i-1 \quad or \quad j = 2(i-I)-1 \\ -1 & if \quad j = 2i \quad and \quad i \leq 2I \\ 0 & \quad otherwise \end{cases}$$

Letting[12] $X(w, \delta) = C\, Z(w, \delta)$, we see that each $x \in X(w, \delta)$ is a $2I$-vector whose first I components give the *net excess density of owners* in each industry associated with some allocation consistent with (w, δ), and whose second I components give the *total density of owners* in each industry associated with the same consistent allocation. This transformation induces a one-to-one mapping between $Z(w, \delta)$ and $X(w, \delta)$ since C is non-singular. Alternatively, one can recover $z \in Z(w, \delta)$ from $x = Cz \in X(w, \delta)$. Therefore, $X(w, \delta)$ is a subset of $B := C\Delta^{2I-1}$. Note that, in an equilibrium (w, δ, ϕ), ϕ induces $z \in Z(w, \delta)$ such that the first I elements are zeroes and the last I elements are δ.

For arbitrary vectors $y \in \mathcal{R}^{2I}$, let $proj_I\, y$ denote the projection of y onto its first I components; and $proj_{-I}\, y$ the projection of y onto its second I components. Then one may observe that

$$proj_{-I}\, B = \left\{ y \in \mathcal{R}_+^I \middle| \sum_{k=1}^{I} y_k \leq 1 \right\} \subseteq [0, 1]^I$$

Next, extend each R_i to the domain $\Theta \times [0, 1]^I$ as follows:

$$R_i(\theta, \delta) := R_i\left(\theta, \left(\min\left\{\frac{1}{2}, \delta_i\right\}\right)_{i=1}^{I}\right)$$

[12] To interpret the transformation that follows, assume that points in Euclidean space are *column* vectors.

These extended R_i functions are still strictly positive, and continue to satisfy Assumption 1. In particular, since the extended functions are bounded, we may define

$$\overline{W} = \max_i \sup_{(\theta,\delta)\in\Theta\times[0,1]^I} R_i(\theta,\delta)$$

The foregoing observations and definitions confirm that $D := \left[0,\overline{W}\right]^I \times [0,1]^I \times B$ is a compact and convex subset of Euclidean space. Let the correspondence $\xi : D \twoheadrightarrow D$ be defined as follows:

$$\xi(w,\delta,x) = \left\{\arg\max_{\tilde{w}\in\left[0,\overline{W}\right]^I} \tilde{w}\cdot proj_I\, x\right\} \times \{proj_{-I}\, x\} \times X(w,\delta)$$

One can easily verify that ξ is well-defined on its domain, and that $\xi(w,\delta,x) \subseteq D$ for any (w,δ,x) in D.

To see that any fixed point of ξ determines an equilibrium of $\mathcal{E}$, suppose that $(w,\delta,x) \in \xi(w,\delta,x)$. We must therefore have

$$\delta = proj_{-I}\, x \tag{3}$$

Then, suffices to show that

$$x^{(I)} := proj_I\, x = 0 \tag{4}$$

since (4) implies that all labor markets can clear given expectations (w,δ). From (3), we thus obtain that δ gives the corresponding vector of industry densities (which coincides, by (4), with the vector of owner densities). Finally, (4) and (3) jointly imply that

$$\delta \in \frac{1}{2}\Delta^{I-1}$$

as required. Hence, (w,δ,ϕ) is an equilibrium for any $\phi \in \Phi(w,\delta)$ generating x.

We now verify that (4) holds. Suppose instead that $x_i^{(I)} > 0$ for some i. The definition of ξ and the fact that (w,δ,x) is a fixed point imply $w_i = \overline{W}$. But then all agents *strictly* prefer being wage-earners in industry i than being firm owners in industry i. This follows because R_i is strictly positive by assumption: no matter how pessimistic agents' beliefs, wage-earners in industry i must expect a strictly positive income when $w_i = \overline{W}$. Since $x \in X(w,\delta)$, this means $x_i^{(I)} \leq 0$, which is a contradiction. Assuming $x_i^{(I)} < 0$ leads to a contradiction by a symmetric argument. Equation (4) is therefore confirmed.

Summarizing, we defined the correspondence $\xi : D \twoheadrightarrow D$, and deduced that for any fixed point (w,δ,x), there is a ϕ such that (w,δ,ϕ) is an equilibrium of $\mathcal{E}$. The final step is to show that ξ does indeed have a fixed point. But ξ satisfies all the conditions of Kakutani's fixed point theorem. The arguments are somewhat lengthy, and may be found in Appendix A. This completes the proof. □

For our purposes, Definition 5 requires some further refinement. In particular, Definition 5 may preclude innovation on the basis of irrational expectations about labor costs. Suppose, for example, that $\delta_i = 0$ in equilibrium. How do we interpret the common wage expectation w_i? A potential entrant into industry i would presumably assess the wage costs of operating in that industry to be equal to the minimum wage necessary to attract some other agent away from his or her current occupation. However, there is no reason why w_i should correspond to this wage rate in the definition of equilibrium given above. In other words, there may be equilibria where some industries do not operate because of unreasonable expectations about the wage rate that would prevail if that industry were opened.

A similar issue is raised in Hart (1980) and Makowski (1980), who analyze general equilibrium models where the set of traded commodities is determined endogenously. The market for a particular commodity may not open because the expected price of that commodity is "unreasonable". More recently, Dubey, Geanakoplos and Shubik (2001) encountered a similar problem. In their GEI model, individuals may choose to default on their financial obligations in some states, and thereby suffer some penalty, such as incarceration. Potential buyers must therefore anticipate the levels of default on different assets. The range of assets actively traded is endogenously determined in equilibrium. However, a market may be inactive on the basis of "unreasonably" pessimistic expectations about price and default risk.

3.1 Innovation-Proofness

We refine the equilibrium concept by ruling out the possibility of an agent strictly improving upon her equilibrium income by starting a firm in an inactive industry, and offering a wage equal to the lowest salary necessary to attract some other agent away from his or her current occupation. Therefore, we require that no lucrative entrepreneurial opportunity is left unexploited.

Definition 6 *Consider an economy* $\mathcal{E}$*, and let* (w, δ, ϕ) *be an equilibrium of* $\mathcal{E}$*. This equilibrium is* innovation-proof *if there does* **not** *exist an industry* i *with* $\delta_i = 0$*, a potential entrepreneur* $k \in [0,1]$*, a potential worker* $k' \in [0,1]$ $(k \neq k')$*, and a wage level* $\hat{w}_i > 0$ *such that*

(a) $\mathbb{E}_{v_{k'}}[\min\{\hat{w}_i, R_i(\theta,\delta)\}] \geq y^{\mathcal{E},\beta_{k'}}(w,\delta)$ *where* $\beta_{k'}$ *is the type of agent* k' *and* $v_{k'}$ *denotes the corresponding upper or lower probability; and*

(b) $\mathbb{E}_{v_k}[R_i(\theta,\delta) - \min\{\hat{w}_i, R_i(\theta,\delta)\}] > y^{\mathcal{E},\beta_k}(w,\delta)$ *where* β_k *is agent* k*'s type and* v_k *denotes the corresponding upper or lower probability.*

The concept of innovation-proofness represents a conceptual modification of the standard logic of price-taking equilibrium. Potential innovators are assumed not only to know the wages in existing industries, but also the reservation wage necessary to start a firm in any new industry. The (unmodelled) process by which this reservation wage information is promulgated within a market economy is clearly different to the (unmodelled) process by which wages in currently active labor markets are made known.

Formulating conjectures about reservation wages in inactive labor markets is fundamental to the process of innovative entrepreneurship. However, unlike writers in the Austrian tradition such as Kirzner, we do not differentiate agents in terms of their ability to formulate accurate conjectures of this sort (their "alertness" to entrepreneurial opportunity). Instead, we assume that all agents have perfect knowledge of reservation wage levels in all potential industries, and equal awareness of the revenue functions of inactive industries. Innovation in our model is therefore driven not by differences in agents' awareness of the entrepreneurial opportunities, but by differences in their responses to the uncertainties that may surround revenue levels in new industries.

In summary, innovation-proofness ensures that equilibria are robust to lucrative technological innovations when all agents have equal access to information about potential revenue functions, and assess the implicit wage rates in inactive industries at the reservation wage of the cheapest potential worker.[13] Since $\alpha \in (0,1)$, a non-zero density of agents would perceive such an opportunity should one exist, and their attempt to seize upon it would disturb the equilibrium. The following Theorem shows that there always exist equilibria with this property.

Theorem 3.2 *Under Assumptions 1 and 2, every economy $\mathcal{E}$ possesses an innovation-proof equilibrium.*

Proof: See Appendix B.

The basic idea of the proof is straightforward. Problems may arise in equilibrium if bears anticipate default on wages in inactive industries. These wages may then be above the reservation level for bears, as potential bear workers may still not find them strictly more attractive than the returns to their current jobs. The proof of Theorem 3.2 uses ε perturbations to the precision of information about risk to ensure that there is always some chance that wages will be paid. This guarantees that wages never strictly exceed reservation levels, and hence that equilibria of the ε-perturbed economy are innovation-proof. Letting $\varepsilon \to 0$, one then obtains a convergent sub-sequence of equilibria whose limit is an innovation-proof equilibrium of the unperturbed economy. This idea resembles the "trembling-hand" approach employed by Dubey, Geanakoplos and Shubik (2001). They induce "reasonable" price and default expectations in inactive markets by introducing ε-traders, who buy and sell small quantities of all assets, meeting all obligations in all states. Letting $\varepsilon \to 0$ induces the required reasonable expectations about the properties of untraded assets.

4 Optimism, pessimism, and occupational choice

In a single technology model, Kihlstrom and Laffont (1979) obtained an equilibrium characterization of firms as having owners who are less risk-averse than their workers. However, in Kihlstrom and Laffont (1982, 1983), this characterization is lost through the introduction of a richer set of institutions for sharing risk. Also, experimental evidence strongly suggests that firm owners are, in practice, no less risk-averse than salaried managers (Wärneryd (1988)). Hence, our model distinguishes agents on the basis of attitude to uncertainty, rather than risk.

Although we too have a restricted set of institutions for sharing risk, the limited liability Assumption 2 entails that wage-labor need not be substantially less risky than firm ownership. A rich structure of firm types is therefore possible in equilibrium.[14] Indeed, an important

[13] Consistent with this motivation, innovation-proofness rules out *entrepreneurial opportunities* that are strictly lucrative to the entrepreneur. However, the analysis of the paper goes through under an even stronger refinement: one that precludes any two agents forming a new firm in *any* industry (inactive or otherwise) such that the welfare of this two-agent coalition is Pareto improved.

[14] See section 5.1.2.

feature of innovation is the inability of the employer to effectively insure workers against uncertainty about the viability of the enterprise. If the firm fails, all are out of pocket. This has important implications for manner in which entrepreneurial alertness affects equilibria, as we shall observe in the following section.

Here, we offer a partial characterization of the occupational choices of Bulls and bears, in the spirit of Kihlstrom and Laffont. In particular, it is possible to show that in all equilibria of our economy, Bulls have a greater tendency to be firm owners than bears. More precisely:

Theorem 4.1 *(a) If bears prefer to be firm owners rather than workers, so do Bulls. Formally:*

$$\underline{\mathbb{E}}\left[R_i(\theta,\delta) - \min\{w_i, R_i(\theta,\delta)\}\right] \geq \underline{\mathbb{E}}\left[\min\{w_i, R_i(\theta,\delta)\}\right]$$

implies

$$\overline{\mathbb{E}}\left[R_i(\theta,\delta) - \min\{w_i, R_i(\theta,\delta)\}\right] \geq \overline{\mathbb{E}}\left[\min\{w_i, R_i(\theta,\delta)\}\right].$$

(b) If Bulls prefer to be workers rather than firm owners, so do bears. Formally:

$$\overline{\mathbb{E}}\left[\min\{w_i, R_i(\theta,\delta)\}\right] \geq \overline{\mathbb{E}}\left[R_i(\theta,\delta) - \min\{w_i, R_i(\theta,\delta)\}\right]$$

implies

$$\underline{\mathbb{E}}\left[\min\{w_i, R_i(\theta,\delta)\}\right] \geq \underline{\mathbb{E}}\left[R_i(\theta,\delta) - \min\{w_i, R_i(\theta,\delta)\}\right].$$

Neither converse is true in general.

Proof: Observe that $R_i(\theta,\delta) - \min\{w_i, R_i(\theta,\delta)\}$, $\min\{w_i, R_i(\theta,\delta)\}$ and $2\min\{w_i, R_i(\theta,\delta)\}$ are pairwise comonotone. Hence

$$\underline{\mathbb{E}}\left[R_i(\theta,\delta) - \min\{w_i, R_i(\theta,\delta)\}\right] \geq \underline{\mathbb{E}}\left[\min\{w_i, R_i(\theta,\delta)\}\right] \tag{5}$$

if and only if

$$\underline{\mathbb{E}}\left[R_i(\theta,\delta)\right] \geq 2\underline{\mathbb{E}}\left[\min\{w_i, R_i(\theta,\delta)\}\right]$$

$$\Leftrightarrow$$

$$\underline{\mathbb{E}}\left[R_i(\theta,\delta) - 2\min\{w_i, R_i(\theta,\delta)\}\right] \geq 0. \tag{6}$$

Therefore, using (6), it follows that (5) implies

$$\overline{\mathbb{E}}\left[R_i(\theta,\delta) - 2\min\{w_i, R_i(\theta,\delta)\}\right] \geq 0,$$

which is equivalent to

$$\overline{\mathbb{E}}\left[R_i(\theta,\delta) - \min\{w_i, R_i(\theta,\delta)\}\right] \geq \overline{\mathbb{E}}\left[\min\{w_i, R_i(\theta,\delta)\}\right].$$

This proves (a). Case (b) is proved in similar fashion. Counter-examples to the converses are easy to construct. □

More precise characterizations are possible only by making additional assumptions about the economy. The identity of entrepreneurs, and in particular of innovators, will depend in potentially complex ways on general equilibrium pressures in a multi-technology economy. Four different firm structures are possible. Two are vertically segregated: the type of the owner is different from the type of the worker. Two are horizontally segregated: worker and owner have the same type. A simple consequence of Theorem 4.1 is to rule out one of the vertically segregated structures, except in trivial cases in which uncertainty is effectively absent.

Corollary 4.1 *In any innovation-proof equilibrium, if*

$$\overline{\mathbb{E}}\left[R_i(\theta,\delta)-2\min\{w_i,R_i(\theta,\delta)\}\right]\neq\underline{\mathbb{E}}\left[R_i(\theta,\delta)-2\min\{w_i,R_i(\theta,\delta)\}\right]$$

then there exist no firms in industry i in which bears hire Bulls.

Proof: If not, we can swap their roles and Theorem 4.1 ensures they are weakly better off. The condition in the Corollary guarantees that the welfare of at least one must have strictly improved. □

This result suggests the following typology of firms. An *entrepreneurial* firm is one in which the owner and worker are both Bulls. In a *traditional* firm, the owner is a Bull while the worker is a bear. Finally, a firm is *bureaucratic* if the owner and worker are bears. In the next section, we explore how precision of information might influence which firm structures emerge in equilibrium.

Another simple consequence of Theorem 4.1 follows if, as in Kihlstrom and Laffont, we assume that there is only one industry in our economy. Then, provided there is non-trivial imprecision in revenue risk, any equilibrium will have some *traditional* firms.

Corollary 4.2 *Assume $I=1$ and that*

$$\overline{\mathbb{E}}\left[R_i(\theta,\delta)-2\min\{w_i,R_i(\theta,\delta)\}\right]\neq\underline{\mathbb{E}}\left[R_i(\theta,\delta)-2\min\{w_i,R_i(\theta,\delta)\}\right]$$

holds. Then, in any innovation-proof equilibrium, all workers are bears if $\alpha\leq\frac{1}{2}$, and all firm owners are Bulls if $\alpha\geq\frac{1}{2}$.

Clearly, when the population is equally split among Bulls and bears, we have a vertically segregated equilibrium in which all firms are traditional in structure. This is the analogue of Kihlstrom and Laffont's result on entrepreneurial psychology.

5 Comparative statics in precision of information

The general equilibrium model with imprecise risk outlined in the previous sections is suitable for studying innovation and diffusion. In particular, as the graph of Γ gets smaller, information becomes more precise. This characterization allows us to study the effects of precision on innovation-proof equilibria. In this framework Bulls continue to use the upper – and bears the lower – envelope of the shrinking set of probabilities generated by the information correspondences.

We consider an example of a simple two-state, two-technology economy, which allows us to make some predictions about the effect that increasing precision of technology has on the nature of a firm in the industry. In equilibrium, different firm structures emerge in response to the underlying technology. Entrepreneurial firms, where a Bull employs a Bull, emerge when information about technologies is very imprecise. Surprisingly, traditional firms, where a Bull employs a bear, are possible in equilibrium only when the information is precise enough.

5.1 An example

Consider an economy where $I = 2$, and $\Theta = \{1, 2\}$. Revenues are defined as follows

$$R_1(\theta, \delta) = 10 - 2\delta_1$$

and

$$R_2(\theta, \delta) = \begin{cases} 21 - 4\delta_2 & \text{if } \theta = 1 \\ 1 & \text{if } \theta = 2 \end{cases}$$

Technology 1 describes a well-established industry that is risk-free. Technology 2 describes a newly available innovation that fails in state 2 (where it has a scrap value of 1).

Let $\varepsilon \in [0, 1]$ index the level of precision in technology 2 risk information. The associated information correspondence implies that the probability of state 1 lies in the interval

$$\left[\frac{3\varepsilon}{4}, \frac{4-\varepsilon}{4}\right];$$

hence the probability of state 2 lies in the interval

$$\left[\frac{\varepsilon}{4}, \frac{4-3\varepsilon}{4}\right].$$

These intervals shrink as ε rises (the precision of information increases with ε). When $\varepsilon = 1$, risk is precise: states 1 and 2 have probability $\frac{1}{4}$ and $\frac{3}{4}$ respectively. When $\varepsilon = 0$, the probabilities of states 1 and 2 can be anything.

Let $\underline{v}_\varepsilon$ and $\overline{v}_\varepsilon$ be the upper and lower probabilities corresponding to a given ε. Note that technology 2 is successful in the sense that when risk is precise all types prefer technology 2 regardless of the number of firms that employ it. That is,

$$\mathbb{E}_{\underline{v}_{\varepsilon=1}}[R_2] = \mathbb{E}_{\overline{v}_{\varepsilon=1}}[R_2] = 16 - 3\delta_2 > 10 - 2\delta_1,$$

for any δ_1 and δ_2. Finally, we assume Bullish optimism is a comparatively rare trait and therefore $\alpha < \frac{1}{2}$.

5.1.1 The innovation-proof equilibria

The absence of risk in industry 1 implies that any participant in that industry – either firm owner or wage earner – in any equilibrium (innovation-proof or not) obtains

$$\frac{1}{2}R_1(\theta, \delta) = 5 - \delta_1 \tag{7}$$

In industry 2,

$$\begin{aligned} \mathbb{E}_{\overline{v}_\varepsilon}\left[R_2(\theta,\delta)\right] &= 1+\overline{v}_\varepsilon(\{1\})(20-4\delta_2) \\ &= 1+(4-\varepsilon)(5-\delta_2) \end{aligned} \tag{8}$$

and

$$\begin{aligned} \mathbb{E}_{\underline{v}_\varepsilon}\left[R_2(\theta,\,\delta)\right] &= 1+\underline{v}_\varepsilon(\{1\})(20-4\delta_2) \\ &= 1+3\varepsilon(5-\delta_2) \end{aligned} \tag{9}$$

Hence

$$\frac{1}{2}\mathbb{E}_{\overline{v}_\varepsilon}\left[R_2(\theta,\delta)\right] > 5 \geq \frac{1}{2}R_1(\theta,\delta)$$

for all δ and all ε; so there is always an occupation in industry 2 that Bulls strictly prefer to either occupation in industry 1. Hence, in any equilibrium all Bulls must be in industry 2. This implies $\delta_2 \geq \frac{\alpha}{2} > 0$ since $\alpha > 0$ by assumption. Moreover, we may assume $w_1 = 5 - \delta_1$ in any equilibrium.[15] Finally, in this example all equilibria are innovation-proof.[16]

In order to describe the remaining features of equilibria, we use the following functions. For each $(\delta_2, \varepsilon) \in \left[0, \frac{1}{2}\right] \times [0,1]$:

1. $\overline{w}_2(\delta_2,\varepsilon)$ is the level of w_2 at which Bulls are indifferent between firm ownership and wage-laboring in industry 2.

2. $\underline{w}_2(\delta_2,\varepsilon)$ is the level of w_2 at which bears are indifferent between owning a firm and laboring in industry 2.

3. $\hat{w}_2(\delta_2,\varepsilon)$ is the level of w_2 at which bears are indifferent between an industry 1 occupation yielding $5-\delta_1$, and accepting a wage-earning position in industry 2.

Details about these definitions, their properties, and the derivation of the equilibrium are provided in Appendix C. Here, we observe that there is a unique $\varepsilon^0(\delta_2)$ such that $\underline{w}_2\left(\delta_2, \varepsilon^0(\delta_2)\right) = \hat{w}_2\left(\delta_2, \varepsilon^0(\delta_2)\right)$, and a unique $\varepsilon^1(\delta_2) > \varepsilon^0(\delta_2)$ such that $\hat{w}_2\left(\delta_2, \varepsilon^1(\delta_2)\right) = \overline{w}_2\left(\delta_2, \varepsilon^1(\delta_2)\right)$. Using these definitions, Figures 1 and 2 illustrate the equilibrium comparative statics in w_2 and δ_2. Observe that they imply a particular pattern of correlation between wages and firm density in industry 2.

If imprecision is high (i.e. $\varepsilon \leq \varepsilon^0\left(\frac{\alpha}{2}\right)$), bears are unwilling to enter industry 2 and $w_2 = \overline{w}_2(\delta_2,\varepsilon)$, leaving Bulls indifferent between ownership and wage-labor. Since owners and workers in these *entrepreneurial* firms are equally optimistic about the new technology, wages are set to share expected returns equally. As ε rises, Bullish optimism about the returns

[15] If $\delta_1 > 0$, we have already established this fact. If $\delta_1 = 0$, then there may be a range of suitable equilibrium values for w_1, but this range will certainly include $w_1 = 5$.

[16] As argued above, in equilibrium $\delta_2 > 0$. If $\delta_1 = 0$, then whatever the equilibrium value of w_1, at least one occupation in industry 1 yields an expected remuneration (to either type) of no less than 5 (see (7)). Hence, each type receives at least this much in their industry 2 occupations – otherwise we would not observe $\delta_1 = 0$ in equilibrium. It is therefore not possible to find any w_1 at which a coalition of two agents could start a firm in industry 1 and Pareto improve on their equilibrium levels of welfare.

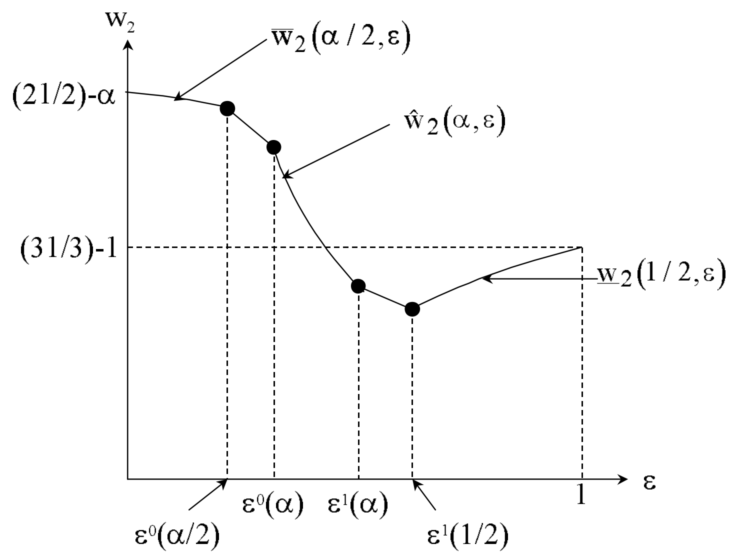

Figure 1: Equilibrium comparative statics for w_2

to technology 2 must be confined within progressively tighter bounds. This means that Bull workers' valuation of their outside option, which is to be an owner in industry 2, is falling. This causes a downward trend in wages.

At somewhat lower levels of imprecision (i.e. for $\varepsilon \in \left(\varepsilon^0\left(\frac{\alpha}{2}\right), \varepsilon^1(\alpha)\right]$), bears become attracted to industry 2 as workers. Bulls are an expensive labor input because of their perceived lucrative outside opportunity as owners of rival firms in industry 2. Since bears are less optimistic about the returns to this occupation, they have a lower reservation wage – one that is tied to the returns in industry 1. Hence wages drop more sharply than before. The downward trend in wages now reflects the falling default risk perceived by bears. After an initial transition phase ($\varepsilon \in \left(\varepsilon^0\left(\frac{\alpha}{2}\right), \varepsilon^0(\alpha)\right)$), all firms in industry 2 have the *traditional* structure: Bulls hire bears.

Finally, when $\varepsilon > \varepsilon^1(\alpha)$, precision has become sufficiently high that bears are happy to own industry 2 firms. This represents the maturity of the new industry, as its comparative advantage over the old industry 1 is revealed. Once $\varepsilon > \varepsilon^1\left(\frac{1}{2}\right)$, all agents are in industry 2. In particular, since $\alpha < \frac{1}{2}$, some bears must own industry 2 firms in equilibrium, employing other bears. Therefore, $w_2 = \underline{w}_2(\delta_2, \varepsilon)$. Since bears perceive the profitability of technology 2 to rise with ε, so must wages. Otherwise bears will prefer ownership to wage-laboring. Meanwhile, the expected surplus to ownership perceived by the Bulls is being eroded; ultimately vanishing when $\varepsilon = 1$.

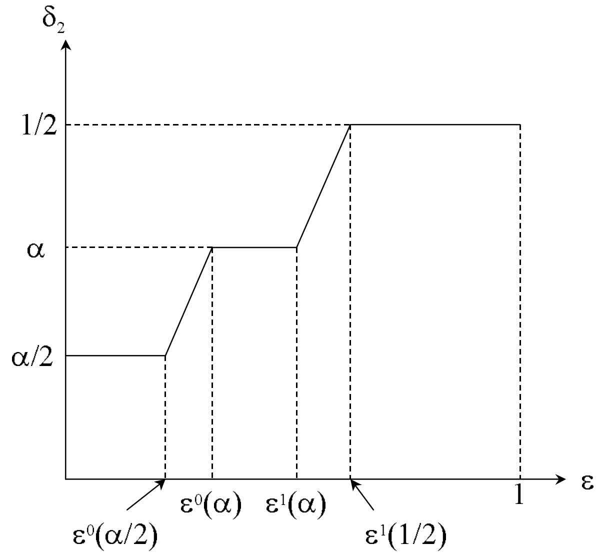

Figure 2: Equilibrium comparative statics in δ_2

5.1.2 Firm formation and imprecision

This simple example provides useful insight into the relationship between the environment in which a firm operates, and its internal structure. Table 1 outlines this relationship, using the taxonomy of Section 4. As we may expect, technology with a high degree of imprecision can only support entrepreneurial firms, since bears are too timid to operate these technologies.

Table 1: Structure of Firms in Equilibrium

Precision of Information	**Firm Structures**
$0 \leq \varepsilon \leq \varepsilon^{0}(\alpha/2)$	Innovative
$\varepsilon^{0}(\alpha/2) < \varepsilon < \varepsilon^{0}(\alpha)$	Innovative and Traditional
$\varepsilon^{0}(\alpha) \leq \varepsilon \leq \varepsilon^{1}(\alpha)$	Traditional
$\varepsilon^{1}(\alpha) < \varepsilon \leq 1$	Traditional and Bureaucratic

Entrepreneurial firms share risk evenly between worker and entrepreneur. Start-ups are often observed to employ workers who share the sentiments of the owners regarding the prospects of the firm. As an early participant in the personal computer industry remarked, "Most of the people in the industry (at that time) were young because the guys who had any real experience were too smart to get involved in all these crazy little machines."[17] Therefore, innovation is constrained not only by the number of adventurous entrepreneurs in the economy, but also the availability of an equally adventurous workforce.

For technologies with intermediate levels of precision, the traditional firm emerges, eventually supplanting the entrepreneurial structure. The advantage of the traditional structure is that it exploits the different attitudes to imprecision of owner and worker. Bullish owners now perceive themselves in receipt of a surplus from "insuring" their workers. Ultimately, the environment becomes sufficiently precise that bureaucratic firms can also be sustained in equilibrium.

Recall that in our model, only the form of the wage contract and the number of technologies are exogenous. In particular, firm formation and occupational choice are determined by the forces of equilibrium. The fact that these forces lead us to a very natural relationship between the precision of the technology and firm culture suggests that the model captures an essential force in the emergence of firms.

5.1.3 Diffusion of innovations and imprecision

Much of the existing literature on innovation is concerned with the process of diffusion of new products and techniques. An aspect of this process which is discussed by a number of authors is the effect of uncertainty reduction on diffusion. As firms innovate, they release information about their novel product or technology. This reduces the uncertainty surrounding the value of the innovation, and hence will tend to promote or discourage further innovators, depending on the nature of the information released. Jensen (1982) and Vettas (1998) are notable examples of theoretical models that accord with the empirical evidence of S-shaped diffusion curves for successful innovations. However, firms are exogenously given in these analyses, with no role for firm formation. The combined role of uncertainty and entrepreneurship has been given scant attention in the formal literature on innovation.

Arguably, Bill Gates, the founder of Microsoft, held optimistic beliefs about the software industry, and built a firm to realise his individual vision. To represent this as "Microsoft", the firm, moving into the software market is misleading. Yet, this is exactly how the formal economic models treat innovation – as a "hard-headed" sideways shift by existing firms into a new and uncertain industry.

Although our model is static, we can use the previous results to study the process of diffusion of successful innovations. Using Figure 2 one may infer an S-shaped diffusion curve. Think of ε as representing not only the precision of information about the new technology 2 but also (and simultaneously) the passage of time. Time $\varepsilon = 0$ denotes the point at which technology 2 becomes available. The temporal interpretation of ε is based on the assumption that utilization of technology 2 gradually releases information to the market, allowing more accurate assessments of its revenue-generating properties.

The analysis suggests both a mechanism of diffusion (uncertainty reduction) and a central agent (the innovative entrepreneur) driving the process. The innovation process cannot be viewed as the diffusion of *firms* from one sector to another. Crucially, innovation involves the

[17] "Triumph of the Nerds: The Rise of Accidental Empires". Transcript available on http://www.pbs.org/nerds/.

break-up of existing firms and the re-constitution of new ones – with structures adapting to the precision of the environment in which the firms operate. Table 1 reveals that significant structural change occurs over the diffusion path. Also, after time $\varepsilon = 0$, all industry 1 firms are bureaucratic, while the firms arriving in industry 2 are entrepreneurial or traditional in structure until time $\varepsilon = \varepsilon^1(\alpha)$. Thus, no firm moves intact from industry 1 to industry 2 over this period.

Diffusion is initially sluggish because of the need to have optimists as both owners and workers. All members of the firm must share an optimistic vision when precision is low, and such optimists are not abundant. Also, as entrepreneurial firms share (upper) expected revenues equally – if wages are too low, Bullish workers will quit to start their own firms – such firms are relatively expensive to run. Diffusion accelerates once precision becomes high enough that bears perceive wages in industry 2 to be sufficiently free of default risk. As the precision of information reaches the critical level $\varepsilon^0(\alpha/2)$, the possibility of operating industry 2 firms with a traditional structure emerges. This gives firm owners in the new sector access to a large, and relatively cheap, workforce. In this phase, Bull owners perceive themselves in receipt of a surplus for "insuring" their bear workers. The final stages of this process may be thought of as Schumpeter's "swarm" of conservative imitators (Schumpeter (1928)). Eventually, the risk of technology 2 becomes so precisely known that some firms become bureaucratic, with bears employing bears.

6 Concluding remarks

The motivation for this paper is the recognition that many important innovations have been initiated by individuals, not firms. We analyze the existence and some properties of equilibrium in a framework that incorporates a notion of imprecise information and entrepreneurial innovation. This framework offers a rich environment in which to understand the relationship between firm structure and the vagueness of the environment in which firms operate.

When imprecision predominates, and there are limitations on the ability of the entrepreneur to shield workers from the consequent uncertainty, firms' objectives ought to be treated as endogenous. The firm captures more than simply a production technology: it is also a mechanism for resolving difference in attitudes to information about risk. When an industry is very new and uncertainty is high, both workers and owners share the same optimistic attitude. As information increases, there are gains from trading across differences in attitude and the nature of the firm is shown to change with the maturation of the industry. It is this change in the nature of the firm in the face of resolving uncertainty that creates the S-shaped diffusion curve in our model.

The PC industry is an excellent example of this process of diffusion and maturation. The early innovators were optimistic entrepreneurs who worked in small partnerships. In the late seventies, the large established IBM eventually recognized the potential in the PC and developed the IBM Acorn – a classic example of the Schumpeterian conservative imitator. This computer was launched in 1981. IBM predicted it would sell half a million by 1984. This prediction proved famously pessimistic: the Acorn sold 2 million units.

Appendix

A Properties of the correspondence ξ

To complete the proof of Theorem 3.1 it is sufficient to show that the correspondence ξ is upper hemi-continuous (u.h.c.), and has non-empty, compact and convex values. It will then follow by Kakutani's fixed point theorem that ξ has a fixed point.

In the course of the arguments, it will be necessary to consider limits of sequences of allocation functions. Although such functions are points in an infinite-dimensional space, the topological arguments are simplified by observing that only finitely many properties of these functions are relevant to the analysis.

Given any allocation function ϕ, define the following $4I$ (Lebesgue) measurable sets:

$$E_i^B(\phi) = \phi^{-1}(i) \cap [0, \alpha) \qquad i = 1, 2, \ldots, 2I$$

$$E_i^b(\phi) = \phi^{-1}(i) \cap [\alpha, 1] \qquad i = 1, 2, \ldots, 2I$$

These sets identify the agents of each type assigned to each of the $2I$ occupations by ϕ. For our purposes, the relevant features of ϕ are simply the Lebesgue measures of these sets. In particular, if (w, δ, ϕ) is an equilibrium, then so is $\left(w, \delta, \hat{\phi}\right)$, where $\hat{\phi}^{-1}(i) \cap [0, \alpha)$ is the interval

$$\left[\sum_{k<i} Leb\left[E_k^B(\phi)\right], \sum_{k<i+1} Leb\left[E_k^B(\phi)\right]\right) \qquad \text{for all } i \in \{1, 2, ..., 2I\},$$

$\hat{\phi}^{-1}(1) \cap [\alpha, 1]$ is the interval

$$\left[\alpha, \alpha + Leb\left[E_1^b(\phi)\right]\right],$$

and $\hat{\phi}^{-1}(i) \cap [\alpha, 1]$ is the interval

$$\left[\alpha + \sum_{k<i} Leb\left[E_k^b(\phi)\right], \alpha + \sum_{k<i+1} Leb\left[E_k^b(\phi)\right]\right) \qquad \text{for all } i \in \{2, 3, ..., 2I\}.$$

We may thus describe $\hat{\phi}$ by specifying the end-points of these intervals.

To do so, let us first define the set

$$\mathcal{T} = \left\{t \in [0,1]^{4I+1} \mid 0 = t_1 \le t_2 \le \cdots \le t_{2I+1} = \alpha \le t_{2I+2} \le \cdots \le t_{4I+1} = 1\right\}$$

For any allocation function ϕ, let $t(\phi)$ be the end-points of the function $\hat{\phi}$ constructed from ϕ as described above. That is: $t(\phi) = \hat{t} \in \mathcal{T}$ if and only if

$$\hat{t}_i = \sum_{k<i} Leb\left[E_k^B(\phi)\right] \qquad \forall i \le 2I + 1$$

and

$$\hat{t}_i = \alpha + \sum_{k<i-2I} Leb\left[E_k^b(\phi)\right] \qquad \forall i \in \{2I+2, 2I+3, \ldots, 4I+1\}$$

Thus, if $t(\phi) = \hat{t} \in \mathcal{T}$, then for each $i \in \mathcal{O}$

$$\begin{aligned} Leb\left[E_i^B(\phi)\right] &= \hat{t}_{i+1} - \hat{t}_i \\ Leb\left[E_i^b(\phi)\right] &= \hat{t}_{2I+i+1} - \hat{t}_{2I+i} \end{aligned}$$

Conversely, given any $t \in \mathcal{T}$, we may construct an allocation function $\phi_{[t]}$ as follows:

$$\phi_{[t]}(j) = i \Longleftrightarrow \begin{cases} j \in [t_i,\, t_{i+1}) & \text{and } i < i^* - 2I;\ \text{or} \\ j \in [t_{i^*},\, t_{i^*+1}] & \text{and } i = i^* - 2I;\ \text{or} \\ j \in (t_k,\, t_{k+1}] & \text{and } i = k - 2I > i^* - 2I \end{cases}$$

where $i^* = \min\{i \mid t_{i+1} > \alpha\}$. It should be clear that, for every $i \in \mathcal{O}$

$$Leb\left[E_i^B(\phi)\right] = Leb\left[E_i^B\left(\phi_{[t(\phi)]}\right)\right]$$

and

$$Leb\left[E_i^b(\phi)\right] = Leb\left[E_i^b\left(\phi_{[t(\phi)]}\right)\right]$$

Let us now return to our consideration of the properties of ξ. Firstly, it is obvious that:

Lemma A.1 *$\xi(w, \delta, x) \neq \emptyset$ for every $(w, \delta, x) \in D$.*

We next show:

Lemma A.2 *$\xi(w, \delta, x)$ is compact for every $(w, \delta, x) \in D$.*

Proof. Boundedness is immediate, so it suffices to prove closedness. The sets

$$\left\{ \arg \max_{\tilde{w} \in [\mathbf{0}, \overline{W}]^I} \ \tilde{w} \cdot proj_I x \right\}$$

and $\{proj_{-I}\, x\}$ are clearly closed; so we need only consider $X(w, \delta)$.

Let $\{x^n\}_{n=1}^{\infty}$ be a sequence in $X(w, \delta)$ with limit $\overline{x}$. Then $\{z^n \equiv C^{-1}x^n\}_{n=1}^{\infty}$ is a sequence in $Z(w, \delta)$ with limit $\overline{z} = C^{-1}\overline{x}$. Associated with each z^n will be an allocation function $\phi_n \in \Phi(w, \delta)$ such that

$$z^n = \left(Leb\left[\phi_n^{-1}(i)\right]\right)_{i=1}^{2I}$$

Consider the sequence $\{t(\phi_n)\}_{n=1}^{\infty}$ in $\mathcal{T}$. Since $\mathcal{T}$ is clearly compact, this has a convergent subsequence with limit $\overline{t} \in \mathcal{T}$. If we can show that

$$\phi_{[\overline{t}]} \in \Phi(w, \delta) \tag{10}$$

and

$$\overline{z} = \left(Leb\left[\phi_{[\overline{t}]}^{-1}(i)\right]\right)_{i=1}^{2I} \tag{11}$$

then the lemma is proved.

Suppose that equation (10) does *not* hold. Then, there must exist some $k \in \{B, b\}$ and some $i \in \mathcal{O}$ such that $i \notin BR^{\mathcal{E},k}(w, \delta)$ and

$$Leb\left[E_i^k\left(\phi_{[\bar{t}]}\right)\right] > 0$$

Consider the case $k = B$ (the argument is similar if $k = b$). Then

$$\begin{aligned} & Leb\left[E_i^B\left(\phi_{[\bar{t}]}\right)\right] > 0 \\ \Rightarrow\quad & \bar{t}_{i+1} - \bar{t}_i > 0 \\ \Rightarrow\quad & t_{i+1}^n - t_i^n > 0 \end{aligned}$$

for sufficiently large n along the subsequence defining $\bar{t}$. But this contradicts the fact that $\phi_n \in \Phi(w, \delta)$ for all n. Hence, (10) must hold.

To see equation (11), suppose that m indexes the subsequence defining $\bar{t}$. Then

$$\begin{aligned} z^m &= \left(\sum_{k\in\{B,b\}} Leb\left[E_i^k\left(\phi_{[t^m]}\right)\right]\right)_{i=1}^{2I} \\ &= \left(\left(t_{i+1}^m - t_i^m\right) + \left(t_{2I+i+1}^m - t_{2I+i}^m\right)\right)_{i=1}^{2I} \\ &\to \left(\left(\bar{t}_{i+1} - \bar{t}_i\right) + \left(\bar{t}_{2I+i+1} - \bar{t}_{2I+i}\right)\right)_{i=1}^{2I} \quad \text{as } m \to \infty \\ &= \left(\sum_{k\in\{B,b\}} Leb\left[E_i^k\left(\phi_{[\bar{t}]}\right)\right]\right)_{i=1}^{2I} \end{aligned}$$

and (11) follows. □

Lemma A.3 *$\xi(w, \delta, x)$ is convex for every $(w, \delta, x) \in D$.*

Proof. Again, the sets

$$\left\{\arg\max_{\tilde{w}\in[0,\overline{W}]^I} \tilde{w}\cdot proj_I x\right\}$$

and $\{proj_{-I}\, x\}$ are obviously convex. Let $\hat{x}, \tilde{x} \in X(w, \delta)$ and consider

$$x = \lambda\hat{x} + (1-\lambda)\,\tilde{x}$$

for some $\lambda \in (0, 1)$.

Define $\hat{z} = C^{-1}\hat{x}$, $\tilde{z} = C^{-1}\tilde{x}$, and

$$z = C^{-1}x = \lambda\hat{z} + (1-\lambda)\,\tilde{z}$$

Let $\hat{\phi}$ and $\tilde{\phi}$ be allocation functions in $\Phi(w, \delta)$ satisfying

$$\hat{z} = \left(Leb\left[\hat{\phi}^{-1}(i)\right]\right)_{i=1}^{2I}$$

$$\tilde{z} = \left(Leb \left[\tilde{\phi}^{-1}(i) \right] \right)_{i=1}^{2I}$$

Now observe that $\mathcal{T}$ is a convex set, and consider the $t \in \mathcal{T}$ defined as

$$t = \lambda \hat{t} + (1 - \lambda) \tilde{t}$$

where $\hat{t} = t(\hat{\phi})$ and $\tilde{t} = t(\tilde{\phi})$. Since

$$\begin{aligned} Leb \left[E_i^B(\phi_{[t]}) \right] &= t_{i+1} - t_i \\ &= \lambda \left(\hat{t}_{i+1} - \hat{t}_i \right) + (1 - \lambda) \left(\tilde{t}_{i+1} - \tilde{t}_i \right) \end{aligned}$$

and

$$\begin{aligned} Leb \left[E_i^b(\phi_{[t]}) \right] &= t_{2I+i+1} - t_{2I+i} \\ &= \lambda \left(\hat{t}_{2I+i+1} - \hat{t}_{2I+i} \right) + (1 - \lambda) \left(\tilde{t}_{2I+i+1} - \tilde{t}_{2I+i} \right) \end{aligned}$$

it is obvious that $\phi_{[t]} \in \Phi(w, \delta)$. We now deduce that for every $i \in \mathcal{O}$

$$\begin{aligned} Leb \left[\phi_{[t]}^{-1}(i) \right] &= (t_{i+1} - t_i) + (t_{2I+i+1} - t_{2I+i}) \\ &= \lambda \left[\left(\hat{t}_{i+1} - \hat{t}_i \right) + \left(\hat{t}_{2I+i+1} - \hat{t}_{2I+i} \right) \right] + (1 - \lambda) \left[\left(\tilde{t}_{i+1} - \tilde{t}_i \right) + \left(\tilde{t}_{2I+i+1} - \tilde{t}_{2I+i} \right) \right] \\ &= \lambda Leb \left[E_i^B(\phi_{[\hat{t}]}) \cup E_i^b(\phi_{[\hat{t}]}) \right] + (1 - \lambda) \, Leb \left[E_i^B(\phi_{[\tilde{t}]}) \cup E_i^b(\phi_{[\tilde{t}]}) \right] \\ &= \lambda \hat{z}_i + (1 - \lambda) \tilde{z}_i \end{aligned}$$

Hence $z \in Z(w, \delta)$ and the proof is complete. □

Lemma A.4 *ξ is upper hemi-continuous.*

Proof. Since ξ is the Cartesian product of three compact-valued correspondences (Lemma A.2), it suffices by Proposition 11.25 of Border (1985) to prove upper hemi-continuity for each of the three component correspondences. The second component correspondence is trivially u.h.c., and the first is u.h.c. by standard arguments, so let us consider the third component of ξ.

Correspondence $X(w, \delta)$ will be u.h.c. if and only if (Border (1985, Proposition 11.11)), for every sequence $\{(w, \delta)^n\}_{n=1}^{\infty}$ in $[0, \overline{W}]^I \times [0, 1]^I$ converging to some limit $(\overline{w}, \overline{\delta})$, and every sequence $\{x^n\}_{n=1}^{\infty}$ satisfying

$$x^n \in X((w, \delta)^n) \qquad \forall n$$

there is a convergent subsequence of $\{x^n\}_{n=1}^{\infty}$ with limit in $X(\overline{w}, \overline{\delta})$.

Let $\{(w, \delta)^n\}_{n=1}^{\infty}$, $(\overline{w}, \overline{\delta})$, and $\{x^n\}_{n=1}^{\infty}$ satisfy the conditions of the preceding paragraph. Define the sequence

$$\left\{ z^n = C^{-1} x^n \right\}_{n=1}^{\infty}$$

and let $\{\phi^n\}_{n=1}^{\infty}$ be an associated sequence of allocation functions. Then the sequence

$$\{t^n = t(\phi^n)\}_{n=1}^{\infty}$$

has a convergent subsequence with limit $\bar{t} \in \mathcal{T}$.

We claim that $\phi_{[\bar{t}]} \in \Phi\left(\overline{w}, \overline{\delta}\right)$. To see this, observe that the correspondences $BR^{\mathcal{E},B}$ and $BR^{\mathcal{E},b}$ are clearly upper hemi-continuous by the continuity of each type's objective function in (w, δ). In particular, upper and lower expectations are continuous. Therefore, if $1 \leq i \leq 2I$ and $i \notin BR^{\mathcal{E},B}\left(\overline{w}, \overline{\delta}\right)$, then it must also be the case that $i \notin BR^{\mathcal{E},B}\left(w^n, \delta^n\right)$ for all sufficiently large n. Hence, we cannot have $\bar{t}_{i+1} > \bar{t}_i$. Analogous conclusions hold for $2I + 1 \leq i \leq 4I$ and $(i - 2I) \notin BR^{\mathcal{E},b}\left(\overline{w}, \overline{\delta}\right)$. So

$$\phi_{[\bar{t}]} \in \Phi\left(\overline{w}, \overline{\delta}\right)$$

as claimed.

Finally, since

$$Leb\left[\phi_{[\bar{t}]}^{-1}(i)\right] = \bar{z}_i \qquad \forall i,$$

it follows that $\{x^n\}_{n=1}^{\infty}$ converges to $\overline{x} = C\overline{z} \in X\left(\overline{w}, \overline{\delta}\right)$ along the subsequence. □

B Proof of Theorem 3.2

First, we introduce a definition that will be used in the proofs. Belief functions may be described by their *Möbius inverse* (see Shafer (1976) and Chateauneuf and Jaffray (1989)). This inverse is the mapping $m : 2^{\Theta} \to [0, 1]$ defined from Γ as follows:

$$m(E) = p\left(\{s \in S \mid \Gamma(s) = E\}\right) \qquad \forall E \subseteq \Theta.$$

Thus, m is the probability induced by p and Γ on the power set 2^{Θ}. Furthermore:

$$\underline{v}(E) = \sum_{A \subseteq E} m(A)$$

One may think of $m(E)$ as the quantity of probability necessarily attached to event E that is not necessarily attached to any of its subsets.

The following is useful.

Lemma B.1 *Suppose economy $\mathcal{E}$ is such that the information correspondence Γ induces upper and lower probabilities $\overline{v}$ and $\underline{v}$ having the following property:*

$$\forall E \in \mathcal{P}^*(\Theta) := 2^{\Theta} \setminus \{\emptyset\} \qquad \overline{v}(E) > 0 \;\Rightarrow\; \underline{v}(E) > 0$$

Then, any equilibrium of $\mathcal{E}$ is innovation-proof.

Proof: Suppose that (w, δ, ϕ) is an equilibrium of $\mathcal{E}$. Suppose further that there exists an industry i with $\delta_i = 0$, such that a wage $\hat{w}_i \geq 0$ and agents $k, k' \in [0, 1]$, $k \neq k'$, may be found satisfying

$$\mathbb{E}_{v_k}\left[R_i\left(\theta, \delta\right) - \min\{\hat{w}_i, R_i\left(\theta, \delta\right)\}\right] > y^{\mathcal{E},\beta_k}\left(w, \delta\right) \tag{12}$$

and

$$\mathbb{E}_{v_{k'}}\left[\min\{\hat{w}_i, R_i\left(\theta, \delta\right)\}\right] \geq y^{\mathcal{E},\beta_{k'}}\left(w, \delta\right). \tag{13}$$

Suppose that $\hat{w}_i \geq w_i$. In this case, we claim that agent k cannot be strictly better off after the re-allocation. This is because occupation $2i-1$ was weakly less desirable to agent k than k's occupation under ϕ when firm owners in industry i had to pay wage w_i. Therefore, owning a firm in industry i and paying the higher wage $\hat{w}_i$ cannot make k strictly better off than in the original equilibrium.

It follows that $\hat{w}_i < w_i$. Thus:

$$\mathbb{E}_{v'_k}\left[\min\{\hat{w}_i, R_i(\theta,\delta)\}\right] \leq \mathbb{E}_{v'_k}\left[\min\{w_i, R_i(\theta,\delta)\}\right] \leq y^{\mathcal{E},\beta_{k'}}(w,\delta).$$

So, equation (13) implies

$$\mathbb{E}_{v'_k}\left[\min\{\hat{w}_i, R_i(\theta,\delta)\}\right] = \mathbb{E}_{v'_k}\left[\min\{w_i, R_i(\theta,\delta)\}\right].$$

Hence, we conclude

$$v_{k'}\left(\{\theta \in \Theta \mid R_i(\theta,\delta) > \hat{w}_i\}\right) = 0 \tag{14}$$

But, equation (12) and

$$y^{\mathcal{E},\beta_k}(w,\delta) \geq \mathbb{E}_{v_k}\left[R_i(\theta,\delta) - \min\{w_i, R_i(\theta,\delta)\}\right]$$

imply

$$v_k\left(\{\theta \in \Theta \mid R_i(\theta,\delta) > \hat{w}_i\}\right) > 0 \tag{15}$$

Observe that equations (14) and (15) imply $v_k = \overline{v}$ and $v_{k'} = \underline{v}$. This contradicts the assumption of the Lemma. □

Corollary B.1 *Suppose economy $\mathcal{E}$ is such that the information correspondence Γ induces the lower probability $\underline{v}$ whose Möbius inverse m satisfies*

$$m(\{\theta\}) > 0 \qquad \forall \theta \in \Theta$$

Then, any equilibrium of $\mathcal{E}$ is innovation-proof.

The assumption on m implies $\underline{v}(E) > 0$ for all $E \in \mathcal{P}^*(\Theta)$ and the result follows from Lemma B.1. □

We are now ready to prove that under Assumptions 1 and 2, every economy $\mathcal{E}$ possesses an innovation-proof equilibrium.

Let $\overline{v}$ and $\underline{v}$ denote the upper and lower probabilities induced by Γ. For the purposes of the present argument it is convenient to make the assumption that all "redundant" elements have been removed from Θ; that is, all θ such that

$$\underline{v}(\{\theta\}) = \overline{v}(\{\theta\}) = 0$$

If m, the Möbius inverse of $\underline{v}$, satisfies the condition in Corollary B.1 we are done. Suppose not; that is, $m(\{\theta\}) = 0$ for some $\theta \in \Theta$.

Consider the space of functions

$$\mathcal{M} = \left\{ f : 2^{\Theta} \to [0,1] \;\middle|\; f(\emptyset) = 0, \quad \sum_{E \in 2^{\Theta}} f(E) \, = \, 1 \right\}$$

Each $f \in \mathcal{M}$ is the Möbius inverse of some lower probability on Θ; conversely, each lower probability on Θ has a Möbius inverse in $\mathcal{M}$ (see Shafer (1976)). We may identify $\mathcal{M}$ with the unit simplex $\Delta^{|\mathcal{P}^*(\Theta)|-1}$. In particular, $\mathcal{M}$ is compact and convex.

Choose some $\tilde{\pi} \in ri[core(\underline{v})]$. Then, $\tilde{\pi}(E) = \underline{v}(E)$ if and only if $\overline{v}(E) = \underline{v}(E)$, and $\tilde{\pi}(E) > \underline{v}(E)$ otherwise. In particular, letting $\tilde{m} \in \mathcal{M}$ denote the Möbius inverse of $\tilde{\pi}$, we must have $\tilde{m}(\{\theta\}) > 0$ for every $\theta \in \Theta$ by our non-redundancy assumption.

Now let us define

$$m^n = \frac{1}{n}\tilde{m} + \left(1 - \frac{1}{n}\right) m$$

The sequence $\{m^n\}_{n=1}^{\infty}$ in $\mathcal{M}$ clearly converges to m and satisfies $m^n(\{\theta\}) > 0$ for every $\theta \in \Theta$. If $\{\underline{v}^n\}_{n=1}^{\infty}$ is the associated sequence of lower probabilities, then $\underline{v}^n \to \underline{v}$ as $n \to \infty$, and for each n and each $E \subseteq \Theta$:

$$\underline{v}^n(E) = \underline{v}\,(E) + \frac{1}{n}\left[\tilde{\pi}\,(E) - \underline{v}\,(E)\right] \geq \underline{v}\,(E)\,.$$

From $\underline{v}^n$ we may construct the associated upper probability $\overline{v}^n$ as follows:

$$\overline{v}^n(E) = 1 - \underline{v}^n(E^c) \qquad \forall E \subseteq \Theta.$$

For each n, define $\mathcal{E}^n$ as the economy with identical α and revenue functions to $\mathcal{E}$, but in which Bulls evaluate their employment options using $\overline{v}^n$, and bears evaluate their options using $\underline{v}^n$. For the purposes of the following argument it is unnecessary to define a sequence of information correspondences generating these beliefs. By Corollary B.1, each $\mathcal{E}^n$ has an innovation-proof equilibrium (w^n, δ^n, ϕ_n). Let (w, δ, t) denote the limit of a convergent subsequence of $\{(w^n, \delta^n, t\,(\phi_n))\}_{n=1}^{\infty}$ (retaining n as index of the convergent subsequence for notational convenience). We claim that $\left(w, \delta, \phi_{[t]}\right)$ is an innovation-proof equilibrium of $\mathcal{E}$.

Since it is obvious that

$$Leb\left[\phi_{[t]}^{-1}(2i-1)\right] = Leb\left[\phi_{[t]}^{-1}(2i)\right] = \delta_i \qquad \forall i \in \{1, 2, ..., I\}\,,$$

it suffices to show that $\phi_{[t]} \in \Phi\,(w, \delta)$, and that $\left(w, \delta, \phi_{[t]}\right)$ is innovation-proof.

Agents of type $\beta \in \{B, b\}$ have objective functions that are continuous in $\left(w^n, \delta^n, v_\beta^n\right)$. Hence

$$\forall \beta \in \{B,\, b\} \qquad \limsup_{n \to \infty}\; BR^{\mathcal{E}^n, \beta}\,(w^n, \delta^n) \subseteq BR^{\mathcal{E}, \beta}\,(w, \delta)$$

If $t_{i+1} - t_i > 0$ for some $i \notin BR^{\mathcal{E},B}\,(w, \delta)$, then $t_{i+1}^n - t_i^n > 0$ must also hold for sufficiently large n. Furthermore, if $t_{2I+i+1} - t_{2I+i} > 0$ for some $i \notin BR^{\mathcal{E},b}\,(w, \delta)$, then again $t_{2I+i+1}^n - t_{2I+i}^n > 0$ must also hold for sufficiently large n. In each case we have a contradiction to $\phi_n \in \Phi(w^n,\, \delta^n)$. Hence, $\phi_{[t]} \in \Phi\,(w, \delta)$.

Suppose that $\left(w,\, \delta,\, \phi_{[t]}\right)$ is *not* innovation-proof. That is, there exist agents k and k', an industry i with $\delta_i = 0$, and a wage $\hat{w}_i$ such that if k owns a firm in industry i and employs k' as a worker at $\hat{w}_i$, both do as well as under the candidate equilibrium, and

$$\mathbb{E}_{v_k}\left[R_i(\theta,\delta) - \min\left\{\hat{w}_i, R_i(\theta,\delta)\right\}\right] > y^{\mathcal{E},\beta_k}(w,\delta) \tag{16}$$

Clearly, it must be the case that $w_i > \hat{w}_i$. Furthermore, equation (16) and the fact that

$$v_{k'}^n(E) > 0 \qquad \forall E \in \mathcal{P}^*(\Theta)$$

imply the existence of some $\eta > 0$ such that, for every n,

$$\mathbb{E}_{v_{k'}^n}\left[\min\left\{c,\; R_i(\theta,\delta)\right\}\right]$$

is *strictly* increasing in c when $c \in [\hat{w}_i, \hat{w}_i + \eta]$. On the other hand, since

$$\mathbb{E}_{v_{k'}}\left[\min\left\{\hat{w}_i,\; R_i(\theta,\delta)\right\}\right] \geq y^{\mathcal{E},\beta_{k'}}(w,\delta) \geq \mathbb{E}_{v_{k'}}\left[\min\left\{w_i, R_i(\theta,\delta)\right\}\right]$$

we see that

$$\mathbb{E}_{v_{k'}}\left[\min\left\{c, R_i(\theta,\delta)\right\}\right] = y^{\mathcal{E},\beta_{k'}}(w,\delta) \qquad \forall c \in [\hat{w}_i, w_i]$$

In particular, we must have $v_{k'} \equiv \underline{v}$ and

$$m(B) = 0 \qquad \forall B \subseteq \left\{\theta \in \Theta \mid R_i(\theta,\delta) > \hat{w}_i\right\} \neq \emptyset$$

Recalling that $\underline{v}^n \geq \underline{v}$, we deduce

$$\mathbb{E}_{v_{k'}^n}\left[\min\left\{\hat{w}_i, R_i(\theta,\delta)\right\}\right] \geq \mathbb{E}_{v_{k'}}\left[\min\left\{\hat{w}_i, R_i(\theta,\delta)\right\}\right]$$

for each n. Letting $\tilde{w}_i \in (\hat{w}_i, \hat{w}_i + \eta)$, we therefore have

$$\mathbb{E}_{v_{k'}^n}\left[\min\left\{\tilde{w}_i, R_i(\theta,\delta)\right\}\right] - y^{\mathcal{E},\beta_{k'}}(w,\delta) > \mathbb{E}_{v_{k'}}\left[\min\left\{\tilde{w}_i, R_i(\theta,\delta)\right\}\right] - y^{\mathcal{E},\beta_{k'}}(w,\delta)$$

for all n. Hence

$$\mathbb{E}_{v_{k'}^n}\left[\min\left\{\tilde{w}_i, R_i(\theta,\delta^n)\right\}\right] - y^{\mathcal{E}^n,\beta_{k'}}(w^n,\delta^n) \geq \mathbb{E}_{v_{k'}}\left[\min\left\{\tilde{w}_i, R_i(\theta,\delta)\right\}\right] - y^{\mathcal{E},\beta_{k'}}(w,\delta)$$

for all sufficiently large n, using the continuity of the functions on the left-hand side of the inequality. Therefore, for all large n,

$$\mathbb{E}_{v_{k'}^n}\left[\min\left\{w_i^n, R_i(\theta,\delta^n)\right\}\right] - y^{\mathcal{E}^n,\beta_{k'}}(w^n,\delta^n) > \mathbb{E}_{v_{k'}}\left[\min\left\{\tilde{w}_i, R_i(\theta,\delta)\right\}\right] - y^{\mathcal{E},\beta_{k'}}(w,\delta)$$

since $\mathbb{E}_{v_{k'}^n}\left[\min\left\{c, R_i(\theta,\delta^n)\right\}\right]$ is strictly increasing in c at $c = \tilde{w}_i$ when n is sufficiently large. But observe that the right-hand side of this inequality is equal to zero, so

$$\mathbb{E}_{v_{k'}^n}\left[\min\left\{w_i^n, R_i(\theta,\delta^n)\right\}\right] > y^{\mathcal{E}^n,\beta_{k'}}(w^n,\delta^n)$$

which is a contradiction. This completes the proof. $\square$

C Derivation of the equilibria in section 5

First, we derive the properties of the wage functions presented in Section 5.

Recall that $\overline{w}_2(\delta_2, \varepsilon)$ is the level of w_2 at which Bulls are indifferent between firm ownership and wage-laboring in industry 2. Therefore, $\overline{w}_2(\delta_2, \varepsilon)$ solves

$$\frac{1}{2}\left[1 + (4 - \varepsilon)(5 - \delta_2)\right] = \mathbb{E}_{\overline{v}_\varepsilon}\left[\min\left\{w_2,\ R_2(\theta, \delta)\right\}\right] \tag{17}$$

Since the left-hand side exceeds 1, we must have $w_2 \geq 1$. Furthermore, we must also have $w_2 \leq 21 - 4\delta_2$. Therefore, if equation (17) holds, w_2 is fully paid in state 1, and workers receive \$1 in state 2. Hence:

$$\begin{aligned}\mathbb{E}_{\overline{v}_\varepsilon}\left[\min\left\{w_2, R_2(\theta, \delta)\right\}\right] &= 1 + \overline{v}_\varepsilon(\{1\})(w_2 - 1) \\ &= 1 + \frac{(4 - \varepsilon)(w_2 - 1)}{4}\end{aligned}$$

and from this we obtain:

$$\overline{w}_2(\delta_2, \varepsilon) = 11 - 2\delta_2 - \frac{2}{(4 - \varepsilon)} \tag{18}$$

If $w_2 > \overline{w}_2(\delta_2, \varepsilon)$, neither type wishes to own a firm in industry 2 (see Theorem 4.1). This is incompatible with equilibrium, since we already know that $\delta_2 \geq \frac{\alpha}{2} > 0$. Hence $w_2 \leq \overline{w}_2(\delta_2, \varepsilon)$ in equilibrium.

In Section 5, we defined $\underline{w}_2(\delta_2, \varepsilon)$ as the level of w_2 at which bears are indifferent between owning a firm and laboring in industry 2. Hence, $\underline{w}_2(\delta_2, \varepsilon)$ solves

$$\frac{1}{2}\left[1 + 3\varepsilon(5 - \delta_2)\right] = \mathbb{E}_{\underline{v}_\varepsilon}\left[\min\left\{w_2, R_2(\theta, \delta)\right\}\right] \tag{19}$$

If $w_2 < 1$, bears strictly prefer any occupation in industry 1 to wage-laboring in industry 2, so we shall assume for the purposes of solving (19) that $w_2 \geq 1$. In this case,

$$\begin{aligned}\mathbb{E}_{\underline{v}_\varepsilon}\left[\min\left\{w_2, R_2(\theta, \delta)\right\}\right] &= 1 + \underline{v}_\varepsilon(\{1\})(w_2 - 1) \\ &= 1 + \frac{3\varepsilon(w_2 - 1)}{4}\end{aligned}$$

so (19) implies

$$\underline{w}_2(\delta_2, \varepsilon) = 11 - 2\delta_2 - \frac{2}{3\varepsilon} \tag{20}$$

If $1 \leq w_2 < \underline{w}_2(\delta_2, \varepsilon)$, neither type will accept a wage earning position in a firm in industry 2 (Theorem 4.1 again). The same is true if $w_2 < 1$. Therefore, $w_2 \geq \max\{1, \underline{w}_2(\delta_2, \varepsilon)\}$ in equilibrium.

Since $\hat{w}_2(\delta_2, \varepsilon)$ is the level of w_2 at which bears are indifferent between an industry 1 occupation yielding $5 - \delta_1$, and accepting a wage-earning position in industry 2, it is clear that $\hat{w}_2(\delta_2, \varepsilon) > 1$. Therefore, $\hat{w}_2(\delta_2, \varepsilon)$ satisfies

$$\mathbb{E}_{\underline{v}_\varepsilon}\left[\min\left\{w_2, R_2(\theta, \delta)\right\}\right] = 5 - \delta_1$$

$$\Leftrightarrow$$

$$1+\frac{3\varepsilon(w_2-1)}{4}=5-\left(\frac{1}{2}-\delta_2\right) \tag{21}$$

Solving (21) yields

$$\hat{w}_2\left(\delta_2,\varepsilon\right)=1+\frac{14+4\delta_2}{3\varepsilon} \tag{22}$$

Using the results above, we have established the following three properties of entrepreneurial equilibria:

$$\delta_2 \geq \frac{\alpha}{2}>0 \tag{23}$$

$$w_1=5-\delta_1 \tag{24}$$

$$\max\{1,\ \underline{w}_2\left(\delta_2,\varepsilon\right)\} \leq w_2 \leq \overline{w}_2\left(\delta_2,\varepsilon\right) \tag{25}$$

In fact, using equations (23), (24), and (25), we may pin down w_2 uniquely (given δ_2). Recalling the definitions of ε^0 and ε^1 from section 5, consider the following three exhaustive cases: (i) $\varepsilon \leq \varepsilon^0(\delta_2)$, (ii) $\varepsilon^0(\delta_2)<\varepsilon \leq \varepsilon^1(\delta_2)$, and (iii) $\varepsilon>\varepsilon^1(\delta_2)$.

In case (i) we must have $w_2=\overline{w}_2\left(\delta_2,\varepsilon\right)$. If $w_2<\overline{w}_2\left(\delta_2,\varepsilon\right)$ then all bears are in industry 1, and Bulls strictly prefer firm ownership to wage-laboring in industry 2. This is incompatible with labor market equilibrium in industry 2 given (23).

The same reasoning implies that $w_2 \geq \hat{w}_2\left(\delta_2,\varepsilon\right)$ in case (ii). Furthermore, since $\alpha<\frac{1}{2}$, we cannot have $w_2>\hat{w}_2\left(\delta_2,\varepsilon\right)$, because all agents would then be in industry 2 occupations causing an excess supply of labor. Hence, $w_2=\hat{w}_2\left(\delta_2,\varepsilon\right)$ in case (ii).

Finally, in case (iii) all agents are in industry 2, so $\alpha<\frac{1}{2}$ implies $w_2=\underline{w}_2\left(\delta_2,\varepsilon\right)$.

These results are summarized in Figure 3; it illustrates the level of w_2 corresponding to the equilibrium value of δ_2. However, Figure 3 is drawn for an *arbitrarily given* δ_2, while δ_2 is yet to be determined, and is clearly not constant in ε.

In fact, the transition points $\varepsilon^0\left(\delta_2\right)$ and $\varepsilon^1\left(\delta_2\right)$ are intervals, as the equilibrium "sticks" at these intersections over a range of δ_2 values. Excluding these transition points, however, one can show that δ_2 is constant within each of the three segments in Figure 3.

To see why, first consider the segment in which

$$w_2=\overline{w}_2\left(\delta_2,\varepsilon\right)<\hat{w}_2\left(\delta_2,\varepsilon\right).$$

Clearly, $\delta_2=\frac{\alpha}{2}$ under these conditions, since all and only Bulls are in industry 2 at this wage. Hence, for $\varepsilon<\varepsilon^0\left(\frac{\alpha}{2}\right)$, industry 2 has density $\delta_2=\frac{\alpha}{2}$, and each firm offers wage $w_2=\overline{w}_2\left(\frac{\alpha}{2},\varepsilon\right)$.

Second, if

$$w_2=\hat{w}_2\left(\delta_2,\varepsilon\right)\in\left(\underline{w}_2\left(\delta_2,\varepsilon\right),\overline{w}_2\left(\delta_2,\varepsilon\right)\right)$$

then $\delta_2=\alpha$. Thus, if $\varepsilon^0(\alpha)<\varepsilon<\varepsilon^1(\alpha)$, industry 2 is characterized by density $\delta_2=\alpha$ and wage $w_2=\hat{w}_2\left(\alpha,\varepsilon\right)$.

Finally, when

$$w_2=\underline{w}_2\left(\delta_2,\varepsilon\right)>\hat{w}_2\left(\delta_2,\varepsilon\right)$$

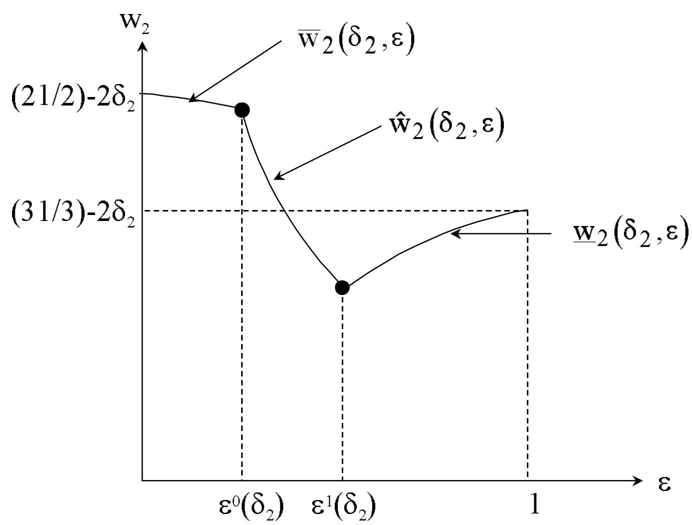

Figure 3: Candidate equilibrium w_2 values

we must have $\delta_2 = \frac{1}{2}$. Therefore, when $\varepsilon > \varepsilon^1\left(\frac{1}{2}\right)$, we have $\delta_2 = \frac{1}{2}$ and wage $w_2 = \underline{w}_2\left(\frac{1}{2}, \varepsilon\right)$.

It is straightforward to verify that $\varepsilon^{\mathbf{0}}\left(\frac{\alpha}{2}\right) < \varepsilon^{\mathbf{0}}(\alpha)$ and $\varepsilon^1(\alpha) < \varepsilon^1\left(\frac{1}{2}\right)$, recalling that $\alpha \in \left(0, \frac{1}{2}\right)$. Suppose, then, that $\varepsilon \in \left[\varepsilon^{\mathbf{0}}\left(\frac{\alpha}{2}\right), \varepsilon^{\mathbf{0}}(\alpha)\right]$. Let $\hat{\delta}_2$ be the corresponding equilibrium density of firms in industry 2. Then it must be the case that $\varepsilon = \varepsilon^{\mathbf{0}}(\hat{\delta}_2)$. If $\varepsilon < \varepsilon^{\mathbf{0}}(\hat{\delta}_2)$, then $\hat{\delta}_2 = \frac{\alpha}{2}$, which yields a contradiction, as $\varepsilon \geq \varepsilon^{\mathbf{0}}\left(\frac{\alpha}{2}\right)$ by assumption. Similarly, if $\varepsilon > \varepsilon^{\mathbf{0}}(\hat{\delta}_2)$, then $\hat{\delta}_2 \geq \alpha$, which again yields a contradiction, since $\varepsilon^{\mathbf{0}}$ is increasing in δ_2.

So $\varepsilon \in \left[\varepsilon^{\mathbf{0}}\left(\frac{\alpha}{2}\right), \varepsilon^{\mathbf{0}}(\alpha)\right]$ implies $\varepsilon = \varepsilon^{\mathbf{0}}(\hat{\delta}_2)$, which further entails that $\hat{\delta}_2 \in \left[\frac{\alpha}{2}, \alpha\right]$. In fact, the equation $\varepsilon = \varepsilon^{\mathbf{0}}(\hat{\delta}_2)$ will have a unique solution for $\hat{\delta}_2$, given the monotonicity of $\varepsilon^{\mathbf{0}}$. This solution will be increasing in ε. As ε is increased over the range $\left[\varepsilon^{\mathbf{0}}\left(\frac{\alpha}{2}\right), \varepsilon^{\mathbf{0}}(\alpha)\right]$, bearish workers from industry 1 gradually replace Bull workers in industry 2, the latter opting to start up their own industry 2 firms instead. As this labor transition takes place, δ_2 rises and δ_1 falls, which raises revenue in industry 1 firms, and depresses revenue in industry 2. This necessitates a fall in w_2 to maintain the required expected income relativities.

Finally, if $\varepsilon \in \left[\varepsilon^1(\alpha), \varepsilon^1\left(\frac{1}{2}\right)\right]$, we can use analogous arguments to conclude that $\varepsilon = \varepsilon^1(\tilde{\delta}_2)$, where $\tilde{\delta}_2$ is the equilibrium density in industry 2. This equation can be solved for $\tilde{\delta}_2$, which will be increasing in ε over the range $\left[\alpha, \frac{1}{2}\right]$. In this case, bears are making the transition to ownership roles in industry 2, with industry 1 gradually vanishing. During this phase, w_2 continues to fall.

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