Title
Spatial Theory for the Integration of Resolution-Limited Data (96-8)

Author
Bruegger, Brad P.

Publication Date
1996-12-01

NCGIA

National Center for
Geographic Information and Analysis

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by
Beat (Bud) P. Bruegger
dipl. Ing. ETH, Swiss Federal Institute of Technology, 1986

A Thesis
The Graduate School
University of Maine

Technical Report 96-8
December 1996

Simonett Center for Spatial Analysis
University of California
35 10 Phelps Hall
Santa Barbara, CA 93106-4060
Office (805) 893-8224
Fax (805) 893-8617
ncgia@ncgia.ucsb.edu

State University of New York
301 Wilkeson Quad, Box 610023
Buffalo NY 14261-0001
Office (716) 645-2545
Fax (716) 645-5957
ncgia@ubvms.cc.buffalo.edu

University of Maine
348 Boardman Hall
Orono ME 04469-5711
Office (207) 581-2149
Fax (207) 581-2206
ncgia@spatial.maine.edu

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A THESIS

Submitted in Partial Fulfillment of the

Requirements for the Degree of

Doctor of Philosophy

(in Surveying Engineering)

The Graduate School

University of Maine

August 1994

Advisory Committee:

Kate Beard, Associate Professor of Surveying Engineering, Chair
David Tyler, Professor of Surveying Engineering
Michael Goodchild, Professor of Geography and Director, NCGIA, UCSB
Wolfgang Kainz, Professor of Geoinformatics, ITC, The Netherlands

a

Lucia

e

Oliver, l’animale

An der Narzißfigur arbeitete Goldmund mit tiefer Liebe, in dieser Arbeit fand er sich selbst, seine Künstlerschaft und seine Seele wieder, ... An seinem Jünger Johannes aber, dessen geliebte sinnende Gestalt ihm immer reiner aus dem Holz entgegentrat, arbeitete er nur in den Stunden der Bereitschaft, mit Hingabe und Demut. ... Nicht er war es, der da stand und aus eigenem Willen ein Bildnis schuf; vielmehr war es der andere, es war Narziß, der sich seiner Künstlerhände bediente, um aus der Vergänglichkeit und Veränderlichkeit des Lebens herauszutreten und das reine Bild seines Wesens darzustellen. ... Ach, daß aus Menschenhänden doch einzig solche Kunstwerke hervorgehen möchten, solche heilige, notwendige, von keinem Wollen und keiner Eitelkeit befleckte Bilder! Aber es war nicht so, er wußte es längst. Man konnte auch andere Bilder schaffen, hübsche und entzückende Sachen, mit großer Meisterschaft gemacht, die Freude der Kunstliebhaber, der Schmuck der Kirchen und Ratsäle -- schöne Dinge, ja, aber nicht heilige, nicht echte Seelenbilder. ... Er wußte es, zu seiner Beschämung und trauer, auch schon im eigenen Herzen, hatte es in seinen eigenen Händen gespürt, wie ein Künstler solche hübsche Dinge in die Welt stellen kann, aus Lust am eigenen Können, aus Ehrgeiz, aus Tändlerei. ... Ach, um hübsche Engelsfigürchen oder andern Tand zu machen, und sei er noch so hübsch, lohnte es sich nicht, Künstler zu sein. ... Für ihn waren Kunst und Künstlerschaft wertlos, wenn sie nicht brannten wie die Sonne und Gewalt hatten wie Stürme, wenn sie nur Behagen brachten, nur Angenehmes, nur kleines Glück.

Herman Hesse

Narziss und Goldmund

At this Narziss-figure Goldmund worked, finding himself again, his soul and best skill in what he did, ... But this figure of St. John the Disciple, whose loved and pensive face emerged before him, clearer and clearer from the wood, he only touched at hours when he was ready for it, utterly self-forgetful and absorbed. ... It was not he that stood before a wood-block, hewing out a portrait with his will; far rather it was the other, was Narziss, who used the skill in his hands to draw aside from the brittle transcience of time into the clear, abiding life of his essence. ... Ah, that such shapes alone might ever emerge from human hands; such sacred, necessary works, not blurred by any vanity or striving! But it was not so, he had long known it. Men could contrive quite different works of art--pretty figures, fashioned with intricate skill, their owners' pride, the ornaments of church and council-house--pleasant toys, yes, but never holy, never the true-born forms of the soul! ... He knew, to his own regret and shame, had felt in his own, juggling hands, how carvers will put forth such trumpery, from idle pleasure in their cunning, vanity, and finnicking ambition. ... What was the use of being a carver, to make polished angels and such trash, no matter how masterly the workmanship? ... For him all art and artistry were worthless unless they shone like the sun, had the might of storms in them--if they brought only pleasant, narrow happiness.

Herman Hesse

Goldmund: Translated from German "Narziss und Goldmund" by Peter Owen Limited

Acknowledgments

I would like to express my gratitude to the people and institutions who contributed to this thesis in various ways. The majority of the presented research was funded by the International Institute for Aerospace Survey and Earth Sciences (ITC), The Netherlands. Many thanks to Manfred Ehlers and Jean-Claude Muller who recommended me for this ITC scholarship. Related research and the early beginnings of the presented research was funded by the National Center for Geographic Information and Analysis (NCGIA), Maine. The Geography Department of the University of Oklahoma let me work on my thesis part-time while employed as a visiting instructor. Peter Krasznai organized a consulting position for me that financed my living and research for about a year. My parents supported me during gaps in the acknowledged funding.

Many thanks go to my committee who gave valuable comments on various versions of this thesis. Kate Beard got me interested in problems of (cartographic) generalization in a research seminar and helped clarify some of the presented ideas in numerous discussions. Kate was a big help in locating relevant literature and was great at encouraging me whenever necessary. Mike Goodchild was a great help at cleaning up the conceptual framework. Dave Tyler gave me a lot of mental support in difficult times. Wolfgang Kainz came all the way from Holland to my defense.

Renato Barrera contributed a lot to my understanding of problems in multiple representation GIS. He has always been a supportive friend and often pointed out relevant computer science literature. David Mark also helped in finding relevant literature. Martin Breunig gave valuable comments to an early version of the thesis. The students of my GIS seminar discussed large parts of my thesis. David Legates helped with some mathematical problems.

Several people have contributed in various forms to putting together the scientific toolbox and the necessary mode of thinking. Werner Kuhn got me interested in formal methods, modeling, and software engineering. Also Andrew Frank and Max Egenhofer contributed to my current view of the world. Max also sat in on my defense. Bob Franzosa taught me topology and mathematical methods. Many of my fellow grad students contributed in discussions. Among them are Douglas Hudson and Taher Buyong.

Very helpful was the administrative and clerical support by Chris LaCasse, Dave Tyler, Karen Boucias, Charles Tarr, Marion Pirike and several others.

Ann Steward and Yousif Hussin have been great friends who gave me a lot of mental support, encouragement, and advice. Peter, Mac, others of the "Latin Mafia", Bheshem (the smiler from Trinidad), Walton, and Kenny kept me sane with their parties. Without Lucia’s love and support, I would never have finished this thesis.

1 Introduction

The introduction states the objective of this thesis and then relates it to problems encountered in current GIS. A brief description of the approach discusses how the thesis objectives can be reached. The terms continuous and discrete model are central to this thesis and are therefore defined in section 1.4. The section on scope lists the major issues that are excluded from this study. The final section gives an outline of the remainder of this thesis.

1.1 Objective

The objective of this thesis is the design of a **spatial theory for GIS** (consisting of representation, meta data, and transfromations) **that allows complete integration of data sets that differ in resolution and format**. The scope is limited to a discrete view of geographic reality similar to "area-class maps", "categorical coverages", and "nominal fields" (see section 1.4 for detail). The spatial theory consists of representations of resolution-limited spatial knowledge, meta data that describe the knowledge content of representations, and transformations between representations of different type, resolution, format (raster or vector). The spatial theory addresses the following problems:

- **What limitations does limited resolution impose on spatial knowledge** that is represented in GIS? This models how the infinitely detailed world is mapped to a finitely detailed, scaled GIS representation.

- **How can such resolution-limited knowledge be represented** in a way that keeps precise track of the contained spatial knowledge and its limitations? Such representation has to capture the uncertainty introduced by approximation of the infinitely detailed world. Meta data has to describe what knowledge is contained in representations.

- **How can the same spatial knowledge be represented in different formats** such as raster and vector?

- **How can spatial knowledge be transformed** to other representation types, levels of resolution, and formats? Such transformations are important tools for the integration of diverse data sets.

These issues must be integrated in a single **consistent spatial theory**. This consistency includes the following properties:

- Meta data such as "resolution" is consistent between different formats such as raster and vector.

- Transformations between representations are completely determined by source and target meta data.
- Transformations correctly propagate uncertainty[1]. The effects of transformations on spatial knowledge must be completely captured by the proposed meta data.
- Representations described by the same meta data must be equivalent (up to different degrees of uncertainty) no matter whether they were derived with a single transformation or an alternative series of transformations. For example, reduction of resolution in a single or in several steps must be equivalent.

The **viability** of the proposed spatial theory is shown by demonstrating the implementability of representations and transformations. The practical applicability of the proposed resolution concept is shown by relating it to the resolution of sensors and by showing that resolution-limited representations can always be visualized within the limitations of display media.

While this thesis focuses on representations and transformations, the **potential impact** of the proposed consistent spatial theory **on the behavior of GIS** is documented with three examples: The first example demonstrates the possibilities of far reaching system support in the fitness for use assessment of data sets and their conversion to a format required by analysis. The second example shows how a resolution-conscious overlay operation can avoid spurious polygons. The third example outlines the potential of the proposed spatial theory for format-independent user interfaces that hide format issues from the user and thus provide the highest possible degree of format integration.

1.2 Problem

This section points out the relevance of data integration and the need for system support. It shows how some major impediments for data integration in current GIS are of conceptual nature and how system support must rely on a consistent spatial theory with compatible representations, meta data, transformations, and uncertainty models.

The capability of integrating diverse data sets is central to GIS [Flowerdew, 1991] [Shepherd, 1991] and is sometimes used as the defining property of GIS: "The benefits of a geographical information system depend on linking different data sets together" [DoE, 1987]; and "A GIS brings information together, it unifies and integrates that information." [Dangermond, 1989].

"Data integration is the process of making different data sets compatible with each other, so that they can reasonably be displayed on the same map and so that their relationships

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1 Note that the treatment of uncertainty in this thesis is limited to the kinds of uncertainty introduced by the scaling process and finite approximation. This excludes uncertainty introduced by, for example, remote sensing classification or finite accuracy in the positioning of sensors.

can sensibly be analyzed" [Rhind, 1984]. This thesis is concerned with making data sets compatible in terms of resolution and format. Most analysis operations require the data sets they relate to be of the same format and comparable resolution [Abel, 1990]. For example, raster overlay requires raster input layers of the same resolution and vector overlay requires vector coverages of comparable "scale". To prepare data sets for analysis, GISs offer transformations across formats and resolution/scale. The use of the appropriate transformations prior to analysis is the user's responsibility.

In current GISs, choosing the necessary transformations can be very problematic. Some examples of such problems are described in the following:

- Assume that two vector data sets shall be overlaid; one being digitized from a 1:25,000, the other from a 1:100,000 map. The GIS offers a Douglas-Poiker line generalization algorithm [Douglas, 1973]. How much do we have to generalize to map from a 1:25,000 to a 1:100,000 scale? The problem is a conceptual incompatibility of the meta data "scale" with the applied transformation.
- In case the same kind of problem requires a larger generalization step, the Douglas-Poiker algorithm may be incapable of performing the required transformation since it produces self-intersecting polygon boundaries that make the data set logically inconsistent [Beard, 1991a]. The problem is that the offered transformation is not generally applicable.
- Assume that a data set digitized from a 1:10,000 scale shall be transformed to 1:50,000 and to 1:100,000. There are two possibilities for computing the latter data set: by applying the generalization algorithm to either the 1:10,000 data set or the 1:50,000 data set. While intuitively, both possibilities should yield the same results, the resulting data sets are likely to be significantly different. Which of the data sets is better suited for the purpose? The problem evident here is again the incompatibility of meta data and transformations. Further, since algorithms such as Douglas-Poiker are incapable of propagating uncertainty, a comparison of the resulting data sets is very difficult.
- Assume now that the two data sets to be overlaid differ in both "scale" and format. Let the raster data-set be more detailed. Users now have the choice of either (i) generalizing the raster data set in the raster domain [Monmonier, 1983] and then converting it to vector, or (ii) converting it first and generalizing in the vector domain. Which of the two possibilities is better, for example in terms of uncertainty? How does raster resolution compare to vector scale? The problem that users face here is caused by the incompatibility of meta data and transformations between the raster and vector domain.
- Assume that a data set has to be converted from a vector to a raster format. Many different conversion algorithms are currently used in GIS and they produce largely different results from the same input [van der Knaap, 1992]. Which of the algorithms is the most suited? The most desirable algorithm would preserve the most knowledge about the world and introduce the least additional uncertainty. Current GIS lack a precise notion of knowledge content that is compatible between formats and they are incapable of propagating uncertainty. The choice of conversion algorithm (in case several are available) is therefore a very difficult problem for the user.

In summary, data integration requires very difficult decisions and considerable effort from GIS users. The problems originate in the incompatibility of representations, meta data, uncertainty models, and transformations.

1.3 Approach

The previous section has given examples of how current GISs provide insufficient support for the integration of diverse data sets and that the solution is closely related with a consistent spatial theory. The goal of a consistent theory can be reached by an approach that (i) models the effect of limited sensor resolution on spatial data explicitly, and (ii) expresses spatial knowledge in a format-independent model that is implemented as raster or vector model only in a second step.

(i) In order to study how limited resolution affects the knowledge contained in GIS representations, this thesis explicitly models how detail is reduced from a discrete geographic reality to different representations. Perception with limited-resolution sensors is used as a major mechanism for such detail reduction. This approach allows a precise definition of the knowledge content of a representation which is crucial for the proposed design of well-defined transformations. In particular, mapping a representation to a coarser resolution becomes well-defined since the source and target knowledge content are precisely known. This compares to current GIS practice where the knowledge content of a representation is only vaguely known to the user and totally inaccessible to machine interpretation. Figure 1.1 illustrates this with an example of two vector representations of the same coast line. What knowledge about the world do the representations really contain? For example, what is found in location p? The vague definition of the knowledge content is closely related to the problem of precisely defining what it means to transform a representation from one scale to another.

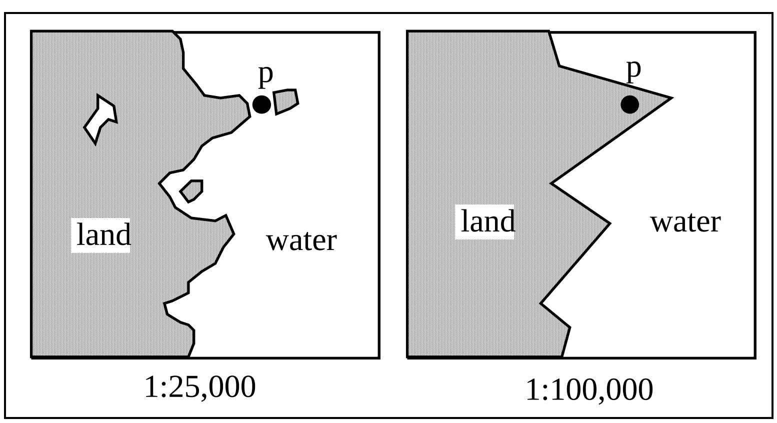

Figure 1.1: What is the precise knowledge content of two conventional vector coverages that show the same coastline at different scales? Is point p land or water?

(ii) A format-independent representation of spatial knowledge is crucial to the integration of raster and vector formats [Breunig, 1992]. The format-independent representations proposed in this thesis can be implemented in either raster or vector format. The use of the same conceptual bases in both formats allows the comparison of knowledge content between raster and vector representations. Transformations across formats can then be defined to preserve the knowledge about the world up to an unavoidable increase in uncertainty. The single conceptual base further allows use of the same meta data in both formats. This avoids the problems of comparing incompatible concepts such as raster resolution and vector "scale".

1.4 Continuous and Discrete Models

The concepts of continuous and discrete models is central to this thesis. They are therefore defined in this section and compared to similar terms used in the literature. The terms model and view of the world in conjunction with continuous and discrete are used as synonyms.

Continuous models describe the world in terms of a property value that is a function of the location in space. This property value is measured on a continuous scale of measurement[2]. Examples of continuous property values include ratio values such as elevation or temperature, or vectors of ratio values such as wind velocity or probability vectors [Goodchild, 1992a]. Usually, a small change in location causes a small change in the property value. However, discontinuities are allowed, i.e., property values can sometimes change abruptly with location. A possible class of continuous models is that of ratio valued fields [Goodchild, 1993a] where the location in space is specified by a point. Another example that is used in this thesis is the class of resolution-limited mixture fields. The continuous values here are mixtures (such as 24% land and 76% water) and locations in space are specified by disks rather than points (see chapter four for detail).

Discrete models describe the world in terms of regions that partition space and are described by a nominal discrete value[3]. Examples of nominal discrete values are landuse classes such as "forest", "grassland", "desert", etc. Other examples are parcel identifiers such as "parcel 12345" or state identifiers such as "Maine". Further, object classes such as "residential building" can be values of discrete models. Values can further be sets of elements that partition element space. For example, if element space is given by all possible mixtures of "land" and "water", possible values are the following three intervals:

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2 Values are measured on a continuous scale of measurement if the set of all possible values is topologically equivalent to R^n.

3 Values are said to be discrete if the set of all possible values is topologically equivalent to a subset of N. Values are said to be nominal, if they are discrete and cannot be ordered.

{• 80% land, < 20% water}, {< 80% land and > 20% water, <80% water and > 20% land}, and {• 80% water, < 20% land}.

Discrete models known from the GIS literature are "area-class maps" [Bunge, 1966] [Mark, 1989a], "categorical coverages" [Chrisman, 1989] [Goodchild, 1992a], and "single-valued vector maps" [Molenaar, 1989]. "Fields with nominal values" [Goodchild, 1993a] are also discrete models since the nominal values imply a partition of space into discrete regions. This thesis proposes a discrete model where regions partition resolution-limited space (rather than Euclidean space) and values are sets of mixtures similar to the ones in the above example. To avoid the implication of Euclidean space, this thesis uses the term discrete model rather than one of the previously proposed terms.

1.5 Scope

To make the research topic manageable, the scope had to be limited in several respects. The major limitation of scope is the focus on knowledge about **discrete models of geographic reality**[4]. This excludes continuous models of reality. One reason for this limitation is that often, the only available knowledge describes a discrete geographic reality. This is for example the case if the knowledge is extracted from maps[5]. Further, resolution limitations in knowledge about a continuous geographic reality have been successfully modeled by linear filter theory (see e.g., [Castleman, 1979]). In contrast, resolution/generalization issues for discrete models of reality are only poorly understood and lack a consistent formal treatment. The spatial theory proposed in this thesis attempts to overcome this shortcoming. Continuous views of reality can be transformed into discrete ones by classification. Classification is common practice, for example, in remote sensing. While the discussion of classification is outside the scope of this thesis, implications of the proposed discrete spatial theory on classification methods are pointed out (see chapter eight).

For reasons of simplicity, this thesis excludes changes over time and is restricted to a two-dimensional space.

While uncertainty is an important issue in scaled approximations of the world, **only** those kinds of **uncertainty** are considered that are **introduced by limited sensor resolution and by finite approximation** that is necessary for computer representation. This excludes uncertainty such as the confusion of nominal classes in the classification of satellite images and the finite accuracy in the position of sensors.

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4 Note that resolution-limited knowledge of a discrete geographic reality can be represented either in a continuous (see section 4.3) or discrete model (see section 4.4).

5 Note that contour lines on maps are an exception since they contain knowledge about a continuous geographic reality.

Further, **resolution** is assumed to be **constant within a data set**. This excludes varying or mixed resolution data sets.

Since the thesis focuses on the effects of limited sensor resolution on the represented knowledge, spatial objects whose geometry is defined in Euclidean space at infinite resolution are somewhat neglected. Such objects include land parcels, census tracts, or states. However, since Euclidean space is a special case of resolution-limited space, the spatial theory allows representation of these objects. Further, the geometry of these objects can be generalized within the theory by artificially simulating the effect of resolution-limited perception. For example, the precise geometry of states can be transformed to a resolution-limited coarser geometry that is displayable at a smaller scale.

Spatial knowledge as used in this thesis describes the **spatial distribution of discrete entities** that inhabit geographic reality. The application behavior of these entities is excluded from the study. For example, the proposed theory captures what mixture of (relatively small) entities fall inside the instantaneous field of view (IFOV) of a sensor, but excludes the problem of how the application behavior of contained entities propagates to the observable overall behavior of the IFOV. An example of a study that deals with this excluded aspect is [Raffy, 1992]. Another example of such exclusions is the modifiable areal unit problem [Openshaw, 1984] since it studies how to derive the application behavior of one set of entities (such as census tracts) from that of another set of entities.

The goal of cartography and thus cartographic generalization [Brassel, 1988] [McMaster, 1988] [Beard, 1988] [Mark, 1989b] [Muller, 1991] is communication of spatial knowledge to human interpreters. The presented study of resolution effects on knowledge and transformations to coarser resolution are closely related to generalization. In contrast to cartographic generalization, however, this thesis focuses on representing spatial knowledge for machine interpretation and analysis rather than human consumption. Similarities of this thesis and cartographic generalization are evident in the fact that the proposed limited-resolution representations guarantee displayability within the limitations of graphic media (see chapter seven). The combination of a transformation to coarser resolution and consequent visualization as presented in this thesis can be seen as a generalization of Perkal's work [Perkal, 1966] that has been applied in line generalization [Chrisman, 1992]. Differences of the approaches are for example evident in "displacements" and "exaggerations" [Brassel, 1988] that are important for human consumption but unsuited for machine interpretation where locational knowledge has to be preserved.

While this thesis studies the integration of data sets that differ in resolution and format, several other aspects of data integration are excluded. Among them are transformations between spatial reference systems and map projections, spatial interpolation, and matching of map sheets [Flowerdew, 1991].

1.6 Outline

The remainder of this thesis is organized as follows: The second chapter discusses related work in the field of spatial modeling. In particular, it reviews concepts that were adopted from the literature for use in this thesis. Also, it points out problems in previous approaches and outlines how these are solved in this thesis.

The third chapter gives an overview of the approach and the components of the proposed spatial theory. The overview provides a framework that demonstrates how different model components that are described in detail in the following chapters fit together. The discussion of the design concepts that underly this thesis show in detail how the properties of format-independence, consistency, and preserved relationship between geometry and attributes are achieved.

The fourth chapter is the heart of the thesis and covers the problem of representing resolution-limited spatial knowledge of a certain discrete world at the format-independent level of infinite models. Starting from a model of geographic reality, it derives different kinds of resolution-limited representations by reducing knowledge content.

Since finite approximation of infinite representations introduces uncertainty, chapter five describes how such uncertainty can be represented. Since finitely representable models are special cases of the infinite model, uncertainty representation is designed in the infinite domain such that the same concepts apply to all possible finite representations.

The sixth chapter defines finite implementations of these models in the form of extended raster and vector data models. These data models are then special cases of infinite uncertain representations; and different formats differ in their choice of parameters used for finite approximation. By demonstrating that the infinite models can be implemented as finite representations, this chapter partly proves the viability of the concepts.

The proposed representations are not as directly related to instructions for graphical output devices as are the classical raster and vector representations. The seventh chapter therefore discusses the visualization of resolution-limited knowledge. By showing that the proposed resolution concept guarantees that representations are displayable within the limitations of graphical displays, the chapter further demonstrates the viability of the proposed concepts.

Chapter eight defines transformations across representation types, levels of resolution, and finite formats. It shows that they are completely defined by their source and target meta data and propagate uncertainty introduced by finite approximation. The transformations are defined in the domain of infinite representations. To show the viability of the concepts, finite implementations of transformations in raster and/or vector models are discussed.

Three examples are used in chapter nine to demonstrate how the explicit modeling of resolution and the consistency of the proposed theory can be used to make GIS more intelligent. The first example demonstrates how meta data on resolution can be used for far-reaching system support in the fitness for use assessment and the preparation of data sets for analysis. The second example demonstrates how overlay that takes resolution into account avoids the problem of spurious polygons. It thus demonstrates how resolution-conscious GISs can become more intuitive and user-friendly. The third example outlines the design of a format-independent user-interface that hides format issues from the user.

A final chapter summarizes the major results, describes possible applications, and outlines future research.

2 Previous and Related Work

This chapter discusses previous and related work that is relevant for this thesis. The first section discusses previous work in the field of data integration. The second section reviews related work about components of a spatial theory, namely representations, meta data, transformations, and uncertainty models. The last section presents relevant literature on general and spatial modeling methodology and on the conceptualization of space.

2.1 Data Integration

This section reviews the relevant literature on data integration. It focuses on system support for making data sets compatible in terms of scale/resolution and format. A general description of the integration problem can be found in [Flowerdew, 1991] [Shepherd, 1991] [Rhind, 1984]. Practical experience with data integration is reported for example in [Rado, 1991] [Salg, 1992].

Several papers have addressed the impact of system architecture on integration. Among them are [Ehlers, 1989] [Breunig, 1992] [Abel, 1990] [Piwowar, 1990] and [Stephan, 1993]. Many aspects of system architecture are complementary to the issues of the proposed spatial theory, since they focus on implementation strategies rather than the underlying spatial concepts.

Some conceptual issues of integration have been discussed by Davis and Simonett [1991]. The article points out some conceptual incompatibilities between models of remote sensing and GIS but does not address the need for a consistent spatial theory explicitly. While most GIS vendors claim raster-vector integration capabilities for their products, a recent request on GIS-L asking for the relevant literature or description of the integration concepts did not produce any conclusive answers [Wilcox, 1992]. This gives evidence for shortcomings in the conceptual understanding that impede a complete integration. Similar evidence is given by Breunig and Perkhoff [1992] who point out the need for a format-independent representation of spatial knowledge that cannot be found in the literature. Ehlers et al [1989]point out that a lack of conceptual understanding is a major impediment for the integration of remote sensing and GIS.

The conceptual problems of multiple representations are very closely related to those of data integration: In both cases the comparison of the knowledge content of representations is crucial. In the field of multiple representations, comparison of knowledge content is used for the identification of equalities among representations for the management of redundancy, the propagation of changes to representations with partially overlapping knowledge content, and the test of consistency among representations. Multiple representations have received attention during the NCGIA research initiative 3 [Buttenfield, 1989a] [Buttenfield, 1993] and thereafter (e.g., [Kidner, 1993]). The need for a consistent spatial theory has been pointed out in a position paper [Bruegger, 1989] for initiative 3 but has not been further investigated.

Goodchild [1993b] emphasizes the role of uncertainty models in integration. Uncertainty models are closely related to a certain representation model since they express uncertainty in terms of representation entities. Since the representation model proposed in this thesis is not comparable to the ones discussed by Goodchild, and due to the limited scope of the uncertainty model of this thesis, Goodchild's framework is not directly applicable to this thesis (see also section 2.2.5).

Since system support for data integration is most relevant to this thesis, the sequel of this section discusses different kinds of support and related literature. Considering the importance of data integration for GIS and the difficulties facing users when dealing with different formats and levels of resolution/scale (see section 1.2), system support in these areas can greatly improve the user-friendliness of GIS. Such support can include (i) fitness for use assessment, (ii) selection of the most adequate data sets, (iii) automatic conversions of data sets in preparation of analysis, and (iv) hiding format differences from the user. These kinds of system support will be discussed in the following. It is evident that they all rely on a consistent theory of representations, meta data, uncertainty model, and transformations.

(i) The fitness for use assessment uses minimal data requirements specified by the user in terms of meta data and determines whether data sets can be converted to the required state. Goodchild addresses fitness for use assessment in one of his challenges [Goodchild, 1992b]. He points out the relation to meta data and argues that they have to be machine readable. In contrast, data transfer standards such as SDTS [Fegeas, 1992] and meta data standards [FCDC, 1992] do not contain machine readable scale/resolution meta data that are compatible across formats and with transformations. For example, the scale of the original paper map is part of a free text entry on the "native data set environment" in the latter standard. Abel [1989] proposes a model for data set management that includes meta data for each data set. While his meta data are machine interpretable, he does not address the issue of compatibility of meta data across formats or of meta data with transformations.

(ii) As outlined in chapter nine, a system can select the most adequate data set in a multiple representation environment [Bruegger, 1989]. Most adequate can then be defined as an optimization that optimizes uncertainty, cost, or processing time within certain constraints on the other criteria.

(iii) The compatibility of meta data and transformations allows automatic preparation of data sets for analysis. Analysis operations typically pose certain requirements of the input data sets. For example, overlay requires a single format and comparable scale/resolution. The analysis requirements can be expressed in terms of meta data. In a consistent spatial theory, analysis requirements and meta data of data sets completely determine the necessary transformations. They can thus be automatically performed by the system. Examples of the automatically performed conversions are described in the integration literature [Abel, 1990] [Piwowar, 1990]. However, the problem of developing a consistent spatial theory has not been adequately addressed in the literature.

(vi) The ultimate degree of format integration is reached when format differences are completely hidden from the user [Maguire, 1991]. This can be achieved by a user interface that provides conceptual, format-independent GIS operations to the user. User commands are then automatically decomposed in a series of executable, formatted GIS operations and, if necessary, format conversions of data sets. For example, the user command "overlay (landuse, planning_zones)" is translated to "vector_overlay(raster_to_vector(landuse), planning_zones)". The translation of conceptual to executable commands is known in computer science as "overloading", the automatic conversion as "coercion" [Cardelli, 1985]. They are common practice in modern programming languages. Applications of these concepts to raster/vector integration in GIS has been reported by Abel and Wilson [1990]. Conceptual difficulties have so far limited a comprehensive application of this concept, however. For example, there are usually several possible decompositions of a user query; but the choice of the one that minimizes uncertainty has not been possible due to the incompatibility of common uncertainty models with transformations and across formats. Further, hiding format differences of data sets that also differ in resolution/scale cannot be solved without a consistent theory that uses compatible resolution/scale measures in the two formats.

In summary, substantial system support is possible for the integration of data sets that differ in scale/resolution and format. Such automation heavily relies on a consistent spatial theory with compatible representations, transformations, meta data, and uncertainty model. The results of this thesis thus enable the design of more user-friendly GIS with significantly improved data integration capabilities.

2.2 Components of Spatial Theory

The objective of this thesis is the design of a consistent spatial theory consisting of representation, meta data, transformations, and an uncertainty model of limited scope. All these components of a spatial theory have received ample attention in the literature. This section reviews the relevance and differences of previous work for the research presented in this thesis.

2.2.1 Representation

This section reviews previous approaches to the representation of resolution-limited knowledge of a discrete geographic reality. The discussion is kept at a conceptual level and points out some problems of previous approaches in the context of resolution-limited knowledge. Two major types of representations are distinguished according to whether they use (i) point sets with a single nominal valued attribute, or (ii) point sets with a more complex attribute.

Both, raster and vector representations (see e.g., [Egenhofer, 1991]) use point sets as basic entities: Vector representations use polygons[6], and raster models use cells. These point sets usually form a partition of space. Further, they have to fulfill certain format requirements. Namely, vector polygons usually must have boundaries composed of straight line segments and the boundary of raster cells must coincide with a predefined grid.

In most (if not all) commercial GISs, representations of discrete models of the world use point sets with a **single nominal valued attribute**. Examples of such attributes are "water", "land", "forest", "grassland", "urban", "parcel no. 1233", "London", "Maine", etc. Since the point sets carry only a single attribute, all their element points are usually seen to carry the same attribute.

Assume that at infinite resolution, a single theme of the (discrete) world can be perceived in the form of a nominal field [Goodchild, 1990]: every point (x, y) then carries a (single) nominal value n. At infinite resolution, the nominal field would directly correspond to a raster representation, since raster cells become indistinguishable from points. Vector representations would group contiguous areas of the same attribute to polygons, that are general point sets at infinite resolution. While in raster representations, the geometry of point sets is predefined, in vector representations, it is determined based on the observed attribute (see section 2.3.1 for a more detailed discussion of these concepts). At infinite resolution, both raster and vector models capture the observed knowledge exactly without introducing any error.

Problems of these representation models show up only at limited resolution where cells have a given size and line segments of polygon boundaries a minimal length. Here, the single attribute cannot capture the world as it is since (a) in the raster domain, cells contain points that differ in their attributes, and (b) vector boundaries are too coarse to follow the natural attribute boundaries and thus cause polygons to contain points of mixed attributes. Since the representations proposed in this thesis are designed in a format-independent domain and since there are different types of representations, this thesis has similarities to both, raster and vector models.

> (a) Raster models use point sets of predefined geometry and usually represent the nominal value that is the modal point attribute in a cell. This modal attribute can be a very weak representation of what is really found in a cell. For example, assume that landuse is observed and that the nominal attribute can fall in any of 10 classes. In this case, a cell that contains 10.1% "forest" and a little less than 10% of the other landuse categories is assigned the modal attribute of "forest". Modal attributes are

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6 Points and lines are not important for the representation of discrete views of the world.

also not uniquely defined. For example, what is the modal attribute of a cell that contains 1/3 forest, 1/3 grassland, and 1/3 water?

While GIS users may know about the limitations of knowledge associated with modal attributes[7], machines (i.e., GIS) interpret cells as if they were homogeneous, i.e., all points in the cell are assumed to carry the modal cell attribute. This homogeneous interpretation of cells is used in all analysis steps and in transformations to coarser resolution. It is easy to imagine how the weak representation of knowledge in the form of modal attributes propagates through GIS operations to a result that contains only very weak statements about the world.

While the representation proposed in this thesis includes a task comparable to assigning an attribute to a whole cell, it avoids the use of modal attributes and thus weak representations of knowledge about the world. Instead, to treat something as "forest" at a higher level of abstraction, a minimum of for example 85% of "forest points" are required. Further, the presence of a maximum of 15% of "inhomogeneity" is represented in a machine readable form. If none of the landuse categories in a point set reach the required 85%, the attribute becomes "transition zone" that expresses the fact that the point set contains a mixture of landuses (see chapter four).

(b) Vector models follow a different approach of representing resolution-limited knowledge. Since their point sets are not predefined, the concept of modal attribute does not apply. Instead, the geometry of point sets has to be determined based on the attributes of points. Figure 2.1 shows the distribution of land and water at infinite resolution. It illustrates that the problem goes beyond just approximating the generally shaped "natural boundaries" at infinite resolution with straight line segments.

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7 My teaching experience actually shows that it is difficult to convince people that a modal landuse can cover less than 50% of a cell.

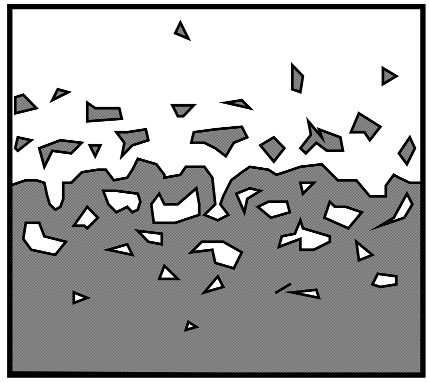

Figure 2.1: A natural boundary at infinite resolution that breaks up into islands and holes.

A major problem of vector representation of limited resolution is the ambiguity of boundary locations. As experienced in cartographic generalization, different cartographers describe the same world with different polygons. If machines shall interpret the represented knowledge during analysis and other GIS operations, a well-defined, objective method for the determination of polygon boundaries is needed.

Perkal [1966] addressed this issue in his paper titled "An Attempt at Objective Generalization"[8]. (See also section 2.2.3 and chapter seven for a more detailed review of Perkal's work). He derives the location of boundaries from the attributes of points. For example, if a disk of diameter epsilon contains only points of attribute "forest", all these points are part of the generalized "forest". Points that do not fall inside such a homogeneous disk of the required size become points of polygon "edges". Polygon edges can thus become areas.

While Perkal's work has found some application in cartographic generalization [Chrisman, 1992], his concepts for an objective choice of polygon boundaries at limited resolution has not found its way into GIS practice. Vector models therefore suffer still from a high degree of subjectivity in the location of polygon boundaries. This subjectivity hampers human and particularly machine interpretation of what knowledge about the world polygons actually contain.

Note that uncertainty models do not solve the problem of subjective polygon boundaries. In the vector domain, uncertainty is usually expressed in terms of locational and attribute uncertainty [Chrisman,

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8 Perkal's objective generalization actually is not formulated in the raster domain but rather uses general point sets of Euclidean space.

1989]. Locational uncertainty is usually expressed by a band or a probability surface that is centered on the represented polygon boundary. Locational uncertainty relates the represented boundary with the "true" boundary [Chrisman, 1991]. For example, a band is assumed to contain the "true" boundary with a certain probability. The problem of this uncertainty model is that the "true" boundary is not really defined. If it is considered to be the "natural" boundary at infinite resolution, the topological complexity of "natural" and represented boundary are largely different (see figure 2.1) which makes the concepts of locational error questionable. If the "true" boundary is limited in its resolution, it is not objectively defined as argued above. The problem is also evident when looking at different subjective boundaries that differ in their topology. In this case, bands or probability surfaces centered on the represented lines are unfit to explain the subjective difference in topology. A similar critique of the discussed error model is found in [Goodchild, 1993b].

To overcome the problem of subjectively defined boundaries at limited resolution, this thesis uses a generalization of Perkal's concepts to objectively derive boundaries from point attributes (see chapter four).

The remainder of this section reviews the second type of representation that uses point sets with **more complex attributes** than just a single nominal value. Namely, it has been suggested to use an attribute for raster cells that consists of a vector of probabilities, that an arbitrary point in the cell belongs to a given category [Goodchild, 1992a].

If these probabilities are unaffected by uncertainty introduced by data acquisition and classification, they express the "mixture" of point attributes within a cell. This approach obviously solves the problems of a modal, single valued attribute. This thesis uses a very similar concept to probability vectors. Instead of assigning probabilities to every point, however, it represents the mixture of point attributes for a whole instantaneous field of view that are not limited to the cells of a regular grid (see chapter four).

It has also been suggested to apply the more sophisticated attribute representation to the determination of boundary locations [Mark, 1989a]. Actually, Mark and Csillag use a special case of the mentioned attribute representation, called "probabilistic epsilon band" that was introduced by Honeycutt [1987]. It is a special case since points in the "fuzzy boundary region" between two polygons can belong only to the categories of these adjacent polygons. The probability vector thus has only two non-zero components. Mark and Csillag [1989a] suggest to use a "contour" of such a probability surface as polygon boundary. Figure 2.2 visualizes the concept.

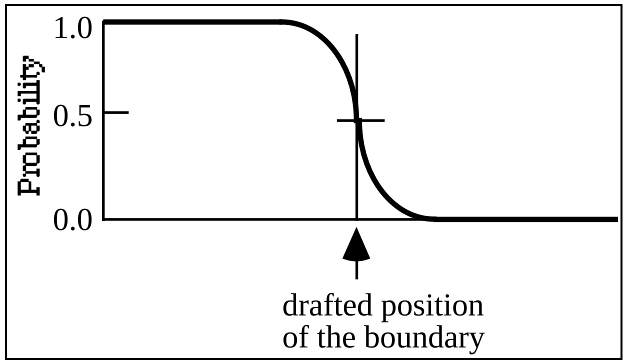

Figure 2.2: Probability of a point being a member of category "A", as a function of position along a profile perpendicular to the boundary between a zone of category "A" on the left and "not-A" on the right. Copied from [Mark, 1989a], figure 2(c), page 73)

This approach seems to have some problems that are pointed out in the following. First, a point in a fuzzy boundary region may well belong to a category different from those of the adjacent polygons. In contrast, this thesis allows the equivalent to general probability vectors (see chapter four).

Second, probability vectors are defined for predefined point sets such as raster cells or instantaneous fields of view. The vector represents the mixture of point attributes for the whole point set. In contrast, the probabilities of single points belonging to a given category is given with 100% probability. It is conceptually unclean to attach a probability vector determined for a predefined point set to all points in this set. This is evident, for example when two overlapping point sets are used to determine the probability vector. In this case, the vectors of the two point sets are likely to be different. A point lying in the intersection of the two point sets would thus be assigned different vectors depending on which point set was selected. If the problem of unique assignment of vectors to points is solved by using a grid of raster cells, the problem of locating a polygon boundary is irrelevant since it does not occur in the raster domain. This thesis solves the problem by assigning mixtures (similar to probability vectors) to whole instantaneous fields of view (i.e., disks) rather than to points (see chapter four). For this reason, resolution-limited space that is a set of disks is used rather than Euclidean space that is a set of points.

2.2.2 Meta Data

A rapidly growing number of data sets available from government agencies and other sources, the ever increasing networking and exchange of data, and the realization that a single data format is not sufficient have increased the importance of meta data in the field of GIS. Some of the recent works in the field include standards such as [Fegeas, 1992] [FCDC, 1992], and discussions in other contexts [Abel, 1989] [Kidner, 1993].

Meta data are defined in the context of a representation model. For example, the concept of scale is closely related to a graphical representation and is not directly applicable to other representation models such as vector. Similarly, raster resolution is defined in terms of entities of the raster model. Since this thesis proposes a new representation model previous work on meta data is not directly applicable.

2.2.3 Transformations to Coarser Resolution

Since the proposed consistent spatial theory includes transformations that map knowledge to coarser levels of resolution, this section compares the proposed transformations to other generalization approaches.

In vector GIS, cartographic generalization performs a similar purpose to the transformation proposed in this thesis. The most common such method in current GIS is the Douglas-Poiker algorithm [Douglas, 1973]. A bibliography of more recent approaches was compiled for the NCGIA initiative 3 [Buttenfield, 1989b]. Line generalization algorithms are defined for the domain of chains of points. The problem consists of either filtering or selecting points in a chain to yield a similar chain with less points and generalized geometry [Buttenfield, 1985]. Since points and chains do not exist as entities in the domain where this thesis defines transformations, the results of previous research in this field are not applicable.

An exception is the work by Perkal [1966] that defines the problem independently of chains of points but has found application in cartographic generalization [Chrisman, 1992]. (See also section 2.2.1 and chapter seven for a detailed review). Perkal's goal was to propose a mathematically well-defined concept for cartographic generalization to solve problems of the existing, vaguely defined approaches. Perkal's model is based on manually sweeping a disk over map graphics to construct areas of a coarser scale. Generalized areas are defined as the union of all disks that are completely enclosed in one of the original areas. While area boundaries in graphical approaches always have a finite width (i.e., line width), Perkal's boundaries become areas themselves. They are either band-like in case of small generalization effect, or larger areas, for example when unresolvably small islands are arranged in dense groups. Discrete views (e.g., for landuse data) proposed in this thesis borrow some of the major concepts from Perkal's work, namely that attributes are determined for disks rather than points and the concept that boundaries can become areas. Perkal's focus on cartographic rather than model generalization explains the major differences of his approach compared to this thesis:

(i) Perkal's disks are described by single homogeneous attributes such as "water" or "land" that can be mapped to single color tones in graphic visualizations. In contrast, this thesis uses inhomogeneous attributes such as "more than 80% water" in order to capture what is known about the world more precisely.

(ii) Perkal uses his generalization process to transform a scaled map to a map of coarser scale. In contrast, this thesis uses a comparable process to transform unscaled geographic reality into a scaled representation. Figure 2.3 illustrates the different uses of Perkal's concepts. t1 and t2 are generalization transformations similar to Perkal's. Both transform an infinitely detailed model of geographic reality to a "scaled" representation. Transformation t3 that maps between two scaled representations is derived from t1 and t2 such that the concatenation of t1 and t3 is equivalent to t2. t1 and t2 are the mechanisms to reduce knowledge content from geographic reality to a "scaled" representation. Meta data describe the information content of representations and are thus closely related to t1 and t2. The figure illustrates how the described relations between transformations and meta data allow determination of transformations such as t3 by source and target meta data.

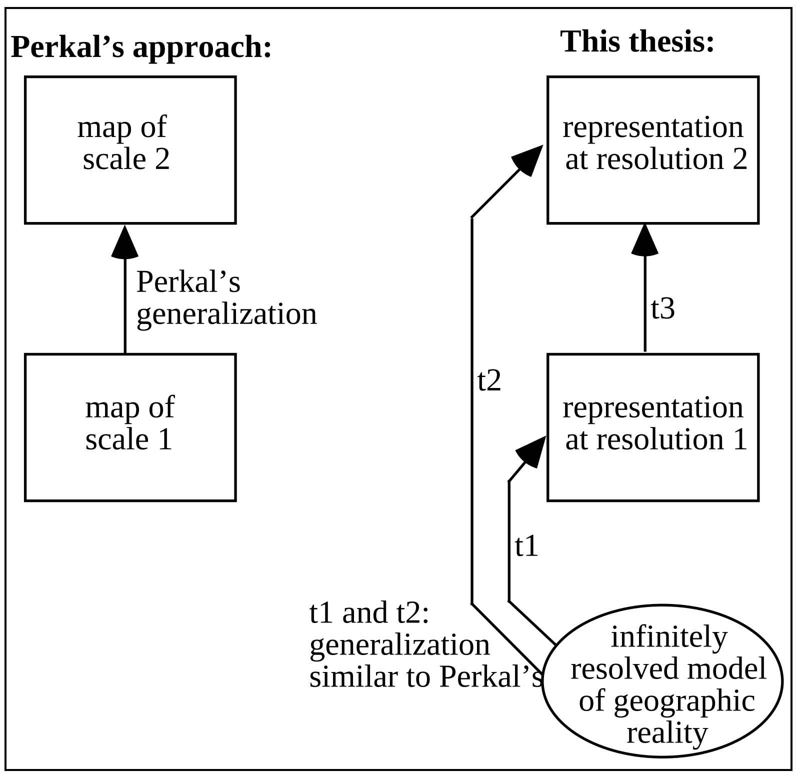

Figure 2.3: Differences of use of comparable generalization concepts by Perkal [1966] and this thesis.

(iii) Due to his graphical focus, Perkal defines his generalization in the visually inspectable Euclidean space: His generalized areas are therefore unions of disks that are Euclidean point sets and therefore visually inspectable. In contrast, this thesis takes Euclidean point sets (i.e., areas) of geographic reality and transforms them to areas of "resolution-limited space". Resolution-limited areas are sets of disks rather than unions thereof, and are therefore not Euclidean point sets and not visually inspectable.

Generalization methods for the raster domain have been described by Monmonier [1983]. These generalization methods are defined in terms of raster cells and zones with a single nominal attribute[9]. Since these entities do not have direct equivalents in this thesis, previous work is largely inapplicable to the problem discussed in this thesis. A major difference in philosophy between this thesis and Monmonier's generalization is evident in the preservation of the relation between geometry and attributes. For example, Monmonier [1983, page 70] proposes to partition and dissolve polygons. The dissolved parts are then "absorbed" in adjacent polygons, but the related change in the attribute of these polygons is not accounted for. Such a generalization procedure therefore fails to preserve the relationship between geometry and attribute. In contrast, this thesis strictly preserves this relationship. A change in geometry is inseparable from a change in attribute (see section 3.6).

The transformation to coarser levels of resolution proposed in this thesis heavily relies on linear filters [Castleman, 1979]. Raster GIS commonly offer finite implementations of such filters. However, these filters are usually applied to images, i.e., raster layers with real or integer cell values. They are not applicable to the nominal cell values associated with a discrete model of the world[10].

2.2.4 Raster-Vector Conversion

Among the transformations across finite formats proposed in this thesis are conversions between raster and vector formats. Such algorithms have been discussed in [Franklin, 1979], [Peuquet, 1981a], [Peuquet, 1981b], [Pavlidis, 1982], [Clarke, 1985], and [van der Knaap, 1992].

One major difference of this thesis as compared to previous work is that the proposed conversions take uncertainty introduced by finite approximation into account. Such uncertainty is represented in the form of a geometric container, i.e., the finitely representable shape totally contains the more general, non-representable shape (see chapter five). This means that reapproximation in a

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9 While Monmonier uses integer values as cell attributes, they are codes for nominal properties since neither order nor ratios are relevant.

10 A moving window filter that determines the modal cell value is not linear.

different finite format must totally contain the area of the original finite representation (see figure 2.4.a). In contrast, format conversions described in the literature attempt to keep the difference between the two different approximations small (see figure 2.4.b).

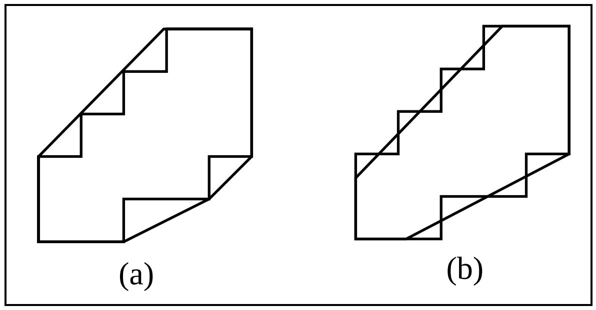

Figure 2.4: Raster to vector conversion as defined in this thesis (a) versus previous approaches [Peuquet, 1981a] (b).

Another major difference is evident in the treatment of objects that are relatively thin or small compared to resolution. Examples of such objects can be rivers and cities. While in the world, these objects are areas, conventional vector models usually represent them at reduced dimension as lines (in the case of rivers) or points (in the case of cities). Since raster representations preserve the areal character of these objects, conventional format conversion has to translate between the different approaches of the two data models [van der Knaap, 1992]. In contrast, this thesis proposes to represent relatively thin or small objects separately from "normal" objects as "stand-alone features" and to preserve the areal character in both, raster and vector representations[11] (see chapters four and five). This approach avoids the problem of possibly arbitrary decisions of how to translate between the different approaches in the classical representation.

2.2.5 Error and Uncertainty Models

The representation and propagation of uncertainty introduced by finite approximation is part of the thesis objective. This section therefore discusses similarities and differences to error and uncertainty models discussed in the literature. The comparison of approaches is discussed at a philosophical level since the treatment of uncertainty in this thesis is drastically limited in scope compared to work described in the literature and due to the difference of the data

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[11] Note that while the proposed vector representations preserve the objects areal character, the proposed vector visualizations show the objects as points or lines (with a certain drawing pen width).

models in which the uncertainty models of this thesis and the literature are defined.

Data quality as captured in error and uncertainty models is of growing concern to the GIS community (see e.g., [Gopal, 1989] [Chrisman, 1991] [Goodchild, 1992a]). A recent survey and taxonomy of approaches is presented in [Goodchild, 1993b]. Due to the largely different scope of the uncertainty model in this thesis and its description in a totally different data model, it would be meaningless to locate the proposed approach in Goodchild's taxonomy of uncertainty models.

The major difference to related models is the **limited scope** of the uncertainty model proposed in this thesis: This thesis considers **only the uncertainty that is introduced in the scaling process and in the finite approximation necessary for representation**. This excludes for example attribute uncertainty introduced by remote sensing classification or positional uncertainty caused by limited accuracy in the positioning of sensors. In contrast, most uncertainty models in the literature attempt to encompass all kinds of uncertainty without isolating the uncertainty discussed in this thesis as a separate type.

Chrisman [1991] defines **error** as the "forcible deviations between a representation and actual circumstances". Since this true error can usually not be determined, uncertainty models often describe the probability of different error magnitudes or of different true configurations of actual circumstances. An example of the former kind of probability are the error ellipses used in surveying engineering; and an example of the latter probability is the uncertainty model by Goodchild et al that assigns different probabilities to different nominal attribute values of points [Goodchild, 1992a].

A second major difference to existing uncertainty models is that this thesis represents knowledge at a limited level of detail such that the "scaling process" does not introduce any error by Chrisman's definition. This is illustrated in figure 2.5. The upper two boxes show the common point set--single nominal attribute approach to modeling knowledge at different scales (see section 2.2.1); the lower two boxes illustrate the proposed representation at limited level of detail. In the classical approach, every point carries an attribute such as "land" or "water". As is shown for point p, these attributes are changed by the scaling process. This change of attribute results in errors according to Chrisman's definition: P, in the 1:100,000 map is represented to be "land", while it is "water" in actual circumstances. In this context, uncertainty models represent, for example, the probability of a point being represented as "land" while the true attribute is "water".

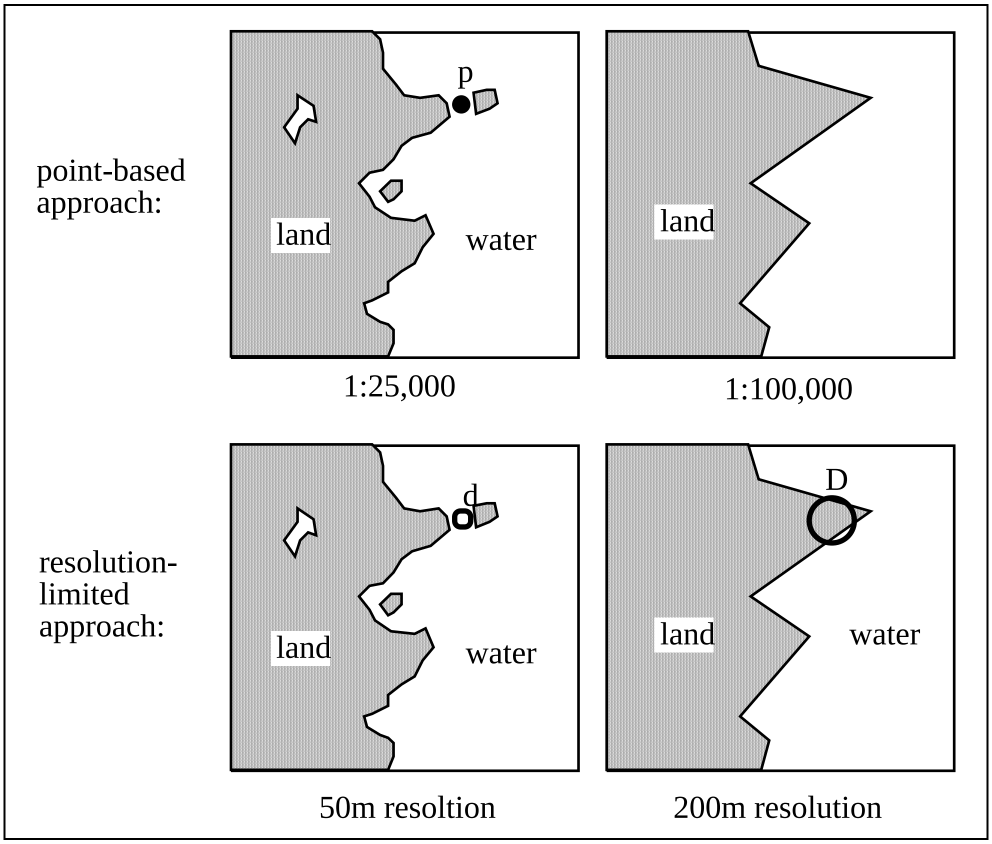

Figure 2.5: Representation at limited resolution compared to the common point-based approach.

In the resolution-limited approach (illustrated in the lower two boxes), no error according to Chrisman's definition occurs. Instead of attaching attributes to points, they are attached to disks whose size is determined by resolution. The disks d and D then correspond to the point p, respectively. Instead of using simple nominal values such as "land" or "water", attributes in the resolution-limited approach express the level of inhomogeneity within a disk. For example, the attribute of disk d is "more than 80% water", where "water" is defined in geographic reality and is therefore invariant with scale. Similarly, the attribute of D is "more than 80% land". While the classical approach showed a contradiction between the knowledge represented at the two different scales, the water at the 50m resolution-limited representation can easily be absorbed in the 20% inhomogeneity of the larger disk at 200m resolution. Due to this absence of contradictions, the scaling process does not lead to errors or uncertainty in the sense discussed above. The representation at limited level of detail thus makes an uncertainty model for scaling effects unnecessary.

While scaling does not result in uncertainty in the proposed model, finite approximation does. This thesis uses two kinds of finite approximations: (i) Continuous functions over two dimensional space that are approximated by sampling and interpolation; and (ii) areas of general shape that are approximated by vector polygons or raster zones[12]. In both cases, the maximal possible approximation error is limited. This is obvious for case (ii) but requires some explanation for case (i): At infinite resolution, the approximated functions can have discontinuities such that approximation errors are potentially unlimited. All finite representations operate at limited resolution, however, where the function is guaranteed to be continuous. Section 4.5.3 will show that this limits the maximal rate of change (i.e., first derivation) of the function. This, in turn, limits the possible values the function can assume between sampling points and thus the maximal deviation between true and interpolated value.

Goodchild states that most errors in the real world are normally distributed and that error magnitudes are therefore potentially unlimited [Goodchild, 1994]. The above argument has shown that errors of finite approximation are theoretically limited and are therefore not normally distributed. This thesis therefore proposes to represent these errors by upper and lower bounds of the possible errors. Alternatively, error bounds could be treated as confidence intervals similarly to error ellipses in surveying. For example, with 90% probability, the actual error stays within the given bounds. Such an extension seems rather difficult and application dependent, however, since it involves assigning probability densities to different configurations of geographic reality.

2.3 Spatial Modeling

This section reviews issues of spatial modeling and modeling in general that are relevant to the presented work. Sinton's concepts for constructing different types of spatial entities are discussed in the first section. They underly the construction of spatial features in chapter four and provides a general understanding of how attributes and geometry are related. The second section reviews different concepts of resolution and points out how they are used in this thesis. Literature on models in remote sensing, discussed in the third section, gives evidence for the practical value of discrete models of reality. It further provides the terms "H- and L-resolution" that are used in this thesis. This thesis adopts the concepts of "objects" and "features" from cognitive research that is reviewed in section four. The structural similarities between the cognitive model and this thesis are also pointed out. The issue of finitizing the inherently infinite spatial domain is an important component of the approach. The review of literature on the topic is found in section five and also gives insight in the origin of formats (raster and vector) which is a prerequisite for the understanding of format-independent models. Finally, in section six, abstraction mechanisms that are used to construct higher-level objects in the model of geograhic reality are reviewed in the last section.

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[12] These areas, vector polygons and raster zones are defined in resolution-limited space rather than Euclidean space.

2.3.1 Sinton's Taxonomy of Spatial Knowledge

A taxonomy of discrete models of spatial knowledge proposed by Sinton [1979] is extensively used in this thesis. It has already been applied implicilty in section 2.2.1 and is the major mechanism to abstract continuous models used to represent resolution-limited knowledge to discrete ones (see section 4.4). This section therefore reviews Sinton's work.

Sinton proposed to distinguish different types of discrete spatial knowledge. These types express different facts about the world and have to be treated differently during analysis. They are derived from the same (continuous or discrete) model of the world by different methods.

Sinton's model of geographic reality defines three components of spatial informations, namely **"location"**, **"theme"**, and **"time"** [Sinton, 1979]. Figure 2.6 shows such a model with a continuous theme. For visualization purposes, location is shown by a single axis rather than two axes .

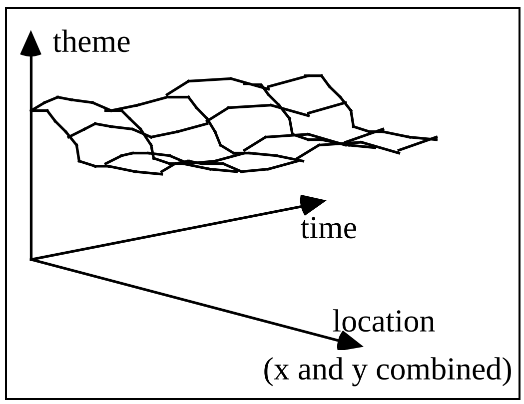

Figure 2.6: Continuous model of the world.

Sinton proposed that discrete representation models are obtained by (1) **fixing** one of these components, (2) **controlling** a second, and (3) **measuring** the third. Since this thesis assumes time to be fixed, only two methods for constructing discrete models are of interest: (i) controlling theme and measuring location, and (ii) controlling location and measuring theme. Examples of these two cases are given in the following:

The first example shows how the classification of landuse from a remotely sensed image corresponds to controlling the theme and measuring the location (see figure 2.7). Albedo represents the theme. It is controlled by defining intervals of albedo values that correspond to discrete landuse classes such as "urban", "forest", and "pasture". These discrete classes are used to discretize the continuous variation of albedo: all locations with an albedo value that falls into the interval of "forest" are assigned the nominal value "forest". Contiguous areas with the same nominal value now form discrete entities (i.e., polygons) whose geometry (or location) can be measured.

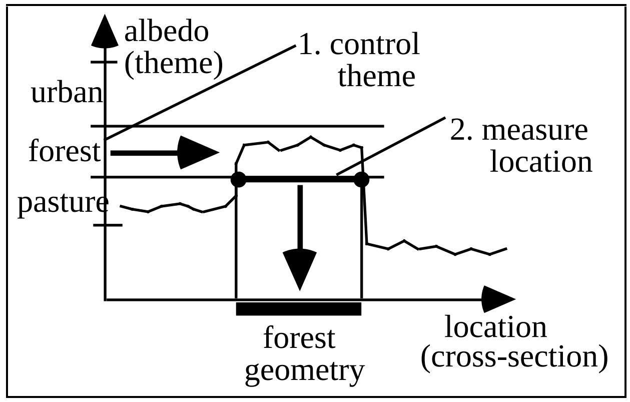

Figure 2.7: Controlling theme to measure location.

The second example shows how the temperature values reported for predefined areal units correspond to Sinton's method of controlling location and measuring theme (see figure 2.8). The theme is given by the continuous variation of temperature over space. In a first step, location is controlled by arbitrarily defining reporting units. Examples of such units are the cells of a regular grid and administrational units such as census tracts, districts, and states. Temperature varies within each reporting unit. The theme is measured by deriving statistical descriptors of this variation. Some of the most commonly used statistics are average, minimum, maximum, and variance.

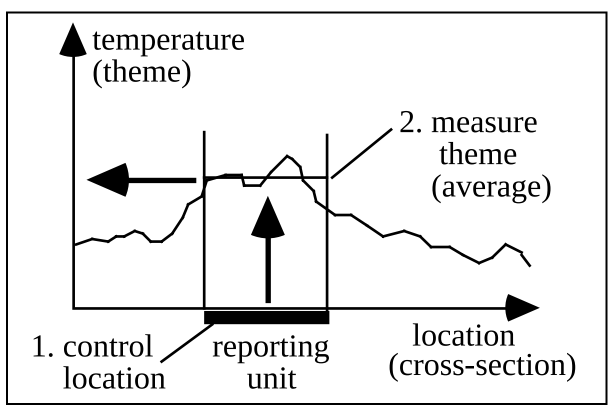

Figure 2.8: Controlling location to measure theme.

Section 2.2.1 gave an example where Sinton's methods were applied to a discrete model of geographic reality. In this model every point in reality was described by a nominal value. In a raster representation, location was first controlled by a grid and the theme was measured in the form of a "modal value" or a "probability vectors". In vector representations, the theme was controlled and the location measured. At infinite resolution, this means that the polygon boundaries separate contiguous areas with the same nominal point attributes. At limited resolution, Sinton's concept is not applicable anymore since the representation of location is too coarse to follow the geometry defined by the theme.

Csillag [1991] surveys some of the relevant research concerning resolution in GIS. He identifies some problems of knowledge representation in this context by questioning the classic understanding of geometry and attribute: "The conventional separation of spatial data into geometry and attributes has not left this community yet. Such a separation is consistent with an entity-relationship model, with geometry defining the objects, which then have attributes and relationships [Mark, 1989a]".

Since geometry and attribute (i.e., location and theme) cannot be treated separately [Csillag, 1991], Sinton's concepts that relate the two aspects are of utmost importance for the understanding of spatial knowledge. In the case of limited-resolution vector representations, this relation between theme and location is lost. This causes the problems of subjectively defined boundary locations (see section 2.2.1) and ill-defined generalization methods[13].

2.3.2 Resolution

Resolution is a central issue for this thesis for obvious reasons. Resolution concepts are necessarily different for continuous and discrete models. This section reviews the concepts of both domains and how they relate across domains. It further points out, which of these resolution concepts are used in this thesis.

In the domain of continuous models, resolution is most commonly modeled by linear systems [Castleman, 1979]. The effect of resolution can then be described by a linear filter. For example, an optical system can be characterized by its "optical transfer function", or by a "modulation transfer function", both of which are basically linear filters [Castleman, 1979]. In image processing, these filters are also called "point spread function" [Castleman, 1979]. In remote sensing, sensor resolution can be described by a linear filter [Davis, 1991]. Here, the filter describes the sensitivity of a sensor cell as a function of the distance from the cell's center point. Figure 2.9 shows two possible filters that describe the

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[13] This problem is solved in this thesis by deriving "mixture fields" as a limited-resolution theme from the original one that leads to coarser geometry of entities. To allow polygonal approximation of this geometry, the attribute values of the "transition zone" (similar to boundary) are slightly modified to preserve the relation between geometry and attribute.

resolution of an imaging system. Filter (a) describes an actual sensor, while filter (b) describes an "ideal sensor".

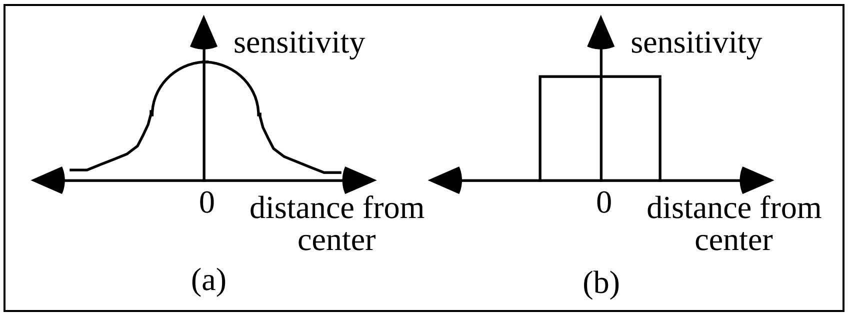

Figure 2.9: The resolution of an actual (a) and ideal sensor (b).

Linear system theory provides the mathematics for comparing and combining linear filters. For example, image restauration attempts to eliminate the effect of a filter by using an approximation of its inverse [Castleman, 1979]. The same concepts can be applied to estimate the image of an ideal sensor from an image perceived by an actual sensor.

In discrete models, resolution is usually expressed in terms of a minimal size of geometric entities. A prime example of this concept is raster resolution, or more generally, the resolution of regular tesselations of space, that is measured by the size of cells. A similar concept has been applied to irregular tessellations [Tobler, 1988]. The shortcoming of these concepts is their limitation to a single tessellation of space. A geometric resolution concept that is free of such limitation is the instantaneous field of view (IFOV) used in remote sensing [Davis, 1991]. It is the smallest area that can be resolved by a sensor cell and variation within this area is unaccessible to the sensor.

An instantaneous field of view can be seen as a special case of a linear filter, namely that of an ideal sensor [Davis, 1991]. This relation can be used to interface between the resolution concepts of continuous and discrete models.

This thesis uses the resolution concepts from both domains, i.e., IFOVs and linear filters. IFOVs are assumed to be disks of constant size. They form the components of resolution-limited space that is used instead of Euclidean space. The linear filter concept is used during the change of resolution in transformations (see appendix B). For compatibility with the concept of IFOV, only filters of ideal sensors are considered.

2.3.3 Models in Remote Sensing

This thesis uses a discrete model of geographic reality that is comparable to discrete scene models in remote sensing. Besides documenting the practical importance of discrete models of reality, this section reviews the terms H- and L-resolution that are defined for these models.

Discrete scene models are commonly used in remote sensing [Strahler, 1986]. Examples of discrete entities used in such models are leaves, branches, plants, crop rows, trees, fields, stands, lawns, cars, streets, gardens, buildings, runways, etc. [Strahler, 1986]. Strahler, Woodcock, and Smith state that implicitly, remote sensing "classification is nearly always based on an H-resolution model" that is a special case of a discrete scene model. They give several examples of supervised and unsupervised classification methods as evidence. They further discuss a large number of works on canopy models as examples of explicitly modeled discrete scene models. Examples of the use of a discreet scene model for theoretical considerations in remote sensing are [Jupp, 1988] and [Jupp, 1989].

Strahler, Woodcock, and Smith [1986] propose the distinction of two different kinds of scene models: H- and L-resolution scene models. This distinction is also used in this thesis. "An H-resolution model is defined as one in which the elements in the scene are larger than the resolution of cells; the L-resolution model presents the opposite case" [Strahler, 1986]. In the context of this thesis, "elements" are the individual discrete objects that inhabit geographic reality and "the resolution of cells" translates to "the size of the instantaneous field of view (or disks or resolution-limited space)" (see chapter four). The concept is illustrated in figure 2.10. The individual discrete objects of geographic reality are shown in gray[14]; the disks that correspond to the instantaneous field of view of the sensor (and thus resolution) are visualized by their boundary circles.

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14 While this thesis requires objects of geographic reality to partition space, they are shown separated for easier visualization.

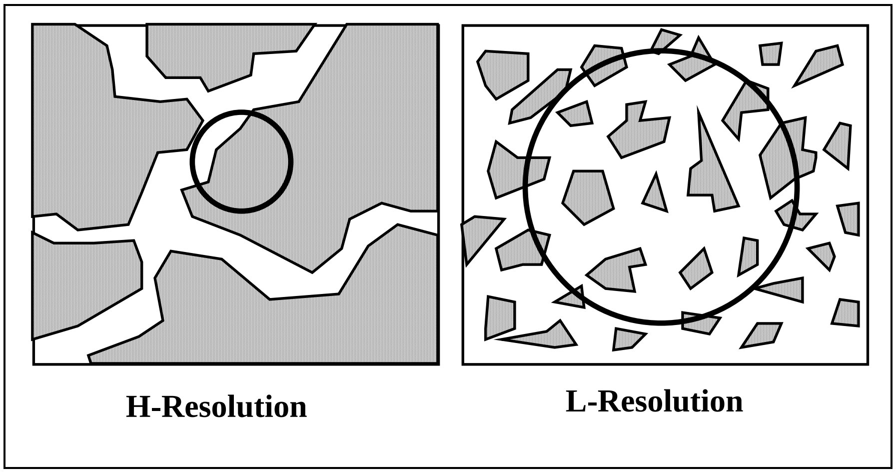

Figure 2.10: H- and L-resolution.

2.3.4 Cognitive Research

This section reviews recent work on spatial cognition since it shows a striking structural similarity to the presented spatial theory and therefore provides the definitions for the terms "object" and "feature".

A cognitive approach to spatial modeling and reasoning has increasingly gained importance in the GIS community [Couclelis, 1992] [Mark, 1989c]. A thorough understanding of how humans perceive and reason about space is a prerequisite for the design of user-friendly GIS, where computer formalisms support human reasoning.

Cognitive research distinguishes four different spaces to express the "scale" at which the world is perceived [Mark, 1989c] [Couclelis, 1992]. Coulelis describes that objects of A- or B-space and features of C- and D-space differ considerably in their properties:

A- and B-space contain **objects**. Typical examples are pens, cups, tables, buildings, etc. They are relatively small compared to the resolution of visual perception. A- and B-space objects are the prototypes for the object concept. Their major properties are homogeneity and sharp boundaries. Both these properties are relevant when objects are physically moved as a whole. Administrative entities[15] such as parcels, census tracts, districts, or states are modeled after objects. They thus have the properties typical for A- and B- space although they can be large compared to visual resolution. Also engineering

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15 or more generally everything that can be "owned"

artifacts are A- and B-space objects since they are homogeneous, have sharp bounaries, and often can be moved as a whole. Objects are seen as having an identity in their own right, independently of purpose and context. Identifiers are thus typical descriptors for objects.

C- and D-space contain **features**. Examples of such features are forests, swamps, etc. They are large compared to the resolution of visual perception and cannot be moved. Typically, features contain a fair amount of inhomogeneities and are separated by fuzzy boundaries or gradual transitions. Features are usually defined in terms of observable properties [Couclelis, 1992]. The definition of features is highly dependent on the characteristics of perception such as resolution and the choice of observable properties. Feature identity is therefore very limited since it is likely a given feature does not exist in a changed context.

This thesis adopts Couclelis' concepts of objects and features. The proposed spatial theory is structurally similar to Couclelis' model since the proposed model of geograpic reality has the properties of A- and B- space while resolution-limited models are closely related to C- and D-space. In the proposed spatial theory, the discrete entities of geographic reality are therefore called objects while features are the entities of resolution-limited models. While objects are homogeneous with a sharply defined geometry, features always contain a certain degree of inhomogeneity and their boundaries are fuzzy[16]. The characteristics of perception on which the definition of features depends is formalized by the concepts of "resolution" and "level of homogeneity".

The identity of objects may or may not propagate from geographic reality to features at limited resolution. In the case where objects are large compared to resoluton (i.e., H-resolution), the geometry of these objects is considered a feature that varies with context. Such features can obviously "inherit" the identifier from the associated object. In the L-resolution case, individual objects are too small to be resolved. Features then become areas that are relatively homogeneous in the distribution of contained object classes. Obviously, they cannot be described by an identifier of an object. Also, features at different levels of resolution cannot be related by a common identifier.

Figure 2.11 illustrates the concepts of how H- and L-resolution features relate to objects of geographic reality. The H-resolution feature is obviously a smoothed version of the infinitely complex geometry of the related object. For simplicity, the possible absorption of inhomogeneities and change of topology have been omited in the figure. The L-resolution feature represents an area that contains objects of the same class. For easier visualization, objects are simply shown as disks and inhomogeneity that is typical for such features is neglected.

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16 Such fuzzy boundaries are represented by an areal "transition zone" in the proposed spatial theory.

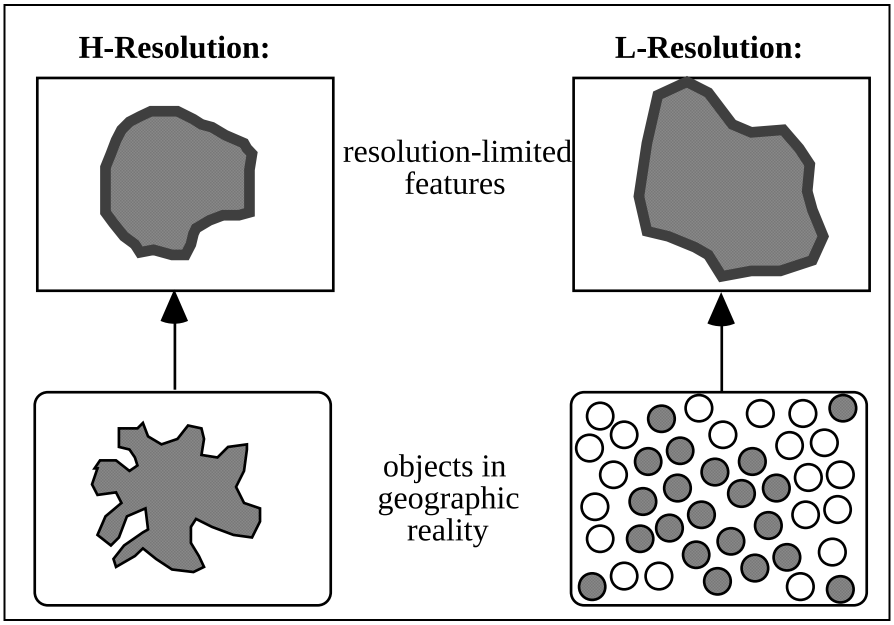

Figure 2.11: H- and L-Resolution features.

Note that an L-resolution feature has no similarities to an aggregation of objects: The feature geometry is based on the mixtures of objects contained in IFOVs. This mixture changes continuously with the location of the IFOV. The definition of the feature geometry based on this mixture continuum is obviously incomparable to an aggregation of discrete entities.

The selection of entities[17] is often used as a generalization operation in the literature (see for example [Brassel, 1988] [Beard, 1988] [McMaster, 1988]). Entities that are not selected are then often dissolved by subdividing the area they occupy and absorbing the resulting segments in adjacent objects (see for example [Monmonier, 1983]). This procedure is incompatible with the spatial theory proposed in this thesis: One problem is that the limited-resolution geometry is determined by a purely geometric method rather than being defined in terms of observable properties or theme (see Sinton in section 2.3.1). Further, the selection is often applied to features rather than objects, for example, in the case of generalizing a landuse map that contains only features. In this case, it cannot be assumed that feature identity is preserved at a coarser level of resolution, i.e., the coarser features cannot be defined in terms of more detailed ones.

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17 Here, the term entities includes objects and features. The generalization literature usually does not distinguish between the two concepts.

2.3.5 Finitization[18] of the Spatial Domain and Data Formats

Recent research has explicitly addressed the problem that spatial models are inherently infinite and cannot be directly represented in finite computers. For example, Smith [1992] designed his logic-based "Term Definition Language" to support the definition of infinite spatial domains such as point sets, and Goodchild [1990] studied different discretizations of an infinite geographic reality that result in different finite data models.

This thesis follows Goodchild's example of capturing the infinite spatial domain in the form of a (non-representable) model of geographic reality and explicitly describing the finitization process that results in finite representations (or data models). The approach of this thesis differs from Goodchild's in using the limitation of resolution (i.e., the scaling process) as a major component of the overall finitization process, while Goodchild's work does not specify a comparable mechanism. Due to this difference, Goodchild's finite data models [Goodchild, 1993a] are not directly comparable to the ones proposed in this thesis.

Goodchild's model of geographic reality is a Euclidean space where every point (x,y) is described by a vector of properties $\{z_1, z_2, ..., z_n\}$. This model obviously contains infinitely many entities (i.e., points) and is format-independent, i.e., neither raster nor vector. Goodchild [1993a] proposes different kinds of discretization methods (or finitizations) that result in different representations that are all formatted, i.e., either raster or vector. While the finitization process used in this thesis results in different, not directly comparable representations, the thesis uses the concept implied by Goodchild's work that infinite models of the spatial domain can be format-independent, while different kinds of finitization necessarily introduce formats.

2.3.6 Abstraction Mechanisms

The model of geographic reality used in this thesis is inhabited by objects. The objects that describe the highest level of detail are atomic, i.e., not further dividable. They are therefore called atoms. For most purposes, the level of detail of an atomic view is extensively high. Abstraction mechanisms known from object oriented modeling [Peckham, 1988] [Brodie, 1984] are therefore used to define higher level objects from atoms and other lower level objects. These abstraction mechanisms are reviewed in this section.

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18 Note that Goodchild uses the term "discretization" rather than "finitization". I prefer the latter term since discrete models can have infinitely many states (for example, an algebraic model of real numbers).

Abstraction mechanisms define many:1 relationships between several lower-level entities and a single higher-level one. Entities here can be either objects or object classes. In the former case, the many:1 relation can be used to define object hierarchies. Four different kinds of abstraction mechanisms are distinguished: "classification", "generalization", "association", and "aggregation" [Peckham, 1988] [Brodie, 1984].

"**Classification** is a form of abstraction in which a collection of objects is considered as a higher level object class. An object class is a precise characterization of all properties shared by each object in the collection. An object is an instance of an object class if it has the properties defined for the class. Classification represents an 'instance-of' relationship between an object class [...] and an object [...]" [Brodie, 1984]. It thus defines a many:1 relationship between many entities of type 'object' and an entity of type 'class'. The criterion for an object to be a member of a class is to exhibit class behavior. Behavior is described in the form of methods and properties that describe the current state of an object. In GISs, classification is frequently based on properties. For example, spatial units can be classified based on their elevation or slope into elevation or slope classes. Note that most programming languages limit the support of classification to that based on methods[19]. In contrast, this thesis uses classification in its most general definition.

"**Generalization** is a form of abstraction in which a relationship between category objects is considered a higher level generic object. This is the 'is-a' relationship" [Brodie, 1984]. Generalization therefore defines a many:1 relationship between many subclasses and a single superclass. All related entities are thus of type 'class'. All subclasses share the class behavior of their superclass. An example of the use of generalization in GISs is the mapping of the classes 'corn fields', 'potato fields', etc. to the class of 'agricultural areas'.

"**Association** is a form of abstraction in which a relationship between member objects is considered as a higher level set object. This is the 'member-of' relationship" [Brodie, 1984]. Association defines a many:1 relationship between entities of type 'object'. The definition of association puts no restrictions whatsoever on the criteria for membership; it could be expressed in terms of a predicate or come from outside the system [Peckham, 1988]. An example of association is the grouping of all instances of the class 'agricultural area' to a

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19 Examples for programming languages that support only method-based classification/generalization are Pascal, C, Eiffel, etc. The reason for this restriction seems to be that only method-based classification/generalization can be supported at compile time while property-based classification/generalization would require run-time support. While method- and property-based classification/generalization is indistinguishable at a conceptual level, they require different mechanisms in a programming language. Note that fields other than programming languages frequently use property based generalization. Examples are climatic classes that are defined in terms of properties such as precipitation, temperature, etc.

single object. Note that in this example, the criterion for membership is defined by a classification.

"**Aggregation** is a form of abstraction in which a relationship between component objects is considered as a higher level aggregate object. This is a 'part-of' relationship" [Brodie, 1984]. Like association, aggregation defines a many:1 relationship between entities of type 'object'. Aggregation is normally used to form composed objects, i.e., objects which consist of several other objects [Smith, 1977]. While an association can be interpreted as an unstructured set of member objects, the set of component objects in an aggregation can be seen to be structured by "labels" which define the components' function in the aggregate object. For example, the aggregate object "car" is composed of wheels, a body, an engine, etc. Each of these components carries a label such as "front-left-wheel", etc.

The behavior of entities related by abstraction mechanisms are often related. "Inheritance" of superclass behavior to subclasses is the best known example. More important for this thesis is **behavior propagation** defined by aggregations and associations [Egenhofer, 1989a]. In propagation, new methods for the higher-level objects are defined in terms of methods and properties of the related lower-level objects. Then, object properties observed by higher-level methods depend fully on the properties of related lower-level objects. For example, the method "mass" of a higher level physical object returns the sum of the masses of all component objects at the lower level. Higher-level behavior is often a statistical measure of the behavior of component or member objects. Examples of such statistics are sums, averages, minima, maxima, and densities.

3 Overview of Approach and Spatial Theory

This chapter gives an overview of approach and model components. The first section discusses modeling in the infinite domain since it is the basis for format-independency. The second section proposes to compose spatial representations from three components. Possible choices for these components and their properties are discussed. This framework is used in section three to define the representations proposed by this thesis. Section four shows how representations and their components are related by abstraction mechanisms. This outlines the structure of how different parts of this work fit together. The abstraction concept is then used in section five to describe the approach of how representations, meta data, transformations, and uncertainty model are defined. The section shows how the approach integrates these model components in a consistent spatial theory. Preserving the relation between attributes and geometry in representations is a crucial consistency requirement. Based on the concepts described in the previous sections, section six shows how this consistency is strictly enforced in the proposed spatial theory.

3.1 Modeling in the Infinite

A major characteristic of the described modeling effort is the use of infinite models. This section compares modeling in the infinite to current GIS modeling practice. In particular, it compares modeling in the infinite to modeling in (finite) spatial data models (or high level spatial data structures) [Egenhofer, 1991]. This allows avoidance of a data model dependent definition of concepts. In this thesis, **finite format** or simply format is used as a synonym of **data model**.

Modeling attempts to describe certain aspects of the world in a formal, and therefore precise language. Conceptual modeling uses languages that are independent of specific hardware and software configurations. Examples of such languages are found in the field of software specification [Liskov, 1986] [Thomas, 1988] [Horebeek, 1989] [Woodcock, 1988]. Applied to spatial knowledge, conceptual modeling is usually related to spatial data models (or high-level spatial data structures) [Egenhofer, 1991] that are unaffected by implementation issues. Spatial data models are explicitly required to be finite for computer implementation [Egenhofer, 1991].

In contrast, the spatial domain is inherently infinite [Smith, 1992] and finitization results in formatted data models [Goodchild, 1990] (see also section 2.3.5). Data formats such as raster and vector can be considered low level artifacts, similarly to implementation issues. In order to exclude low level influences and be format-independent, this thesis therefore initially designs all models in an infinite domain.

Spatial modeling in the infinite is not as radically new as it may seem. Rather, the design of finite data models has always relied on Euclidean geometry as a conceptual basis. Examples of spatial modeling work that explicitly uses the infinite domain of Euclidean space are Goodchild's geographic reality [Goodchild, 1990], and Egenhofer's point-set topological relations [Egenhofer, 1993]. Also, the design of format conversions has always relied on Euclidean space to embed the finite domains of raster and vector models.

The use of "off the shelf" mathematics such as Euclidean geometry has been seen as the underlying mathematical basis rather than modeling in the infinite. This thesis uses an infinite model (i.e., resolution-limited space) that is similar to Euclidean space but not commonly used in mathematical geometry. While this approach definitely differs from most previous spatial modeling, the use of an infinite modeling domain is not new.

A fair deal of spatial modeling work is performed in the finite domain of a spatial data model such as raster or vector. Examples are vector line generalization methods (see [Buttenfield, 1989b] for a bibliography) that are expressed in terms of the entities of a finite model. Another example is Goodchild's error model that is defined in the raster domain [Goodchild, 1992a]. Models that are developed in a finite domain are restricted to the format of their data models. In contrast, this thesis models in the infinite and explicitly includes different finitizations in its modeling effort. This guarantees that concepts defined in the infinite domain are compatible across different finite domains. For example, the resolution concept defined in raster is compatible with its vector counter part.

The computer implementation of an infinite model requires two major steps. The first finitizes the infinite model to a finite data model. The second step implements this data model in a low-level data structure, which is common practice in modern GIS development [Egenhofer, 1991]. This thesis describes infinite spatial models in chapters four and five, and describes different finitization options in chapter six.

3.2 Components of Spatial Representations

In the context of this thesis, representations of spatial knowledge can be seen as consisting of three components (see figure 3.1): (i) a model of space that defines **geometric** entities, (ii) a model of **geographic** content that defines (geographic) entities that inhabit the world, and (iii) relations between geographic objects and geometric entities. This section discusses different choices and characteristics of these three components. Section 3.4 will use the framework introduced here for the definition of the representations proposed in this thesis.

Figure 3.1 visualizes the framework of spatial representations. The entities defined by the models of space and geographic content are shown as disks. A set of relations links certain (but generally not all) geometric entities with geographic entities.

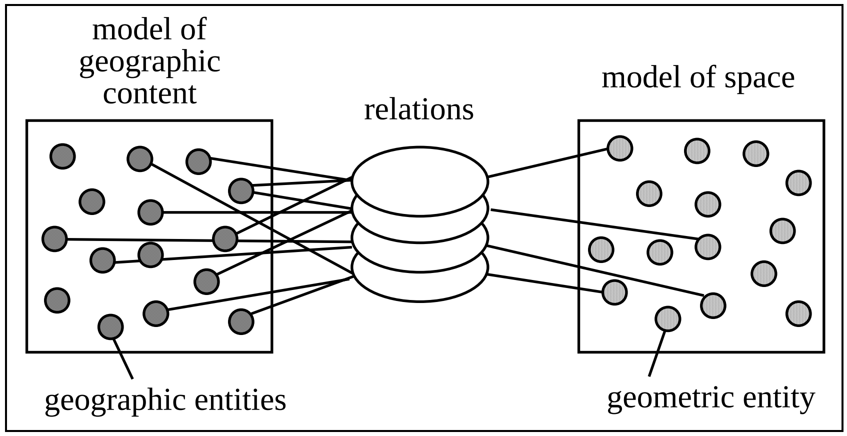

Figure 3.1: The three components of a spatial representation.

The goal of **models of space** is the definition of geometric entities and operations on them. Examples of geometric entities are points, point sets, raster cells, raster zones, etc. Operations include the distance between two points, the area of a point sets, and a test for contiguity of a raster zone. Models of space thus describe purely geometric aspects without dealing with the geographic world itself. Examples of models of space are Euclidean space, regular tessellations such as raster models [Peuquet, 1984], and irregular tessellations such as simplicial complexes [Egenhofer, 1989b]. This thesis proposes an alternative model of space called "resolution-limited space". It is similar to Euclidean space but is composed of infinitely many disks of constant radius rather than Euclidean points.

The geometric entities defined by models of space are either atomic or sets of atomic entities. Examples of atomic entities are points in Euclidean space, cells in raster models, or simplices in simplicial complices. Sets of such entities are then point sets, raster zones, and simplicial complices, respectively. Since resolution-limited space uses disks of constant radius as atomic entities, it similarly defines sets of disks that are called "regions".

Models of geographic content define the geographic entities that can inhabit geometric space and their behavior. Geographic entities can be either objects or features (see section 2.3.4 for a definition of these terms). Examples of geographic objects are trees and background that are used in remote sensing scene models for forestry [Jupp, 1988], or land parcels in cadastral applications. Examples of geographic features are landuse classes such as "residential", "agricultural", and "industrial". The choice of geographic entities is obviously highly application dependent.

In case of using objects (rather than features), geographic content can be defined at multiple levels of abstraction[20]. The objects that compose the lowest level of abstraction are called "atomic objects" or simply "atoms". Higher-levels of abstraction describe geographic content in terms of higher-level objects that are defined by abstraction mechanisms (see section 2.3.6) in terms of atoms. Models that use multiple-levels of abstraction are sometimes called "nested models" [Strahler, 1986]. While atoms are usually considered to be mutually exclusive, higher-level objects can be allowed to share atoms.

The behavior of geographic entities can either be described by a simple nominal value or by a set of methods and properties as is common in an object-oriented approach [Meyer, 1988]. Examples of a nominal behavior description of objects are the identification numbers of land parcels or the vegetation type of features in area class maps [Mark, 1989a]. Examples of objects with an object-oriented behavior description are "land parcel" with methods such as "split", "neighbor", etc. and properties such as "identification number", "owner", "value", "area", etc. Further, the properties of vegetation features can include identifiers such as "spruce forest", "grassland", etc. (that are comparable to a single nominal value) and additional properties such as "associated albedo range", "susceptibility to wild fire", or "suitability potential for urban development". Since behavior description with a single nominal value is a special case of the object-oriented approach, this thesis will use the latter approach.

Relations between geographic and geometric entities describe how geographic entities inhabit geometric space. Two kinds of relations are possible: (i) those that relate every atomic geometric entity in space to geographic entities, and (ii) those that relate selected sets of atomic geometric entities to geographic entities.

(i) The former kind of relations describe a view where every location in space is inspectable and inspection determines the (potentially several) geographic entities found. For example, in a nominal field, every point (x,y) of Euclidean space is related to a single geographic entity that is identified by a nominal property value [Goodchild, 1990].

(ii) In the second case, only a subset of all possible sets of atomic geometric entities are related to geographic entities. According to Sinton (see section 2.3.1), this subset can either be defined (1) arbitrarily in the geometric domain, or (2) derived as the geometries of geographic entities from the domain of geographic content[21]. An example of case (1) are raster cells (defined as point sets in Euclidean space). An example of case (2) are the geometries of objects such as "parcels" or features such as "forest".

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20 Abstraction mechanisms are incompatible with the feature concept since they rely on the universal identity of objects that exists independently of levels of abstraction. Features in contrast are defined in terms of resolution-dependent observable properties and their identity is therefore restricted to a single level of abstraction.

21 Called "thematic" domain by Sinton.

Relations between geometric and geographic entities can assume different formats depending on the cases distinguished above and the number of geographic entities attached to a relation. The different possibilities are described in the following:

If a geometric entity is atomic and attaches to only a **single** geographic entity, the relation takes the form of **containment**. Such a relation expresses that the geometric entity is contained in the geographic one, or inversely, that the geographic entity contains the geometric one. For example, in the model of geographic reality proposed in this thesis where atoms are mutually exclusive, Euclidean points relate to atoms by containment.

If relations link atomic or arbitrary non-atomic geometric entities with (generally) **several** geographic entities, they take the form of **mixtures**. The concept of mixtures is similar to probability vectors [Goodchild, 1992a] (see also section 2.2.1). They describe what percentage of the geometric entity's area each geographic entity covers. For example, if the model of geographic content contains the two entities "land" and "water", a possible mixture would be (21.7% land, 78.3% water). In case of non-atomic arbitrary geometric entities, relations of type "mixture" are related to those of type "containment" since the area covered by a geographic entity is usually determined by an integral over all atomic geometric entities that are "contained" in the geographic entity.

The third kind of relation links a **single** geographic entity to its **geometry** which is a non-atomic geometric entity. Relations of type "geometry" are related to those of type "containment", since every atomic geometric entity contained in the "geometry" of a geographic entity is "contained" in this geographic entity.

3.3 Overview of Representations

This section uses the framework of the previous section to define the representations used in this thesis. These representations differ in their choices of model of space and geographic content, and their kinds of relations. The following defines the representations "geographic reality", "mixture field", "feature partition", and "stand-alone feature" (see chapter four for more detail and appendix C for an algebraic specification):

Geographic reality is a representation that uses Euclidean space and a multi-abstraction model of geographic content called "**content of geographic reality**". The geometric entities are points and point sets; and the geographic entities are atoms and higher-level objects. Relations of type "containment" link every point in space with a single atom. Relations of type "mixture" and "geometry" can be derived from these basic relations. Since each point is contained in a single atom, the atom geometries partition Euclidean space. Figure 3.2 illustrates the model of geographic reality.

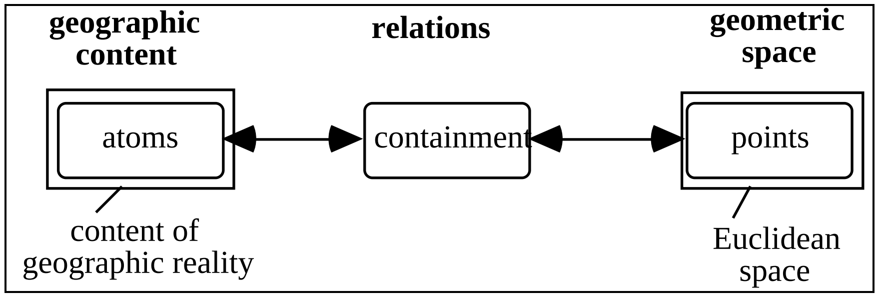

Figure 3.2: Basic relations in geographic reality.

Mixture fields use a Euclidean model of space. Their model of geographic content is a subset of that of geographic reality: It only contains a **subset of higher-level objects** that form a **partition** of Euclidean space. This model of geographic content is therefore called "**object partition**". The relations of mixture fields link point sets that are **disks** of a given, constant size to the geographic objects. The relations thus are **mixtures**. The combination of a geometric disk and a mixture can be seen as a disk-shaped **instantaneous field of view** (IFOV) of an imaginory sensor that can directly observe mixtures[22]. Note that in mixture fields, the mixture changes continuously with location of the IFOV (or disk). Mixture fields are therefore continuous models[23]. Figure 3.3 illustrates the structure of mixture fields.

An equivalent but more compact representation of mixture fields uses **resolution-limited space** rather than Euclidean space (see section 4.2 for detail). In resolution-limited space, disks are the atomic geometric entities. Resolution-limited space thus defines only a subset of the Euclidean geometric entities. The atomic entities of this subset are exactly those entities that are used by the relations of mixture fields.

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22 Such an imaginary sensors can be implemented with an actual sensors and a module that classifies the vector of observable properties to a mixture of geographic entities.

23 To clarify the claim of mixture fields being continuous, the following compares them to elevation models where every point in space is described by an elevation: Such elevation models contain infinitely many discrete entities, namely points, and are comparable to IFOVs in mixture fields. The property of points, namely elevation, changes continuously with location and is comparable to the mixtures of IFOVs.

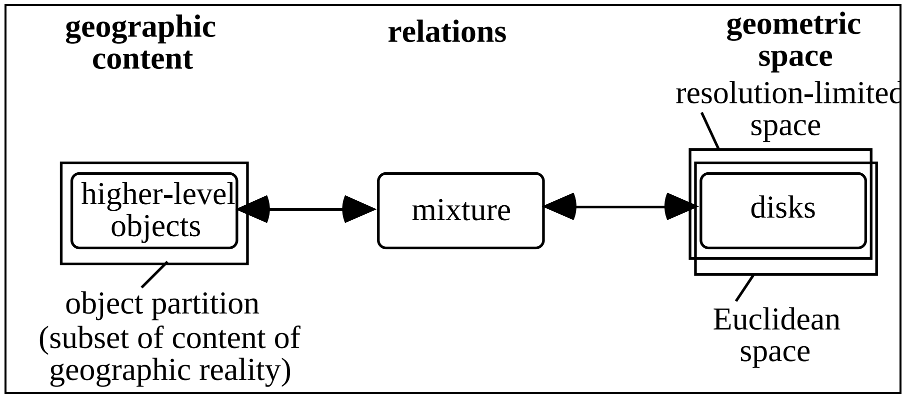

Figure 3.3: Mixture field.

Feature partitions are representations that use resolution-limited space as models of space and "**mixture partitions**" as models of geographic content. Mixture partitions consist of sets of mixtures called "**mixture classes**". A mixture class is the defining (non-spatial) attribute of a feature. The mixture classes of a mixture partition form a partition of the set of all possible mixtures. In consequence, also the geometry of the related features forms a partition of geometric space. Examples of mixture classes are defined by the mixture classes [more than 80% water], [between 80% and 20% water/land], and [more than 80% land]. They are comparable to the categories "water", "transition zone"[24], and "land" in scaled maps where small inhomogeneities in the theme can be absorbed. The relations of feature partitions link mixture classes to their **geometries**. In resolution-limited space, these geometries are sets of disks whose mixtures belong to the same mixture class. Sets of disks rather than point sets are used since disks, not points, are described by a mixture. While mixture fields were continuous models, feature partitions are discrete models. Figure 3.4 visualizes the structure of feature partitions.

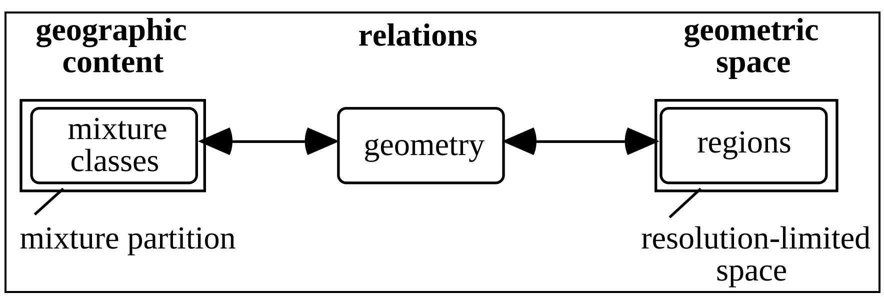

Figure 3.4: Feature partition.

In the infinite domain, **stand-alone features** are special cases of feature partitions (see chapter four). They are treated as entities of their own since in the domain of finite representations, they significantly differ from the partitioning features of feature

24 In the context of maps, the transition zone is equivalent to feature boundaries.

partitions. The major difference between partitioning and stand-alone features are their approaches to modeling the uncertainty introduced by finite approximation[25] (see chapter five for detail). In the context of the discussion in this chapter, stand-alone features are parts of feature partitions that use a mixture partition with only two mixture classes, namely that composed of mixtures containing a non-zero percentage of a certain geographic object, and the class of mixtures that does not contain this object at all.

3.4 Reduction of Knowledge Content

The representations used in the proposed spatial theory are related by an abstraction process. This process reduces the content of represented knowledge from geographic reality to more abstract representations. Since knowledge content is an important concept for fitting representations, meta data, and transformations into a single consistent theory, the related abstraction process is discussed in this section and visualized in figure 3.5.

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[25] The uncertainty model used for feature partitions is designed to only represent the knowledge that is certain. This characterizes a worst-case approach. Stand-alone features are designed for the representation of relatively small or thin objects that would disappear (be absorbed) at low resolution in ordinary feature partitions. In order to preserve the knowledge about such small features at these low resolutions, a best- (rather than a worst-) case approach to uncertainty modeling has to be used.

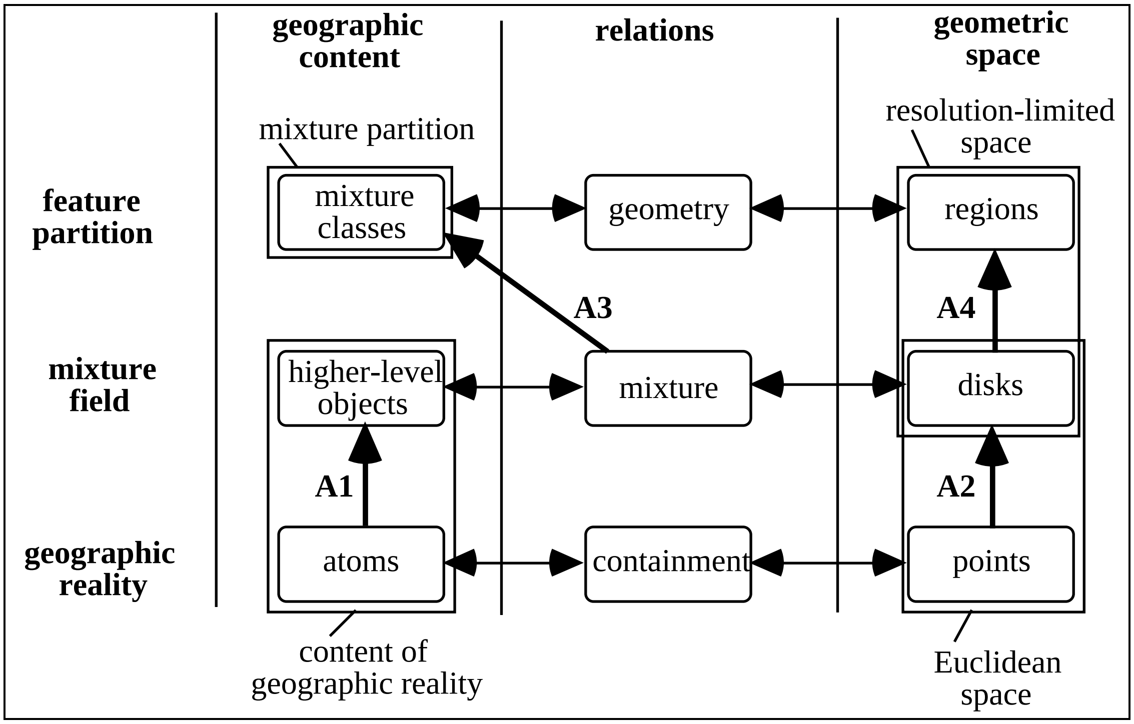

Figure 3.5: Abstraction in the proposed spatial theory.

The highest level of detail is represented by **geographic reality.** Here, geographic content is described by atoms and higher-level objects. Their spatial distribution is known "point sharp" since it is expressed by basic relations that link every point of Euclidean space to the atom that contains this point.

Mixture fields are derived from geographic reality by the abstraction mechanisms A1 and A2. A1 defines higher-level objects by aggregation and/or association of atoms. While higher-level objects already exist in geographic reality, the reduction of knowledge content is caused by "forgetting" atoms and other lower-level objects and relating only the remaining higher-level objects to geometric entities[26]. A2 abstracts in the geometric domain by defining disks as associations of points. Note that disks are entities of both, Euclidean and resolution-limited space.

The reduced knowledge content of mixture fields is evident in several aspects: Both, the number of geometric entities and geographic objects are reduced compared to geographic reality. While in geographic reality, an unlimited number of relations between arbitrary point sets and geographic objects can be derived from the basic relations, mixture fields contain only a largely reduced number of possible relations. These reductions of entities

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26 This argument is based on the philosophy that the derivation of knowledge from represented (basic) knowledge does not increase the knowledge content of a representation. Methods used for such derivations are thus not treated as knowledge. Instead, the whole set of possible methods is assumed to be available at all times.

result in "forgetting" the "point sharp" distribution of geographic objects. For example, the point sharp geometry of an object cannot be derived from knowledge contained in a mixture field.

Feature partitions are derived from mixture fields by the abstraction mechanisms A3 and A4. A3 creates mixture classes from individual mixture values. It can be seen as an association of mixture values that is defined by a classification based on mixture properties (see section 4.1 for detail). While mixture fields distinguish individual mixtures, only mixture classes are distinguished at the level of feature partitions; and individual mixtures of the same class are considered to be equivalent. Abstraction mechanism A4 is an association of disks to regions, i.e., sets of disks. The relations of feature partitions link mixture classes to their geometries. These geometries are a small subset of all possible regions (i.e., sets of disks) of resolution-limited space. Again, the reduction of information content is evident in a decrease of geometric and geographic entities, as well as relations between them.

3.5 Approach to Consistent Spatial Theory

This section describes how representations, meta data, and transformations are integrated in a consistent spatial theory. The argument is based on the abstraction process introduced in the previous section. The first part of this section discusses consistency in the context of infinite models where no uncertainty caused by finite approximation is present. The second part describes the extension that allows the representation of such uncertainty.

Figure 3.6 visualizes the situation of certain, infinite models. R1, R2, and R3 are three different resolution-limited representations, i.e., either mixture fields or feature partitions. A1, A2, and A3 are the abstraction processes that map geographic reality in the corresponding representation (see previous section). The level of abstraction is defined by the parameters of the abstraction process. Among others, these parameters include the resolution, i.e., size of disks, and the representation type (i.e., mixture field or feature partition).

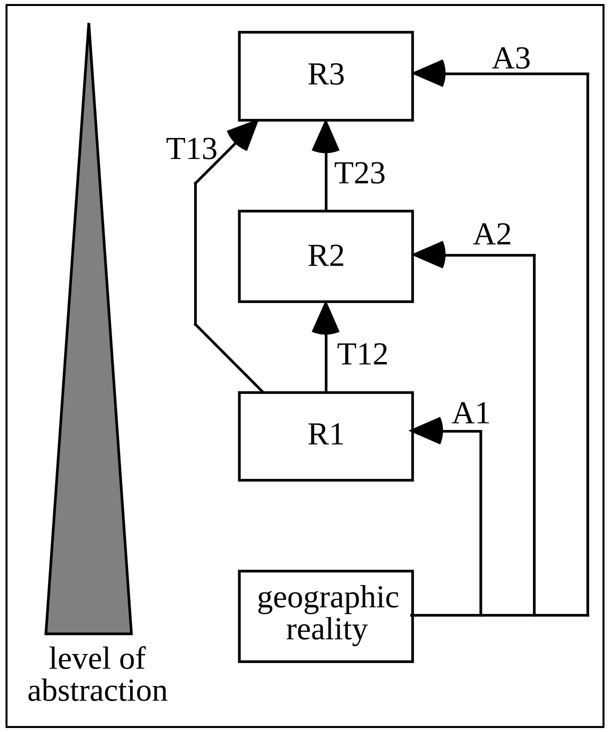

Figure 3.6: Approach to a consistent spatial theory.

In this thesis, **representations** are defined as the result of an abstraction process (see chapter four for a detailed discussion). The abstraction processes used in chapter four can be described by parameters that are used as **meta data** (see chapter eight for an overview of parameters). Since this meta data uniquely determines the abstraction process associated with a representation, it precisely specifies its knowledge content. The complete consistency of meta data and representations is evident from the design of these two components.

Resolution-limited representations are related by transformations such as T12, T23, and T13 in figure 3.6. These transformations are defined to map from the knowledge content of the source representation to that of the target representation. For example, T12 is defined such that T12(A1(geographic reality)) is equal to A2(geographic reality). It is obvious from this definition, that transformations are uniquely determined by the source and target meta data of the related representations.

One problem observed in current GISs is that a series of transformations yields a different result from that of a single transformation that is conceptually equivalent. For example, using a Douglas-Poiker algorithm [Douglas, 1973] to generalize from a "1:25,000" to a "1:100,000 scale" in a single step yields different results than using the same algorithm twice with an intermediate result at a scale of 1:50,000[27]. In contrast, the proposed transformations always yield the same result, no matter whether a single or multi-step approach is taken. This is evident when looking at the transformations T12, T23, and T13 of the above figure: A two-step transformation composed of T12 and T23 is then equivalent to the single-step transformation T13. In the proposed spatial theory, these transformations are defined by the following equations: (i) T12(A1) = A2, (ii) T23(A2) = A3, and (iii) T13(A1) = A3. Using (i) to substitute A2 by T12(A1) in (ii) yields: T23(T12(A1)) = A3. This shows that T23(T12) is really equivalent to T13.

So far the discussion has shown how representations, meta data, and transformations combine to a consistent spatial theory. The remainder of this section discusses how the uncertainty model fits in. In the proposed spatial theory, all uncertainty is caused by finite approximation of the infinite domain. The certain spatial theory is extended to handle uncertainty in three steps: (i) add finite approximation as an additional step of the abstraction process (see chapter five), (ii) modify the certain representations such that they incorporate uncertainty in the entities used for representation (see chapter five), and (iii) modify the definitions of transformations such that they propagate this uncertainty (see chapter eight). It is evident from these steps that the extension with uncertainty modifies components of the certain spatial theory rather than defining additional ones.

The above approach to achieving a consistent theory thus still applies. However, one additional issue deserves attention, namely the **transitive closure** of uncertainty representation under transformations. It does not go without saying that the uncertainty introduced by (finite implementations of) transformations can be completely captured by the uncertainty model of representations. If it cannot, uncertainty has to be represented in the form of lineage data that represents the uncertainty of the initial representation and a history of transformations that this representation was subjected to. Obviously, such uncertainty data increases in volume over time. The longer the history, the more difficult human and machine interpretation of the uncertainty data becomes. Considering these potential difficulties, the uncertainty model in this thesis was defined to be compatible with transformations such that their effect on uncertainty can be completely captured by the proposed representations. This property is sometimes called transitive closure [Gill, 1976]. Transitive closure is achieved by keeping precise track of the effect that transformations have on the knowledge content of target representations.

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[27] More precisely, the Douglas-Poiker algorithm uses the width of strips as control, rather than map scales. The algorithm then finds subchains of points that are completely contained by strips centered on subchain endpoints and limited in width. In the single step generalization, all points of the original chain have to be contained in such strips. In the second step of the two-step procedure, only those points that were not eliminated in the first step have to fit in the strips. Obviously, this can lead to narrower strips for equivalent subchains, or longer subchains that still fit into strips of the same width.

3.6 Approach to Format Integration

This section describes how representation concepts can be designed format-independently and compatible with both raster and vector representations. For this purpose, the **abstraction process** from geographic reality to representations (see previous section) consists of **two steps**. Figure 3.7 shows the situation using the example of a feature partition. The **first step** of the abstraction process is solely concerned with the resolution-limiting **scaling process**. It results in an intermediate representation that can only be described with an infinite number of parameters and thus cannot be implemented in finite computers. The figure illustrates the situation by showing a feature geometry of general shape.

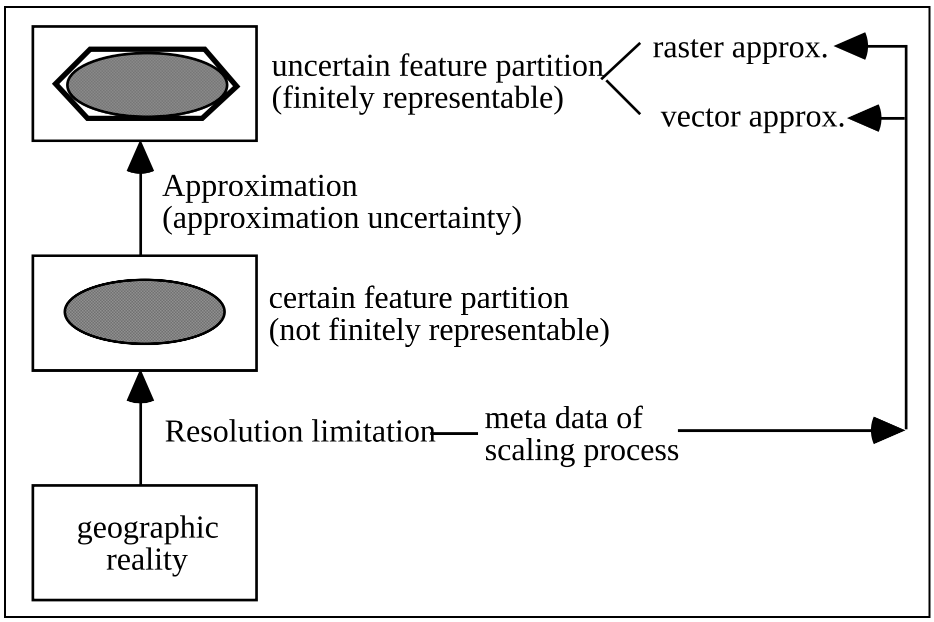

Figure 3.7: Two-step abstraction process to achieve format integration.

The **second step** of the abstraction process approximates the intermediate representation. **Approximation** allows transformation of the general feature geometries to shapes that can be represented by a **finite number of parameters**. The resulting representations are thus suited for computer implementation. Different kinds of approximations are possible that result in different finite parametrizations. The most prominent examples are **raster** and **vector** approximations.

This approximation process introduces **approximation uncertainty**--the only kind of uncertainty modeled in this thesis. Since the intermediate representation lacks such uncertainty, it is called a **certain feature partition**; the result of the approximation is accordingly called **uncertain feature partition**. Certain representations will be discussed in detail in chapter four, uncertain ones in chapter five.

Compatibility between finite formats requires use of the **same concepts** in both, the raster and vector domain. These concepts include entities used for the representation of spatial knowledge (for example, disks) and meta data (for example, resolution). In the proposed spatial theory, all these concepts are defined in the first step of the abstraction process and are unaffected by the abstraction process. They therefore apply equally to raster and vector representations and total compatibility between formats is achieved.

3.7 Preserving the Relation between Geometry and Attibute

The discipline of spatial modeling has become increasingly aware of the dangers of treating geometry and attributes separately [Csillag, 1991]. Sinton [1979] has shown that to preserve the relation between geometry and attribute, one has to be defined in terms of the other (see section 2.3.1). This section describes how Sinton's concepts are strictly applied in this thesis to preserve the relation between geometry and attributes. Namely, two kinds of entities are constructed with Sinton's concepts: instantaneous fields of view in mixture fields, and features in feature partitions (see section 3.3). Further, the relation between geometry and attribute is strictly preserved in approximations that are necessary for finite representation.

Mixture fields are composed of (infinitely many) instantaneous fields of view (IFOV) whose geometries are disks and whose attributes are mixtures such as (21.6% water, 78.4% land). These IFOVs are defined by Sinton's concept where location is controlled and theme measured. Control and measurement is performed in the model of geographic reality. In more detail, the control of location defines the disk shaped geometries of all potentially possible IFOVs[28] in Euclidean space. Potentially, several objects of geographic reality fall partly or completely inside the geometry of each IFOV. This theme is measured by determining the mixture of objects inside the disk.

The second kind of discrete entities constructed by one of Sinton's concepts are **"features"**. Feature geometry consists of a set of disks in resolution-limited space and the attribute of a feature is a mixture class. Here, Sinton's control and measurement are performed in a mixture field. In more detail, the theme is controlled by defining sets of mixtures. The location is then measured by determining the sets of disks whose mixtures fall into the defined mixture class. Note that the geometries of features consist of sets of disks rather than point sets like in previous approaches.

Chapter five describes approximations of mixture fields and feature partitions in order to model physical representations in finite computers. The relationship between geometry and attribute is strictly preserved in the proposed approximation process. This is most

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28 of a single sensor with constant resolution.

evident in feature partitions where a change in the feature geometry due to approximation always goes along with a change of attribute (namely the attribute of the transition zone).

4 Representation of Resolution-Limited Knowledge

This chapter describes in more detail the representations that were outlined in the previous chapter. An algebraic specification of these representations is given in appendix C. The proposed representations are **infinite** models that are format-independent (see sections 3.1 and 2.3.5). The representations are therefore unaffected by uncertainty introduced by finite approximation (that will be discussed in chapter five). The following three representation types are discussed: **geographic reality** (section 4.1), **mixture fields** (section 4.3), and **feature partitions** (section 4.4). In the absence of uncertainty that is assumed in this chapter, **stand-alone features** are the features of a special case of feature partitions[29].

Geographic reality is considered to be infinitely resolved since it represents spatial knowledge "point sharp". In contrast, mixture fields, feature partitions, and stand-alone features are considered resolution-limited since they are based on disks rather than points. **Resolution-limited space** is proposed in this chapter (section 4.2) since it is a major component of resolution-limited representations. Geographic reality serves as source of all knowledge since the resolution-limited representations are derived from it by abstraction processes (see sections 3.4 and 3.5). Since these abstraction processes are inseparable from the representations they result in, abstraction and related representations are discussed together.

The chapter is structured as follows: (the major concepts are shown in bold)

- Section 4.1 describes **geographic reality**. Its components are **geographic objects** that are either **atoms**, **crisp objects**, **class associations**, or **administrative objects**. **Object Partitions** are defined as sets of geographic objects that partition space.
- Section 4.2 defines **resolution-limited space** that will be used in the design of the resolution-limited representations "mixture field" and "feature partition". The geometric entities defined by resolution-limited space are **disks** and **regions**, i.e., sets of disks.
- Section 4.3 defines **mixture fields**. They represent knowledge about the spatial distribution of the geographic objects of an object partition in the form of a continuous model. The representation is based on **mixtures** that measure the percentage of disk area covered by each geographic object in the related object partition. This thesis distinguishes between **classified** and **unclassified** mixture fields.
- Section 4.4 derives the discrete models called **feature partitions** from mixture fields by an abstraction process modeled after Sinton's concepts. Sinton's theme control takes the form of defining sets of mixtures, called **mixture classes**, that define the geometries of **features**. Sets of mixture classes that partition **mixture space** (i.e., the set of all possible mixtures) are called **mixture partitions** and define sets of features that compose **feature partitions**. A special kind of mixture partition can be defined by a single value called **level of**

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29 This is only the case when no uncertainty is present. In chapter 5, feature partitions and stand-alone features will be treated differently.

homogeneity. The **transition zone** is a mixture class that separates all other mixture classes in a mixture partition. **Stand-alone features** are defined as a special case of features.

- Section 4.5 investigates some quantitative properties of mixture fields and feature partitions. For this purpose, it defines a distance function for mixtures called **mixture-distance**. The discussed properties will be used in later chapters of the thesis.

4.1 Model of Geographic Reality

This section describes geographic reality by first illustrating its role in the spatial theory and then discussing its components, namely "atoms", and the higher-level object types "crisp objects", "class associations", and "administrative objects". These higher-level objects are derived from atoms by abstraction mechanisms (see sections 2.3.6 and 3.4). The final section defines "object partitions" that are used later in the definition of mixtures.

4.1.1 Purpose

The model of geographic reality has a central role in the proposed spatial theory and is important for data integration that is based on this theory. It represents the most detailed knowledge about the world available in the spatial theory. Since all resolution-limited representations are derived from it, geographic reality can be seen as the "parent" of all possible representations.

Geographic reality (and parts thereof) are only rarely physically represented in computers. Examples of such representations are remote sensing scene models used for analytical simulations (see, for example, [Jupp, 1988] [Jupp, 1989]) and administrative objects (see section 4.1.5). In the majority of cases, the model of geographic reality is only of theoretical relevance: It is assumed that a geographic reality exists and that all representations are derived from it. How this geographic reality actually looks is irrelevant. The assumption of its existence is sufficient for the definition of meta data and transformations (see section 3.5).

Data integration is based on a notion of how different representations relate to each other. In the proposed theory, these relations are modeled by transformations and determined by meta data. They are based on the assumption, that data sets result from the same geographic reality. Only under this assumption are meta data comparable and transformations meaningful.

4.1.2 Atoms

The smallest granularity in geographic reality is defined by atoms. Atoms are discrete objects that are not further dividable in the spatial theory. In this thesis, they are assumed to be mutually exclusive. Since they are not further dividable, they are considered to be pure (rather than a mixture of smaller entities).

In the model of geographic reality, **atoms** are characterized by the following properties:
- a unique **identifier**,
- their **geometry** or location that is defined by a point set in Euclidean space,
- a certain **behavior** that can be expressed in the form of methods and properties. Part of this behavior consists of **relations** and interactions with other atoms.

The **geometry of atoms** is required to form a **partition**[30] **of Euclidean space**.

This model of atoms is very general and can represent the needs of many applications. The following examples illustrate this:

An application that studies bio-diversity may use point sets that are described by a habitat type as atoms. In order to satisfy the requirements of atoms, the regions must form a partition and for the purposes of the application, the habitat type must be considered absolutely homogeneous in a region.

In a cadastral application, atoms could be land parcels. Higher level objects such as school districts, election districts, census tracts, townships, counties, states, and countries, are all composed from these atoms. The limitations of this model are evident when the change of ownership splits parcels, which is not allowed for atoms since they are considered to be undividable.

A more flexible model of geographic reality could therefore consider every point in space as an atom[31]. This model could support cadastral applications where ownership is expressed in terms of Euclidean geometry (for example, by polygons with known vertex coordinates).

The model of geographic reality proposed by Goodchild [1993a] also uses points as atoms. They carry observable properties that can be represented by a vector $(z_1, z_2, ..., z_n)$. These points can then be grouped to (finitely many) discrete objects based on their observable properties, as is done, for example, in remote sensing classification.

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30 A set X is said to be partitioned by a family of sets $\{S_i\}$ if (i) the sets in this family are mutually disjoint and (ii) the union of the sets in the family is equal to X [Gill, 1976].

31 More precisely, the atom geometry would be a singleton point set.

Jupp, Strahler, and Woodcock [1988] [1989] use individual trees and "background" as atoms of their scene model. While the geometries of trees do not form a partition of space, this requirement is satisfied by using "background" as an atom of its own.

Similarly, a model of geographic reality could use physical atoms and "void" as model atoms. The physical atoms could then have the geometry of disks, in accordance with Bohr's atom model.

These examples illustrate that the choice of model of geographic reality is highly application dependent, and that the concept of geographic reality is general enough to accommodate a wide variety of models.

4.1.3 Crisp Objects

Abstraction mechanisms (see section 2.3.6) are used to define higher-level objects from lower-level ones. The lower-level objects used in abstraction mechanisms can either be atoms or objects defined by abstraction mechanisms indirectly in terms of atoms. This thesis distinguishes three kinds of higher-level objects: crisp objects, class associations, and administrative objects. This section discusses crisp objects.

Crisp objects are **aggregations** (or sometimes associations) of objects that are **based on relations or interactions between component objects**. It is these relations and interactions that are used as "glue" to form a higher-level object from its components. Figure 4.1 illustrates the concept of crisp objects. Lower-level objects are visualized as disks. Relations/interactions between these objects are shown as lines. The components of the crisp object are marked by a black filling.

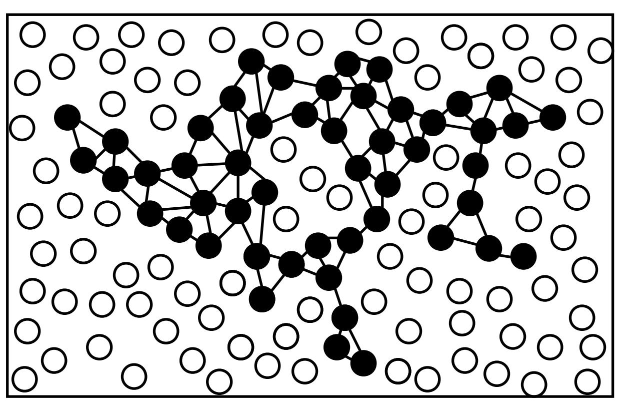

Figure 4.1: Crisp object. The black component objects form a crisp object based on relations/interactions that are visualized by lines.

Examples of crisp objects are houses that are composed of walls, a roof, doors, windows, etc. In this case, the relations between components is one of physical attachment. Another example are trees that are composed of leaves, branches, etc. These components interact, for example, by exchanging liquids and physical attachment. Finally, "Sherwood forest" is a crisp object that consists of trees that interact due to their mutual proximity.

A typical property of crisp objects is their **limited spatial extent**. In contrast, class associations such as the set of all trees usually have an unlimited spatial extent (see section 4.1.3). This limited spatial extent is caused by the fact that relations and interaction often go hand in hand with spatial proximity. Limited spatial extent is a requirement for the concept of geometry of an object.

Due to their character of individual spatially limited entities, crisp objects are typically used for **H-resolution** representations (see section 2.3.3) where the limited-resolution geometry is represented by features (see section 4.4) of the same identifier. Figure 4.2.a illustrates this case by showing a crisp object with its (infinitely resolved) geometry and identification number, and an instantaneous field of view that is used to derive the simplified feature geometry. L-resolution representations usually do not use crisp objects since it is too cumbersome to describe the content of an instantaneous field of view in terms of many individual objects (rather than a mixture of object classes). Figure 4.2.b illustrates this by showing some crisp objects with their identification numbers in relation to an instantaneous field of view.

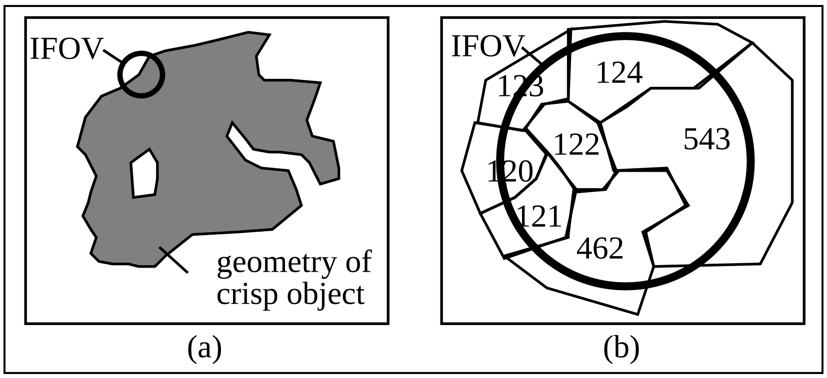

Figure 4.2: H- (a) and L-Resolution (b) representation that uses crisp objects.

4.1.4 Class Associations

A **class association** is an association of all crisp objects that belong to the same class. This association is thus **based on** (class) **behavior** rather than relations between components. Examples of class associations are associations based on the classes of "buildings", "agricultural fields", "trees", "lakes", etc. Figure 4.3

illustrates the concept. The black disks visualize the component objects that belong to a single class.

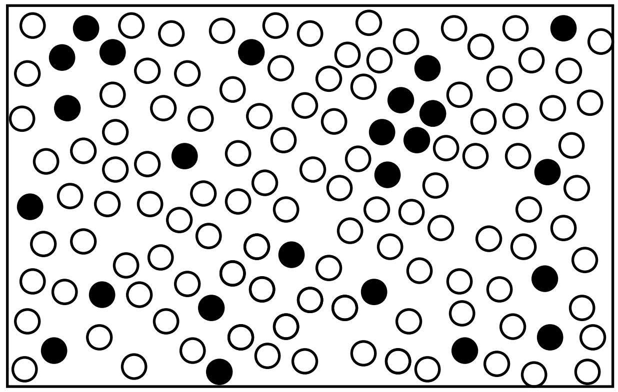

Figure 4.3: Class Association. The black disks visualize objects that belong to the same class. The association of these objects forms the higher-level object of a class association.

While crisp objects correspond to the typical objects of human cognition, class objects are not usually used in cognition and therefore not seen as individual objects. However, in the sense of object-oriented modeling, they are individual objects. The **spatial extent** of class associations is **often unlimited**. It is often not meaningful to speak of the geometry of a class association. This is evident from figure 4.3 above. Class associations are therefore usually not used for H-resolution representations, but are ideally suited for **L-resolution representations**. There, the content of an instantaneous field of view is expressed as a mixture of object classes (or more precisely class associations) rather than individual crisp objects (see figure 4.2.b above).

4.1.5 Administrative Objects

Administrative objects are **associations of atoms based on containment in a defining point set**. Their name is motivated in administrative units such as parcels, districts, counties, or states that are defined by point sets such as polygons. While in the context of this thesis, they behave similar to crisp objects, they are distinguished since they are defined geometrically rather than in terms of non-spatial interactions between components.

If atoms are points, containment in the defining point set is always determined. If atom geometries are larger point sets, containment is not determined for atoms whose geometries only partly intersect the defining point set. In this case, they are contained if their center of gravity lies in the defining point set (see algebraic specification in appendix C). However, applications that use administrative objects usually use points as atoms that avoid the discussed problem.

Administrative objects are a rare examples of entities of geographic reality that can be physically represented in GIS. Most other objects have a geometry that is too complex to be represented on finite machines. Administrative objects are an exception since they are defined by point sets such as polygons that are representable with a small number of parameters. Since this thesis focuses on the effects of limited resolution, the physical representation of administrative objects is not of interest. The generalization of administrative objects to resolution-limited representations is relevant, however. For this purpose, administrative objects can be treated like crisp objects.

4.1.6 Object Partitions

This section shows how higher-level objects in geographic reality can overlap and defines object partitions as subsets of objects that partition space.

The previous three sections have shown different applications of abstraction mechanisms that define higher-level objects. Recursive application of abstraction mechanisms defines object hierarchies. Figure 4.4 shows the possibility of multiple such hierarchies co-existing in geographic reality[32]: The lower row shows a set of lower-level objects. They are the components of higher-level objects shown in the upper row and connected to their components by lines. Although the figure shows only two levels of the hierarchies, it is obvious that multiple, overlapping hierarchies exist. The objects of different hierarchies overlap since they share lower level atoms. This is evident in the figure for adjacent pairs of white and gray higher-level objects.

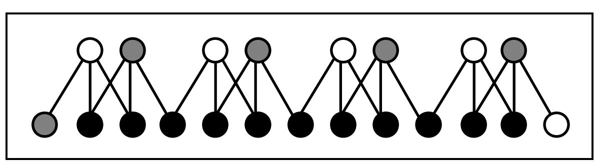

Figure 4.4: Multiple object hierarchies.

Object partitions are subsets of all geographic objects that partition space. This partition is evident in both the geometric and content domain: In the geometric domain, the geometries of objects contained in an object partition form

32 In set theory this would be expressed as a "partial order" [Gill, 1976]. The related theory is not relevant for this thesis, however.

a partition of Euclidean space. Since objects are (indirectly) defined as sets of atoms, an object partition also partitions the set of all atoms.

Figure 4.4 above shows two object partitions, one consisting of the white, the other of the gray disks. Note that object partitions can contain crisp objects, class associations, administrative objects, or a mixture thereof.

4.2 Resolution-Limited Space

Resolution-limited space is a model of space that captures the resolution limitations of sensors. The use of resolution-limited instead of Euclidean space leads to simpler, more compact spatial representations since it provides disks as higher-level entities. This section first discusses how resolution-limitations can be modeled in existing models of space and then introduces resolution-limited space as a more elegant model.

4.2.1 Resolution-Limitation and Existing Models of Space

Sensor resolution can be expressed in terms of their instantaneous field of view (IFOV; see section 2.3.2). The IFOV defines the smallest geometric entity that can be inspected. All larger inspectable entities are composed of IFOVs and have to be observed one IFOV at a time. These geometric entities can therefore be modeled as sets of IFOVs.

This thesis assumes that the relations between geometric entities of mixture fields and geographic entities (see section 3.2) originate from an imaginary sensor that observes geographic reality. The imaginary sensor then directly observes mixtures (see section 4.3.1 for detail). Obviously, this **limits the geometric entities** that can be related to geographic objects of geographic reality. Geometric entities that can potentially be inspected by the imaginary sensor are called **resolvable geometric entities**. For simplicity, this thesis assumes that a representation results from a single sensor whose IFOV is disk-shaped and of constant size.

In Euclidean space, an IFOV of the imaginary sensor is a disk, i.e., a point set. Larger resolvable geometric entities are then sets of disks or sets of point sets. Euclidean space defines an infinite number of point sets and also infinitely many sets of point sets can be constructed. The resolvable entities are only a small subset of all entities. The design of mixture fields and feature partitions later in this chapter requires the availability of the **set of resolvable geometric entities**. In Euclidean space, all these entities have to be constructed from points and **rules** for such construction must be provided. **Geometric relations** between resolvable entities (such as the distance between two disks) must be **expressed in terms of**

points (e.g., as the distance between the center points of the disks) rather than in terms of resolvable entities directly. Further the construct of **sets of sets of points** that is necessary for larger resolvable entities is somewhat complex.

Resolution-limited space, in contrast, defines only a **subset of the Euclidean geometric entities**, namely **those that are resolvable**. This allows the design of simpler and more compact spatial representations, since rules for the construction of resolvable entities are unnecessary, geometric relations can be expressed directly in terms of resolvable entities rather than being expressed in terms of points, and sets of sets of points are replaced by the simpler construct of sets of disks.

Regular tessellations [Peuquet, 1984] also provide only a subset of the Euclidean geometric entities. For example, raster cells are comparable to disks and raster zones to sets of disks. However, regular tessellations only provide a subset of all resolvable geometric entities, namely those whose locations fall on some regular grid. Since sensors are not restricted to these locations, different data sets are bound to be based on different grids which causes problems during data integration. For the design of a spatial theory that serves as a framework for data integration, regular tessellations are therefore unsuitable.

4.2.2 Definition of Resolution Limited Space

This section defines the entities and operations of resolution-limited space. An algebraic specification of resolution-limited space is given in appendix C.

The **atomic geometric entities** of resolution limited space are **disks of the same constant size**. The diameter of these disks is called **resolution** of the space. For every location described by a coordinate pair (x, y)[33], there exists exactly one disk. **Non-atomic entities** of resolution-limited space are **regions** that are arbitrary **sets of disks**. Since all entities are parameterized in the resolution of the space, **different resolutions define different resolution-limited spaces**.

The semantics of resolution-limited geometric entities and their operations are specified by **implementation in Euclidean space** (rather than axiomatically). While the relations of resolution-limited space to Euclidean space can be "forgotten" whenever the scope is limited to a single resolution-limited space (identified by its resolution), they are important when resolution-limited spaces of differing resolution are compared.

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33 x and y are arbitrary real numbers.

Resolution-limited space defines operations on its geometric entities. Examples of such operations are the **distance** between two disks (see section 4.5 for an application), the **medial axis** of a region (see chapters 6 and 7 for applications), and the **boundary** of a region (see chapter six for an application).

To keep the design of resolution-limited space simple, **all operations are defined as analogies to Euclidean operations**--as suggested by the use of the same operation names. The mappings from resolution-limited disks to Euclidean points and vice versa are important components of the implementation of resolution-limited operations in term of Euclidean operations. These two mappings are "centerPoint" (that maps a disk to a point) and "disk (resolution)" (that maps a point to a disk centered on it). These mappings can be applied to single disks and points, as well as sets thereof. For example, the centerPoint of a region (i.e., a set of disks) is the set of all centerPoints of disks contained in the region (i.e., a point set).

Operations in resolution-limited space can then be implemented by the following three-step method: (1) map the arguments from resolution-limited space to Euclidean space by using centerPoint, (2) perform the Euclidean operation with the same name, and (3) if the result of the Euclidean operation is a point or a point set, then this result has to be mapped back to resolution-limited space with "disk (resolution)". For example, the resolution-limited distance between two disks is implemented as follows:

r-l.distance(d1, d2) = Eucl.distance(centerPt(d1), centerPt(d2))

(where "r-l" is the prefix of operations of resolution-limited space, "Eucl" that of Euclidean space, d1 and d2 are disks, and "centerPt" is an abbreviation for "centerPoint").

Similarly, the resolution-limited operation "boundary" is implemented as:

r-l.boundary(region) = disk(resolution) (Eulid.boundary(centerPt(region)))

From an axiomatic point of view, the relation between the algebras of resolution-limited and Euclidean space is described by an isomorphism [Gill, 1976]. Isomorphisms are defined by two bijections[34] between entities and operations of two algebras, respectively. In this thesis, the bijection between entities is "centerPoint" with its inverse "disk(resolution)"; the bijection between operations relates operations of the same name. The isomorphism requires that for every n-

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[34] A mapping from the elements of set A to those of a set B is a bijection if and only if every element of A is mapped to exactly one element of B (a one-to-one mapping) and every element in B is the result of a mapping of an element of A (i.e., A is mapped "onto" B). Bijections always have inverses [Gill, 1976].

ary[35] operation $o_1(e_{11}, e_{12}, ..., e_{1n})$ of resolution-limited space (identified by index 1), an n-ary operation $o_2(e_{21}, e_{22}, ..., e_{2n})$ of Euclidean space (identified by index 2) exists such that

$$cp(o_1(e_{11}, e_{12}, ..., e_{1n})) = o_2(cp(e_{21}), cp(e_{22}), ..., cp(e_{2n}))$$

(where cp is an abbreviation for centerPoint).

This axiomatic specification of the semantics of resolution-limited space is equivalent to the above implementation in Euclidean space.

4.3 Resolution-Limited Mixture Fields

Mixture fields represent all possible knowledge that can be collected with imaginary resolution-limited sensors. This sensor is responsible for the abstraction process and related reduction of knowledge content between geographic reality and mixture fields (see sections 3.4 and 3.5). The first section therefore discusses sensorial perception of geographic reality. The second section uses this to define mixture fields.

Section 4.3.1 discusses real and imaginary sensors, and section 4.3.2 distinguishes unclassified and classified mixture fields. These concepts are related as follows: **Real sensors** perceive **unclassified mixture fields** such as remotely sensed images. A classification process that is based on "mixed pixels" transforms such unclassified mixture fields to classified ones. **Imaginary sensors** are a combination of a real sensor with an automatic classification process. They can therefore directly perceive **classified mixture fields**.

4.3.1 Sensorial Perception of Geographic Reality

The **abstraction process** from geographic reality to mixture fields is modeled by perception with **imaginary sensors**. The instantaneous field of view (IFOV) of such sensors is disk shaped. Imaginary sensors can, among other properties, **directly observe mixtures**. An individual sensor is therefore identified by its **resolution** (i.e., the diameter of the disk) and an **object partition** that specifies the kinds of mixtures it can observe. In every location that is specified by a disk, an imaginary sensor observes the following properties:

- a **mixture** based on the sensor's object partition

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35 An n-ary operation has n arguments.

- **additional observable properties** $p_1, p_2, ..., p_n$.

A **mixture** is a vector that represents the **areal percentages of the IFOV that every object of the associated object partition covers** (see appendix C for an algebraic specification). For example, if the object partition contains the class associations "land" and "water"[36], then a possible mixture can be represented by the vector (21.3% land, 78.7% water). Figure 4.5 illustrates this. Water is shown as light, land as dark gray polygons.

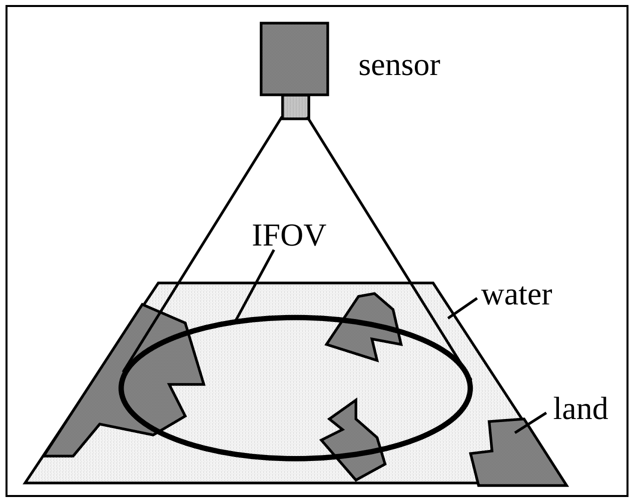

Figure 4.5: Imaginary sensors directly observe mixtures.

The percentages in a mixture always add up to 100% since the considered objects form a partition of space. The object partition that is associated with the mixture determines the level of abstraction at which geographic reality is perceived (in terms of the object hierarchy). Perception is possible from atom level up to that of large crisp objects or highest-level class associations. The **components** of a mixture are the **geographic objects** of the associated object partition. These geographic objects can either be crisp objects, class associations, or administrative objects.

In addition to a mixture, imaginary sensors observe other properties in their IFOV. While actual sensors cannot observe mixtures, these **additional properties** represent actually observable properties such as albedo, temperature, etc. They are included in the proposed spatial theory to interface mixture fields with actually observable fields.

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36 Such class associations can for example be based on points as atoms. Points then either belong to the class "water" or "land".

The relation between properties observed by real sensors and mixtures observed by imaginary sensors is expressed by a **classification** as it is known from image processing and remote sensing (see, for example, [Castleman, 1979]). A mixture then corresponds to a "mixed pixel" or a probability vector of remote sensing [Goodchild, 1993b]. The derivation of mixtures from actually observable properties usually relies on the assumption that these properties are functions of the desired mixture. For example, the albedo of a mixture may be a weighted sum of the characteristic albedos of the geographic objects of the mixture, where mixture percentages are uses as weights.

From an object-oriented point of view, an IFOV defines an association of contained objects. The observable properties then describe the behavior of this association. Behavior propagation [Egenhofer, 1989a] (see also section 2.3.6) describes the relation between the behavior of member objects and that of the association as a whole.

If the granularity defined by member objects is fine compared to the size of the IFOV (L-resolution), behavior propagation results in a precise prediction of the observed property. A mixture of class associations then provides all the knowledge about member objects necessary for behavior propagation, namely the ratio of different object types and their class behavior.

In contrast, if the granularity is relatively coarse (H-resolution), it is difficult to decide how objects that fall only partly into the IFOV contribute to overall behavior. This is problematic since the relative large size of such objects suggest that their contribution to overall behavior is significant. In the H-resolution case, also mixtures are a poor description of the member objects of an association.

Due to the problems of classification in the H-resolution case, this thesis interfaces only to observable fields of the L-resolution case. From this point of view, it is not surprising that most classification methods in remote sensing are based on L-resolution [Strahler, 1986]. While L-resolution representations can originate from actual perception of the world with consequent classification, H-resolution representations are considered to be derived from L-resolution ones by the application of abstraction mechanisms to geographic objects: Lower-level objects are relatively small and therefore define an L-resolution situation--while, after abstraction, the higher-level objects are relatively large as is characteristic for H-resolution.

4.3.2 Definition: Mixture Fields

A **mixture field** is always associated with an individual imaginary sensor. It represents **all possible knowledge that can potentially be acquired from geographic reality by this imaginary sensor**. Imaginary sensors limit knowledge content by imposing resolution and perceiving only the objects contained in their associated object partition.

To contain all possibly acquirable knowledge, a mixture field must represent the mixture and observable properties of every location in space. Locations here are specified by a disk; and the set of all possible locations is given by the atomic entities of resolution-limited space. In summary, a **mixture field** can be seen as **a mapping that assigns a mixture and other observable properties to every disk of resolution-limited space** (see appendix C for an algebraic specification of mixture fields). Since mixture percentages and observable properties change continuously with location, mixture fields are **continuous models**.

Since mixture fields represent all potentially acquirable knowledge, their **acquisition requires an infinite effort**. Actual (finite) acquisitions of mixture fields are based on sampling the field in finitely many locations and interpolating the properties of all other locations. Chapter six discusses such finitizations of mixture fields from a representation point of view. The concepts are equally applicable to data acquisition, however.

This thesis distinguishes between two types of mixture fields, **classified** and **unclassified mixture fields**. **Classified** mixture fields use **two or more geographic objects in their object partition** that is associated with its mixtures. **Unclassified** mixture fields use a **single geographic** object in their object partition. Unclassified mixture fields can be observed by real sensors; Classified mixture fields are observed by imaginary sensors that classify directly observable properties to mixtures. While predominantly classified mixtures fields are important for modeling the scaling process in this thesis, unclassified mixture fields are discussed to show the connection to current practice. The term "unclassified mixture field" rather than just "field" is used to point out that they are a special case of mixture fields and to contrast with the common understanding of fields [Goodchild, 1993a] where resolution is not modeled explicitly.

In **classified mixture fields**, the mixtures are the most important properties; the other observable properties are far less important since they can be derived by methods[37] from mixtures. Unlike actual property values, these methods are the same for every location in space. Examples of (finitized versions of) classified fields are the result of the classification of vegetation classes in remote sensing using mixed pixels.

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[37] Methods are used here in the sense of class methods of object-oriented programming. They are procedures rather than represented data values.

In **unclassified mixture fields**, the object partition contains only the class association of the super class of all geographic objects. The degenerated mixture of unclassified fields therefore contains no actual information. The actual values of the other observable properties in every location are therefore far more important. Examples of (finitized versions of) unclassified mixture fields are remotely sensed images where the properties are represented by the albedo of different spectral bands, and elevation models.

Unclassified fields are only marginally relevant for this thesis. Their major purpose is to provide an interface to actually observable properties and to define a resolution concept for such data. Note that this resolution concept is solely defined by the size of IFOVs and is unaffected by the sampling density (that will be discussed in chapter six).

4.4 Resolution-Limited Feature Partitions

Resolution-limited feature partitions are representations that are **derived by an abstraction process from mixture fields** (see section 3.4). The abstraction process is modeled after **Sinton's concept** where mixtures (the theme) are controlled in order to measure the location of features that are resolution-limited regions (see section 2.3. for a discussion of Sinton's concepts, and section 3.7 for details of how it is applied to mixture fields). While mixture fields are continuous models (see section 4.3.2), Sinton's concept is used to define discrete features that compose a **discrete model**. The first section discusses limitations of feature partitions that are described in the form of requirements that have to be satisfied to yield a meaningful representation in this format. The theme control of mixtures is then discussed in the second section. Since the theme is controlled by defining mixture partitions, section three discusses their representation and limits the focus of this thesis to one kind of mixture partition. The fourth section then defines feature partitions that result from mixture control. Their major properties are then discussed in section five. The sixth section defines stand-alone features as a special case of features. An algebraic specification of concepts introduced in this chapter is given in appendix C.

4.4.1 Requirements for Feature Abstraction

Mixture fields are well suited to represent continuous change in mixtures. Feature partitions, in contrast, require that **mixtures are almost constant over large areas**, i.e., within features, and then **rapidly transition between features** in the so called **transition zone**. If this requirement is not satisfied, the definition of features based on a mixture field is problematic and feature partitions are therefore poor representations of the world. In other word, some cases require continuous models and would be misrepresented by discrete models.

Figure 4.6 shows such a situation that fulfills the requirements of feature partitions. A two-component mixture of "land" and "water" is represented by its land component (while its water component is implied as the complement of the land component). For easier visualization, the figure shows a cross section through space. The locations of the features "land", "water", and "transition zone" are indicated by bars below the location axis. The figure shows how the mixture within the features "land" and "water" are relatively homogeneous, and how the transition between land and water is rapid. Further, the mixtures associated with the feature land are different from those of the features transition zone and water.

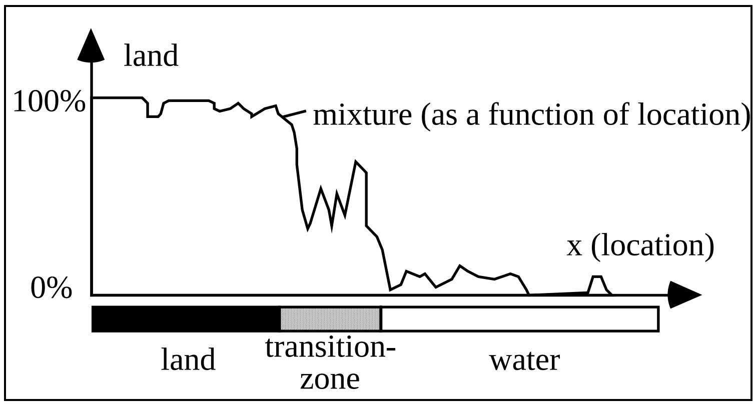

Figure 4.6: Feature partitions require large areas of relatively homogeneous mixture that are separated by transition zones with a rapid transition of mixture values.

4.4.2 Theme Control of Mixtures

The fundamental properties of classified mixture fields are mixtures. They are therefore chosen as the controllable theme. In contrast, other observable properties can be derived from mixtures by class methods (see section 4.3.2) and are therefore less interesting for theme control. This thesis assumes that unclassified mixture fields are not directly abstracted to feature partitions, but that they are first classified to classified mixture fields as an intermediate step of abstraction.

The control of mixtures requires awareness of the set of all possible mixtures that is also called **mixture space**. To understand mixture space, mixtures can be represented as points in a coordinate system that uses mixture percentages as coordinates. Every object in the object partition related to the mixture space then defines a coordinate axis. Since all mixture percentages must be between 0% and

100%, all possible points must fall in the first quadrant of the coordinate system. Further, since the mixture percentages add up to 100%, all points must lie in a hyper-plane that intersects the axes at the 100% mark. Figure 4.7 illustrates this for the case of an object partition that contains the three class associations "urban", "rural", and "forest". All possible mixtures are then points in a triangular section of the plane.

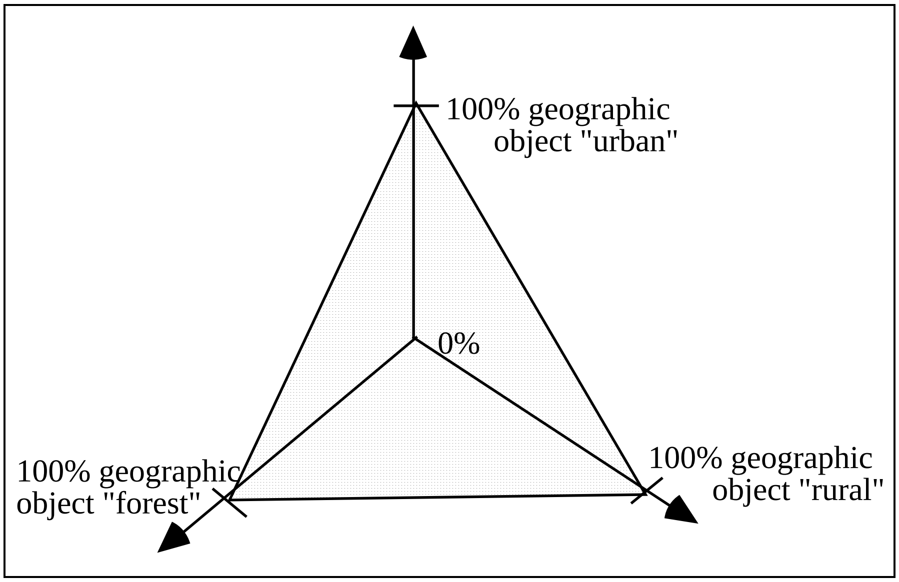

Figure 4.7: The set of all possible three component mixtures can be represented by a triangular section of a plane.

The **theme control** of mixtures is now based on the arbitrary definition of sets of mixtures, called **mixture classes**[38]. Mixture classes are comparable to the intervals of albedo used in remote sensing classification. A single mixture class can then be used to "measure" the location of the associated feature. Since feature partitions are composed of several features, several mixture classes have to be defined. Further, since the features of feature partitions are required to partition resolution-limited space, the **mixture classes** defined for theme control have to **partition mixture space**. A family of mixture classes with this property is called a **mixture partition**. Mixture partitions completely describe theme control.

Mixture partitions must **always** contain one mixture class called **transition zone**. Note that the transition zone is an **area** rather than a line **in mixture space**. This is partly motivated by the fact that the mixtures of transition zones are

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[38] The classification implied by the term mixture class is based on values of mixture percentages. All mixtures that belong to a mixture class have certain class properties in common, for example, their mixture percentages of "land" are higher than 80%. While in the strict sense of object-oriented modeling, a set of mixtures is an association of class members rather than class itself, this thesis uses the term mixture class for simplicity.

significantly different from those of other features (see, for example, figure 4.6 in section 4.4.1 above). It is also important for the visualization of feature partitions within the limitations of graphic media, since this visualization relies on a minimal width of the transition zone (see chapter seven).

The areal character of the transition zone is motivated also by the requirements of representing spatial knowledge: It was argued earlier, that the modal mixture component is a weak representation of the available spatial knowledge since all locations well inside the relatively homogeneous feature are described by the less discriminating attribute found in the transition zone (see section 2.2.1). For example, in figure 4.6 above, the land feature is characterized by all mixtures that contain more than 80% of the class association "land". In contrast, in a mixture with as little as 50.1% land, land is still the modal component[39]. To describe the whole land feature by a land percentage of "more than 50%" would drastically reduce the knowledge about what is found in the interior of the feature (as compared to a percentage of more than 80%).

The philosophy of modal mixture components requires linear transition zones. For example, in the above example, the transition zone contains only the mixture point[40] (50% land, 50% water). The requirement of an areal transition zone thus avoids the weak representation of available knowledge described above.

4.4.3 Representation of Mixture Partitions

Figure 4.8 visualizes one kind of mixture partition. Here, all mixture classes (but the transition zone) are defined by containing more than a given percentage of a geographic object (or mixture component). This minimal percentage will be called **level of homogeneity** or simply **homogeneity**. While the level of homogeneity may be different for different mixture components, this thesis will use the simplest case where a single level of homogeneity is used. The mixture partition is then completely determined by the level of homogeneity. Individual mixture classes are then defined by their **predominant geographic object** (or mixture component).

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39 In mixtures with more than two components, this percentage can go well below 50% (see section 2.2.1).

40 In two-component mixtures, a mixture line degenerates to a point.

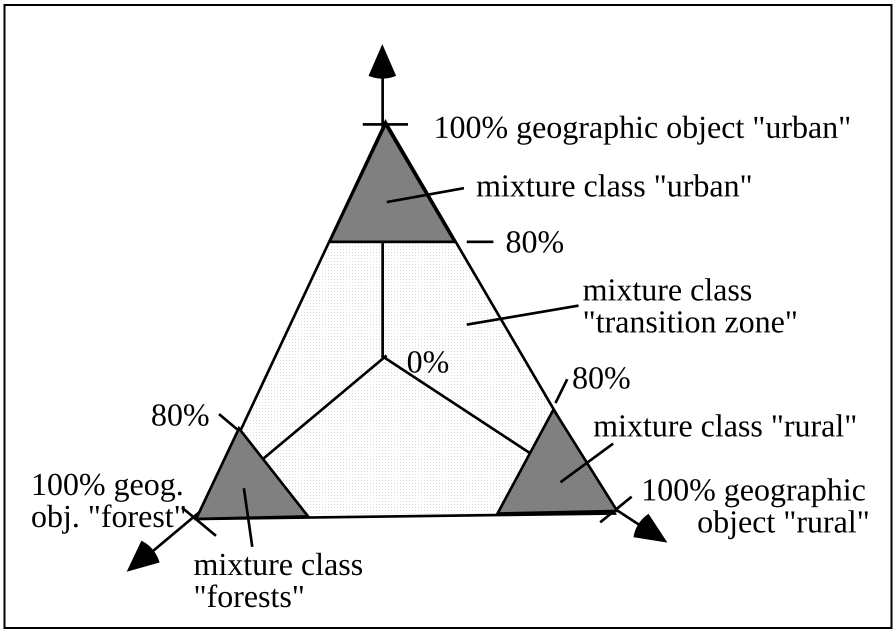

Figure 4.8: Mixture partition that is defined by a level of homogeneity of 80%.

A more general kind of mixture partition is visualized in figure 4.9. Mixture classes are now defined by a mixture ball. The concept of mixture balls will be discussed in detail in section 5.1.2. For the purpose of this discussion, mixture balls are special kinds of polygons in mixture space. They are determined by a single mixture that is the center point of the mixture ball and a mixture radius that is expressed by a percentage. In the visualization of figure 4.9, mixture balls are not round but rather hexagons. The mixture class "pine forest" shows that the previously discussed kind of mixture partition already used mixture balls; namely those centered on the "corners" of mixture space and mixture radii that are 100% minus level of homogeneity. The more general kind of mixture partition allows an arbitrary number of mixture balls, in arbitrary locations of mixture space, and with arbitrary radii. The mixture classes are still separated by transition zone, however. While the previous kind of mixture partitions could be represented by a single level of homogeneity, the more general kind is represented by the center points and radii of mixture balls.

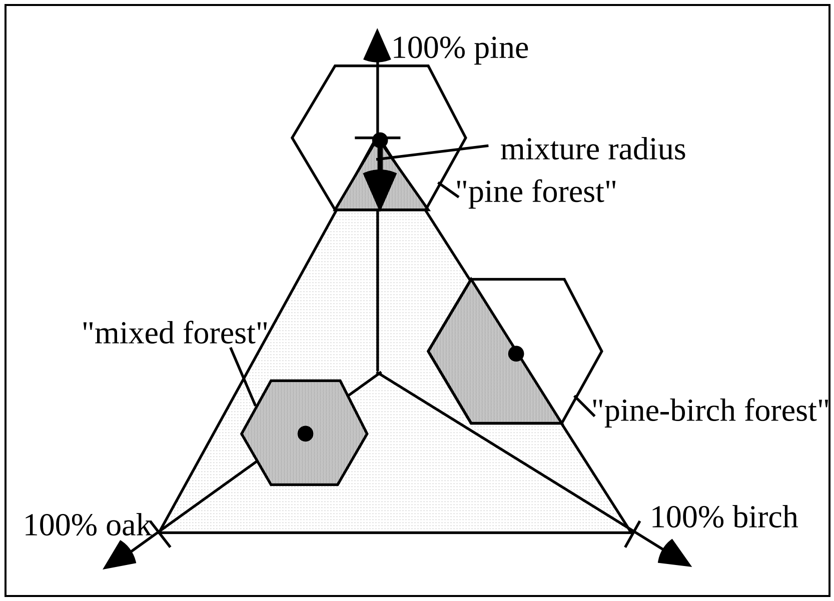

Figure 4.9: Example of a more general kind of mixture partition.

The most general, finitely representable kind of mixture partition is defined by polygons in mixture space. While the concept of feature partitions works with any kind of mixture partition, this thesis uses only those mixture partitions that are representable by a single level of homogeneity for a detailed discussion. This kind of mixture partition covers the needs of most GIS applications, and can be represented by a single value (i.e., the level of homogeneity). A later extension to include more general mixture partitions is possible. While leaving the representations of feature partitions unaffected, it would require changes in the meta data of feature partitions and in the transformations between feature partitions of different resolution.

4.4.4 Definition of Feature Partition

Feature partitions are representations that are derived from classified mixture fields by an abstraction process. The abstraction process incorporates two abstraction mechanisms, one that associates mixtures to mixture classes, and another that associates disks to regions (i.e., sets of disks). A **feature** is described by an **attribute** that is a **mixture class** and a **geometry** that is a **region**. The relation between attribute and geometry is given by Sinton's concept of controlling theme and measuring location (see sections 2.3.1 and 3.6). Namely, a disk is part of a feature's geometry if its mixture[41] is an element of a the feature's mixture class.

A **feature partition** is a **set of features** whose mixture classes partition mixture space and consequently, whose geometries partition resolution-limited space. Feature partitions are uniquely determined by (i) the mixture field from which they were derived by an abstraction process and (ii) the mixture partition that is the parameter of the abstraction process. Since this thesis restricts its scope to mixture partitions that can be represented by a single level of homogeneity, a **feature partition** is **identified by** the **resolution** and **object partition** of the underlying mixture field and the **level of homogeneity** of the mixture partition.

4.4.5 Properties of Feature Partitions

This section discusses some properties of feature partitions that are defined by a level of homogeneity[42]. In particular, it discusses the "absorption" of "foreign" geographic objects in features that have a predominant geographic object. For example, the feature with a predominant class association "water" can absorb small pieces of the class association "land" (i.e., islands). The section further investigates how the width of the transition zone is affected by the configuration of objects in geographic reality. A comparison to a crisp-boundary approach illustrates how the areal transition zone allows the use of expressive attributes.

Geographic objects that are different from the predominant geographic object of a feature but are contained in disks of the feature's geometry are called **inhomogeneities**. If such inhomogeneities fail to show up as features of their own, they are said to be absorbed in the feature that covers the location of the inhomogeneity.

The maximal size of such inhomogeneities is limited by level of homogeneity and spatial resolution (i.e. the size of disks). Figure 4.10 illustrates this. Assume that the feature of interest is defined at a level of homogeneity of 80%. The inhomogeneity in (a) is then too large to be absorbed in the feature since it covers more than 20% of the disk area; in case (b), the inhomogeneity is small enough to be absorbed in the 20% allowed inhomogeneity. It is obvious how resolution, i.e. the size of disks, together with the level of homogeneity limits the maximal size of absorbable inhomogeneities. A more quantitative study of this issue will be given in section 4.5.

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41 The underlying mixture field specifies a disks mixture.

42 see limitation of considered kinds of mixture partitions in section 4.4.3.

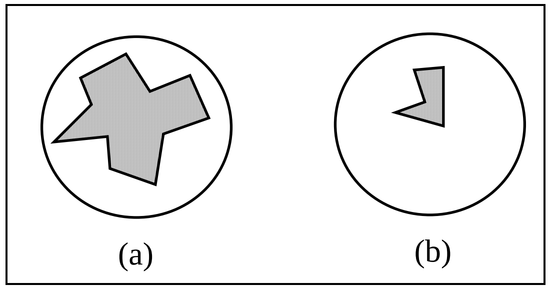

Figure 4.10: Inhomogeneities that can be absorbed in a feature are limited in size by resolution and level of homogeneity. Large inhomogeneities cause the disk's mixture to fall outside the feature's mixture class (a), while small homogeneities can be absorbed (b).

Figure 4.11 illustrates the effects of absorbing inhomogeneities on the geometry of features as compared to that of objects in geographic reality. The top row shows three different configurations of the class associations "land" and "water" in geographic reality. The bottom row shows the corresponding limited-resolution features "land" and "water". The visualization uses a projection of limited-resolution space to Euclidean space, where disks are projected onto their center point. The transition zone is shown as a thick line. Column (a) shows how small islands can be totally absorbed in the feature "water". In (b), long and thin areas of the object "land" are absorbed in the feature "water". Column (c) demonstrates how groups of islands can become a contiguous part of the feature "land" if they are close enough together, and how small islands close to the coast can result in a feature topology that is different from the object topology in geographic reality.

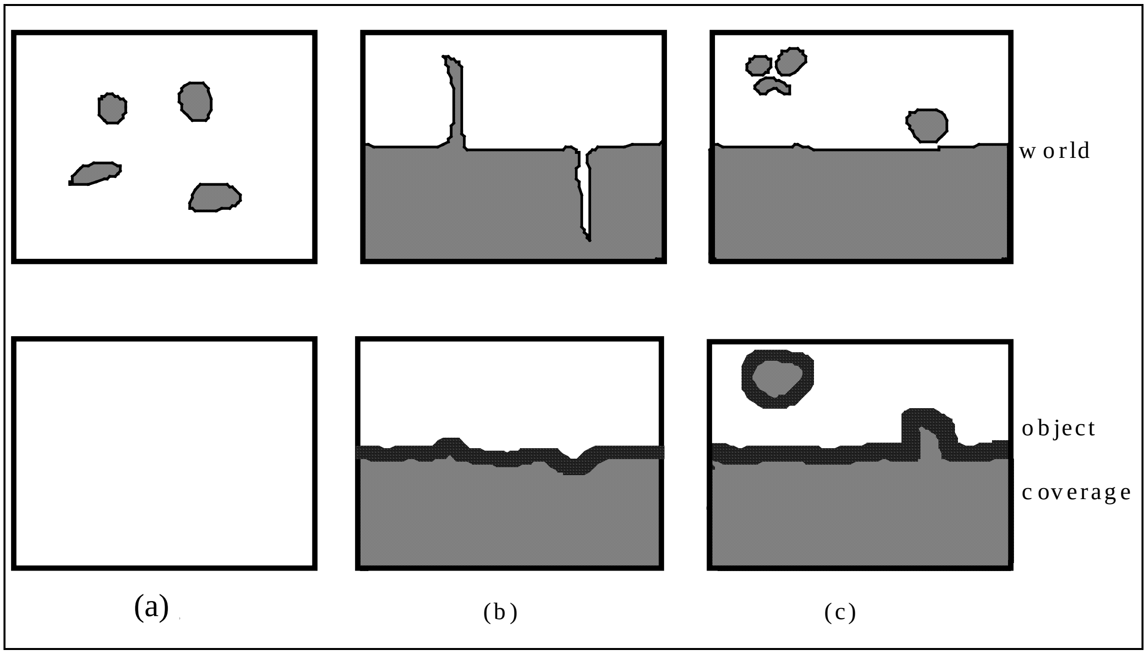

Figure 4.11: Examples of the absorption of inhomogeneities.

If inhomogeneities are relatively small, they are usually totally absorbed by features; if they are relatively large, they show up as features of their own. In between these two cases, inhomogeneities show up as transition zone only. Figure 4.12 shows two examples. Land is shown in light gray, water in white, and transition zone in dark gray. In case (a), islands cause a transition zone inside the feature "water"; in case (b), a complexly shaped shoreline causes a large transition zone.

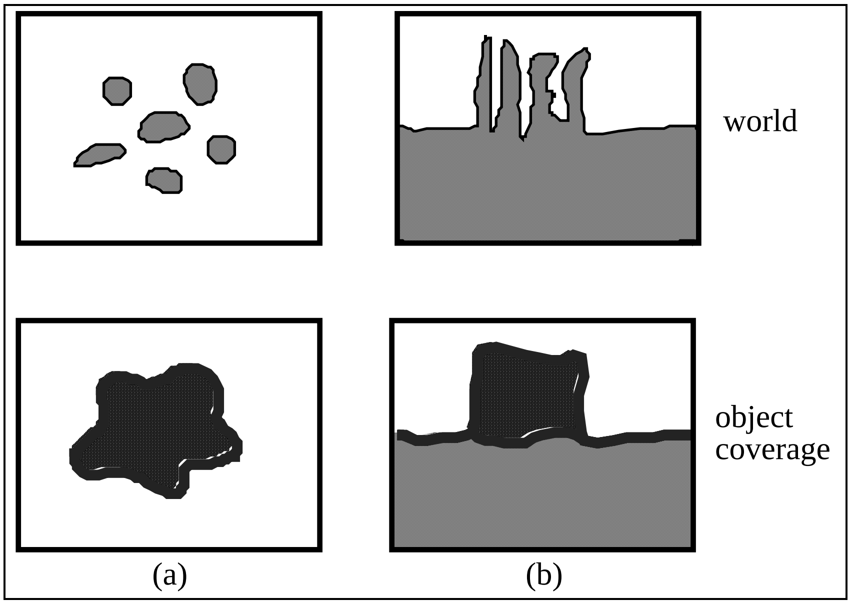

Figure 4.12: Inhomogeneities that are too large to be totally absorbed and too small to show up as features of their own, are absorbed in the transition zone.

The width of the transition zone[43] depends on the configuration of objects in geographic reality. This is illustrated in figure 4.13 with an example of the distribution of land and water. The three frames in the top row of the figure visualize land and water in geographic reality. We are interested in the mixtures of disks that fall on the indicated cross section. These mixtures are shown in the bottom row of the figure[44]. The frames in the top row show the last disk that contains 100% land and the first disk that contains 100% water. The curves in the bottom row show how the mixtures change in between these positions. The lines of 80% land and 80% water indicate the edge of the transition zone between the mixture classes "land" and "water". The locations where the mixture curve falls between these lines defines the feature "transition zone" that is marked by shaded bars. The transition zone width can obviously vary from less than a disk diameter (a) to significantly more (c).

43 Since a transition zone is a resolution-limited region rather than a point set, its width has to be measured in terms of resolution-limited distance. In the visualization used in the figures, every disk is projected to a Euclidean point and the resolution-limited width projects to a "normal" Euclidean width.

44 While the mixtures describe disks, they are visualized as the attributes of points along the cross section.

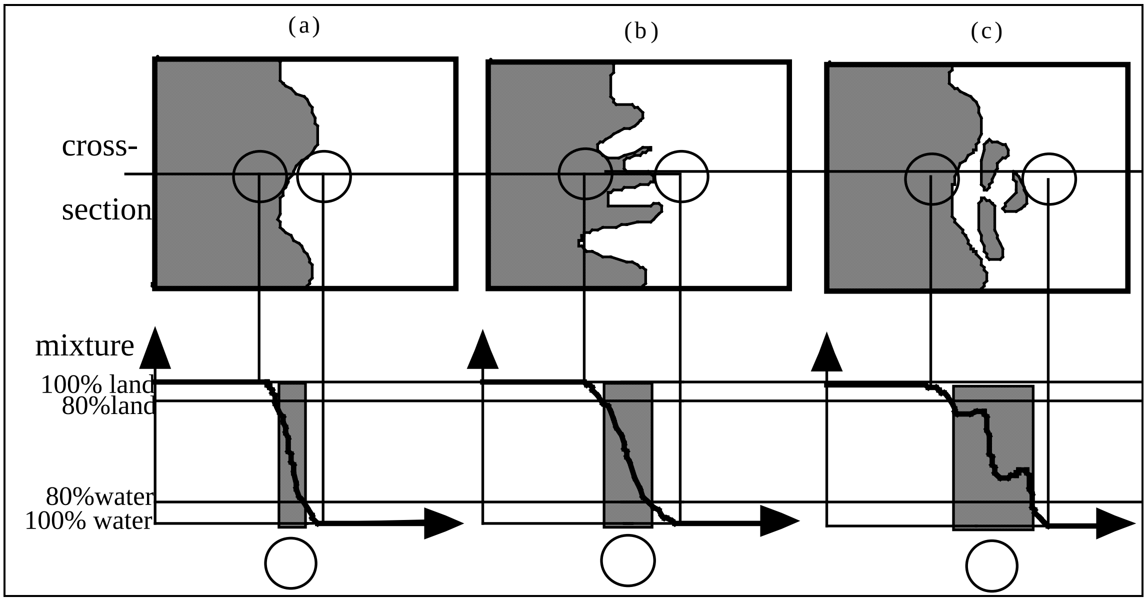

Figure 4.13: The width of the transition zone depends on the configuration of objects in geographic reality.

A major incentive for an areal transition zone is that it allows expressive feature attributes. The following comparison of an areal transition zone with a crisp boundary shows this (see figure 4.14). The fills suggest that the areal transition zone allows much stronger attributes than the crisp boundary approach. In case of an **areal transition zone**, the attribute meaning is determined by the **level of homogeneity**. In particular, features "land" and "water" contain more than, for example, **80%** of the geographic objects "land" and "water", respectively. To achieve a **crisp boundary**, features have to be defined in terms of the **modal mixture component** rather than a level of homogeneity. Every disk that contains more land than water is then part of the feature "land". In the two-component mixture of "land" and "water", the feature attribute "land" then expresses that disks contain more than 50% of the geographic object "land". In the case of mixtures of, for example, ten components, a mixture percentage of **as little as 10.1% can be modal** if all other components are less than 10%. This shows that a modal attribute expresses much weaker knowledge about what can be found in the world than an attribute that is based on a level of homogeneity. A geometrically attractive crisp boundary thus has to be paid for by weak attributes, while an areal transition zone preserves much more of the knowledge of the underlying mixture field.

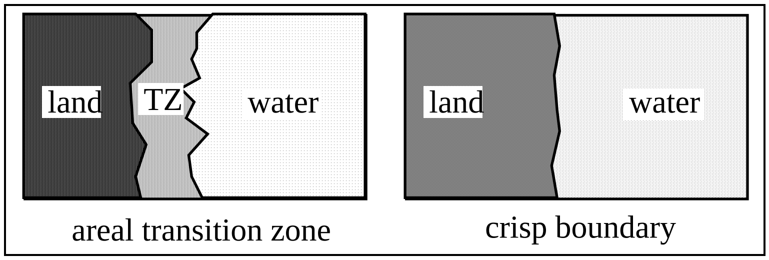

Figure 4.14: Comparison of areal transition zone and crisp boundary.

4.4.6 Special Case: Stand-Alone Features

Feature partitions contain several features that form a partition of space. Stand-alone features are the features of **special feature partitions**[45] **that contain only two features**, namely the **stand-alone feature** itself and the **transition zone**. Like all features in this thesis, stand-alone features are defined by a **predominant geographic object** and a level of homogeneity. In contrast to partitioning features, the **level of homogeneity** of stand-alone features is **0%**, i.e., every disk that contains more than 0% of the predominant object in its mixture is part of the stand-alone feature.

The term "stand-alone" expresses that stand-alone features exist independently of other features[46]. In contrast, general feature partitions contain several features that are closely related by a mixture partition. This thesis uses the term **partitioning features** for the features of general feature partitions. Partitioning features are then a contrast to stand-alone features.

If uncertainty introduced by finite approximation is not considered (as is the case in this chapter), stand-alone features are represented and otherwise treated like partitioning features. The difference between stand-alone and partitioning features is the way in which they model the mentioned uncertainty. Here, stand-alone features must follow a different approach than partitioning features in order to preserve the knowledge they represent (see chapter five for detail).

Stand-alone features are designed to preserve knowledge about very small geometric objects. Normal features with their high level of homogeneity require a minimal size of the predominant object before it shows up as a feature. If the

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45 Here, these feature partitions are defined by the more general kind of mixture partitions that use mixture balls (see section 4.4.3).

46 The transition zone is irrelevant in absence of other features and is only needed to show that stand-alone features are part of a special case of feature partitions.

predominant object is very small compared to resolution, knowledge about its presence in geographic reality is not represented. Maps give many examples, where small objects are relevant in spite of their size. For example, maps often show cities, rivers, and roads, that are too small to be preserved in the scaling process. Cartography uses methods to preserve the knowledge about such objects and displays them as graphic point or line symbols of the minimal allowable size. Since all objects are two- (or three-) dimensional in geographic reality, showing cities as (zero-dimensional) points and roads as (one-dimensional) lines is often called dimension change [Nickerson, 1986].

The preservation of such knowledge requires mixture classes that contain mixtures with arbitrarily small percentages of the small geometric object. A disk that is part of one stand-alone feature can then easily also be part of another partitioning or stand-alone feature. This is evident in the fact that a disk can simultaneously contain more than 0% of a small geographic object and more than say 80% of another, larger geographic object. This discussion shows that stand-alone features usually overlap with other features rather than forming a partition of space.

4.5 Some Quantitative Properties of Mixture Fields and Feature Partitions

While some qualitative properties of resolution-limited mixture fields and feature partitions were pointed out above, this section discusses quantitative properties. A first section investigates the size threshold for inhomogeneities to be absorbed at a given resolution. Then, a distance function for mixtures is defined in order to express how similar or dissimilar mixtures are. This distance function is used in the third section to define the maximal difference in the mixture of two disks of a mixture field as a function of the spatial distance between them. The fourth section applies this maximal rate of mixture change to determine the minimal width of the transition zone in feature partitions.

The following discussion uses the calculation of intersection areas of disks and half planes in different configurations. Appendix "A" documents how to calculate these intersection areas.

4.5.1 Absorption of Inhomogeneities

While it is not possible to look at all possible configurations of inhomogeneities, the two examples of circular- and bar-shaped inhomogeneities give an idea of the maximal dimension of inhomogeneities that can be absorbed. The two configurations are illustrated in figure 4.15: Inhomogeneities are shown in gray and two resolution-limited disks without shade are shown for reference.

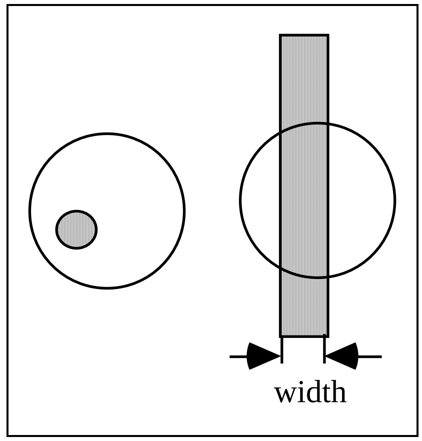

Figure 4.15: Circular and bar-shaped inhomogeneities.

Section A.3 in the appendix provides the formulas and table for the calculations of circular inhomogeneities. Table 4.1 lists the maximal size of circular inhomogeneities that are still absorbed as a function of the level of homogeneity.

Level of homogeneity [%]	**Maximal radius of circular inhomogeneity** [% resolution-limited disk]
95	22
90	32
85	39
80	44

Table 4.1: Maximal radius of absorbed circular inhomogeneities as a function of the level of homogeneity.

Since a ratio of radii is difficult to visualize, figure 4.16 shows the cases of 95 and 80% homogeneity.

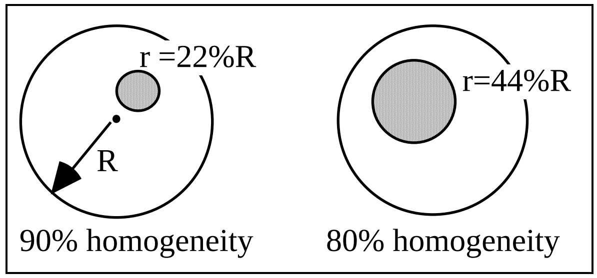

Figure 4.16: Maximal circular inhomogeneity still absorbed at 95% and 80% homogeneity.

The maximal size of bar-shaped inhomogeneities can be derived from section A.2 in the appendix. The inhomogeneity obviously has its maximal effect if it is centered on a resolution-limited disk. Table 4.2 shows the maximal width of bars that are still absorbed as a function of the level of homogeneity.

Level of homogeneity [%]	**Maximal width of absorbed bar** [% of radius of resolution-limited disk]
95	8
90	14
85	24
80	32

Table 4.2: Maximal width of absorbed bar-shaped inhomogeneities as a function of the level of homogeneity.

Figure 4.17 shows two cases from the above table graphically.

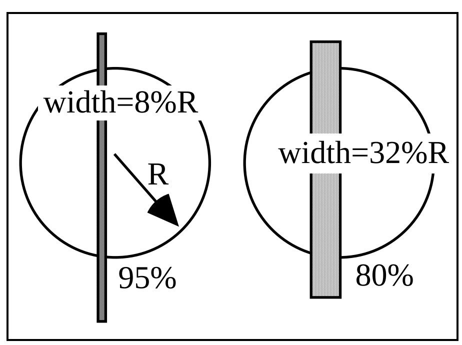

Figure 4.17: Maximal bar-shaped inhomogeneity still absorbed at 95% and 80% homogeneity.

4.5.2 Distance Function for Mixtures

Finite resolution limits the maximal rate with which mixtures can change. In order to capture this maximal rate, mixture change first needs to be quantified. This is done in this section by defining a distance function for mixtures.

The following simple example illustrates the idea of a distance between mixtures. Let us consider mixtures with only two components, for example, the class associations "land" and "water". Two mixtures with the same ratio of land and water are obviously equivalent, and therefore have a distance of 0%. Mixtures containing 100% land and those containing 100% water are examples of the maximal possible difference between mixtures, expressed in a distance of 100%.

In mixture fields, mixtures are always associated with disks. The idea behind quantifying mixture-distances is to consider one disk and measure the minimal area that has to be replaced with a different mixture component in order to reach the mixture of the second disk. For example, let us consider two disks that contain a mixtures of land and water ratio of (10% , 90%) and (50%, 50%), respectively (see figure 4.18). In order to get from a (10% , 90%) mixture to (50%, 50%), water in 40% of the first disk has to be replaced by land. Since 40% of the disk area had to change, the distance between these two mixtures is 40%.

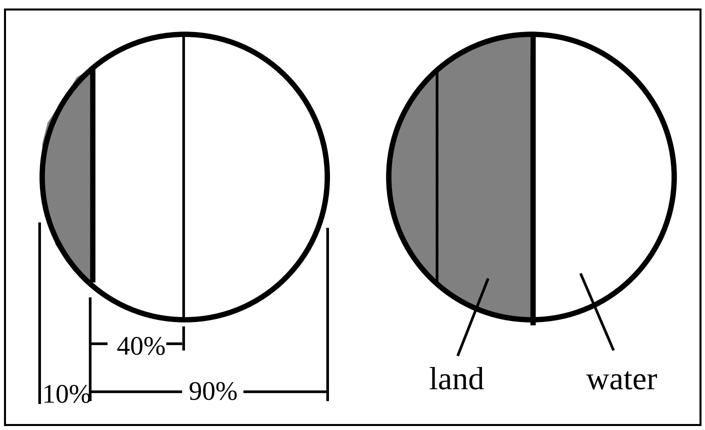

Figure 4.18: Two disks with mixtures of land and water of (10%,90%) and (50%, 50%), respectively.

Let us now apply this concept to general mixtures with arbitrarily many components. The doubled disk area that has to change is then found by summing up the absolute values of percentage differences over all mixture components. In the above example, we have to sum up |10-50| for land and |90-50| for water. This adds up to 40 + 40 = 80, i.e., twice the mixture distance as predicted.

The following captures the distance between two mixtures m_a and m_b: Let m_a and m_b be mixtures of n components. m_a can then be written as $[a_1, a_2, ..., a_n]$ and m_b as $[b_1, b_2, ..., b_n]$. The mixture-distance md is then determined as follows:

$$md(m_a, m_b) = 1/2 \sum |a_i - b_i|$$

Mixture-distances are obviously limited to the interval between 0% and 100%. They are thus always positive, as expected from a distance. They are also symmetric, i.e., $md(m_a, m_b)$ is the same as $md(m_b, m_a)$, and satisfy the triangular inequation[47], i.e., $md(m_a, m_c) \varepsilon\ md(m_a, m_b) + md(m_b, m_c)$ for arbitrary mixtures m_a, m_b, m_c.

4.5.3 Maximal Rate of Mixture Change in Mixture Fields

Since close disks of resolution-limited space intersect, their possible mixture difference is limited. This can be expressed as a maximal mixture-distance between two disks as a function of their spatial distance. This maximal rate of mixture change is relevant to possible finite representations of mixture fields since it allows interpolation of mixture values between sampling locations (see chapter six). It will further be used in the following section to determine the minimal width of transition zones in feature partitions.

Figure 4.19 illustrates two close disks with their intersection area. Obviously, the geographic content of the intersection area must be the same for both disks. The disks can thus only differ in the geographic contents found in their non-overlapping parts. Since mixture-distance is defined as the area that changes its thematic content, it must be restricted to the non-overlapping part of the disks.

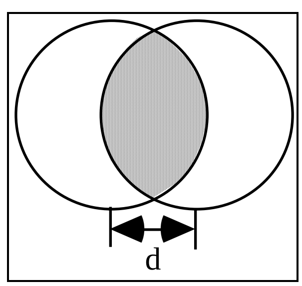

Figure 4.19: Two adjacent disks with their intersection area (gray) that can be derived form the distance d between their center points.

[47] The proposed mixture distance has all the properties of distance functions since it is a special case of the well-known city-block metric.

Table 4.3 shows the maximal mixture change as a function of the spatial distance between disks. Section A.2 in the appendix describes the underlying area calculations. Note that a distance of 200% of the radius between disks means that they just touch at a point of their boundary.

spatial distance between disks [% of radius]	**maximal mixture-distance** [%]
0	0
20	14
40	26
60	36
80	50
100	60
120	72
140	80
160	90
180	96
200 or more	100

Table 4.3: Maximal rate of mixture change.

Figure 4.20 shows this maximal rate of mixture change as a function. A straight line between the end points of the function is added for reference. The disk configurations are visualized for the distances of 0, 100, and 200% of the radius.

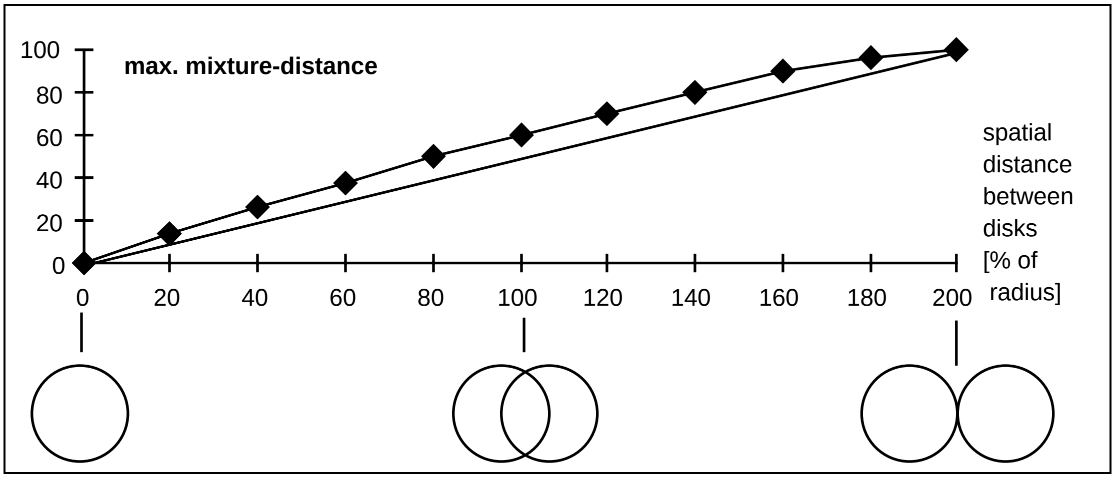

Figure 4.20: Maximal rate of mixture change.

4.5.4 Minimal Separation of Features

The maximal rate of mixture change implies that in a feature partition, the features with a predominant geometric object are always separated by the transition zone by a minimal spatial distance. Such separation properties are relevant to the finite representations of features (see chapter six), as well as their visualization (see chapter seven).

Features are defined by mixture classes, i.e., sets of mixtures. The minimal mixture-distance between two mixture classes is then the minimal distance between element mixtures. Figure 4.21 shows an example of illustration. The mixture classes A and B are defined as part of a mixture partition. The partitioned mixture space contains all mixtures with the three components c1, c2, and c3. The mixture class A contains all mixtures with at least 80% c1; B contains all mixtures with at least 40% c1, at least 40% c2, and at most 10% c3. The more c1 a mixture in B contains, the closer it is to A. The mixture of B with the maximal amount of c1 is P1 with 60% c1, 40% c2, and 0% c3. P2 in A is the closest to P1, since it contains the minimal possible amount of c1 and the maximal possible amount of c2. P2 thus contains 80% c1, 20% c2, and 0% c3. The mixture change from P1 to P2 consists of replacing 20% of c2 with 20% of c1. The minimal mixture distance between A and B is therefore 20%.

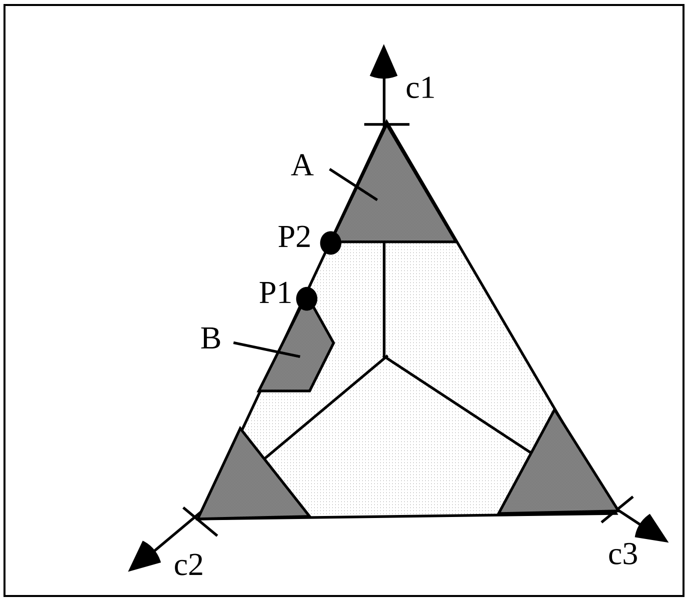

Figure 4.21: Example of minimal mixture-distance between mixture classes.

In case of feature partitions that use a mixture partition defined by a level of homogeneity per predominant geographic object, the minimal mixture-distance can be expressed in a simple formula. Let A and B be mixture classes with a level of homogeneity of H_a and H_b, respectively. The minimal mixture distance mmd between A and B is then given by the following formula:

$$mmd = H_a + H_b - 100\%$$

For example, if A has a level of homogeneity of 70%, and B one of 90%, the minimal mixture distance between A and B is 60%.

The maximal rate of mixture change relates mixture-distances and spatial distances (see preceding section). A minimal mixture-distance between mixture classes therefore implies a minimal spatial distance between the according features or equivalently, a minimal width of the transition zone. For example, two features that are separated by a minimal mixture-distance of 60% are separated by at least a disk-radius in the spatial domain (see table 4.3 above).

5 Representation of Uncertain Spatial Knowledge

The previous chapter has shown that mixture fields and feature partitions are the results of abstraction processes applied to geographic reality (see figure 5.1). Both these results are infinite representations and can therefore not be physically represented in a finite computer. In order to allow physical representation, the infinite representations have to be **approximated** by finite representations. Approximation leads to differences between actual properties and represented properties. This constitutes **approximation error** (in the sense of Chrisman's definition of error [Chrisman, 1991]). While it is not possible to represent individual errors[48], an **uncertainty model** represents the possible magnitudes of error.

Approximation is modeled as an **additional step in the abstraction process** (see figure 5.1). Section 5.1 discusses the approximation of mixture fields. This abstraction step maps the certain mixture fields of chapter four to **uncertain mixture fields** (see figure 5.1). **Uncertain feature partitions** can be constructed in two ways, either by propagation of uncertainty from uncertain mixture fields, or by an approximation abstraction of certain feature partitions (see figure 5.1). Section 5.2 will first discuss the former possibility and then reason that the latter possibility results in a representation of the same format. While **stand-alone features** were treated as a special case of partitioning features in the previous chapter, they have to be treated different from partitioning features in the uncertain case. This allows preservation of knowledge that would otherwise be lost at relatively coarse resolution. This is not evident in figure 5.1 where stand-alone features are derived from feature partitions of the same resolution. It will become clear only when considering transformations to coarser resolution (see chapter eight). Then, small partitioning features get lost while, due to their different uncertainty model, small stand-alone features are preserved.

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48 If actual errors were represented, the representation would allow the reconstruction of the true property (as represented property + error). This is obviously not possible in finite representations.

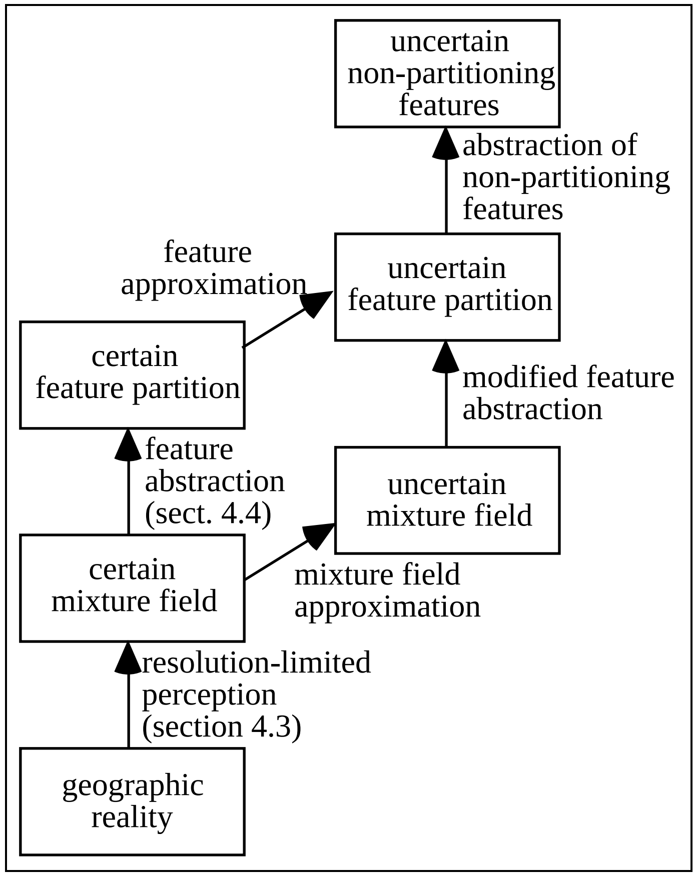

Figure 5.1: Abstraction steps between geographic reality and uncertain representations.

While the abstraction process from geographic reality to certain representations was described in terms of standard abstraction mechanisms (see sections 2.3.6 and 3.4); the final step in the complete abstraction process that describes the approximation cannot easily be modeled by one of these abstraction mechanisms applied to entities of the spatial theory. However, several different states of certain representations map to a single state of an uncertain representation. This many-to-one mapping shows that the approximation process is an abstraction [Liskov, 1986].

This chapter models uncertainty that is introduced by approximation at a general level. All possible **finite representations** (including the ones proposed in chapter six) are then **special cases of the general uncertain representations**, since they are all approximations of certain representations. Figure 5.2 illustrates how finite representations approximate the general case using the example of a mathematical function: A general function is visualized in (a), while (b) shows a function that can be described with a finite number of parameters, namely the values in sampling points. The figure illustrates how finite approximation introduces error that can be represented in the form of an uncertainty model.

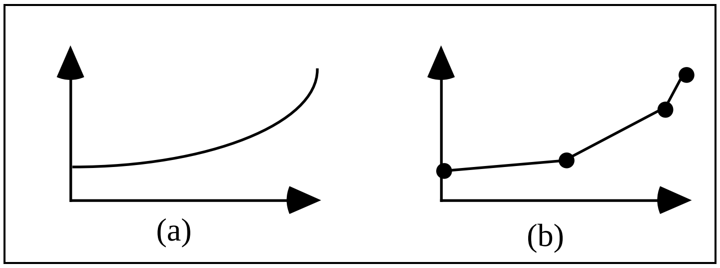

Figure 5.2: Functions with a finite number of parameters (b) can only approximate general functions (a) and therefore introduce error.

5.1 Uncertain Mixture Fields

This section describes the approximation of certain mixture fields that results in uncertain mixture fields. The first section describes the concept of representing mixture uncertainty. The next section is concerned with finding a representation for such uncertainty. The final section shows how different finite parametrizations are special cases of uncertain fields.

5.1.1 Basic Concept

The representation of certain mixture fields was described in section 4.3.3. For every disk in resolution-limited space, the actual mixture was known. In uncertain mixture fields the actual mixtures are unknown. Instead of single mixtures, intervals[49] (or more generally sets) of mixtures are used in uncertain mixture fields. It is then known for **certain that the actual mixture lies somewhere in this set of mixtures**[50].

Figure 5.3 illustrates this concept. For easy visualization, the figure shows only a cross-section through space on the horizontal axis, and a mixture of only two components (for example, "land" and "water") on the vertical axis. The mixture axis then shows the percentage of one component, e.g., "land" and implies the second component as its complement. The certain mixture field is visualized in (a), while the uncertain case is shown in (b). The certain representation relates a

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49 Strictly, the concept of "interval" can only be applied to two-component mixtures. It is used here to facilitate understanding.

50 Note that the approximation errors in the case of mixture fields are the result of interpolation between sampling locations. Due to a maximal rate of mixture change as a function of location change (see section 4.5), the maximal approximation error is theoretically limited.

single mixture with every location (e.g., (65% land and 35% water)). The uncertain representation relates an interval[51] of mixtures with every location (e.g., the interval between the mixtures (64% land and 36% water) and (68% land and 32% water)). A single mixture interval corresponds to a vertical line segment in the figure (marked by arrows). The entirety of all intervals is shown as a "band" of varying width. The uncertain representation can be described by a finite number of parameters, for example, the mixture intervals in sampling locations (marked by points in the figure). Mixture information between sampling locations is then derived by interpolation; and mixture uncertainty increases with the distance from the sampling location.

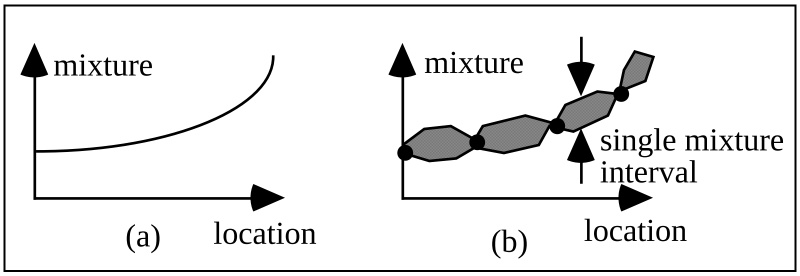

Figure 5.3: A certain (a) and uncertain mixture field representation (b).

5.1.2 Representation of Uncertain Mixture Fields

This subsection is concerned with the representation of intervals and sets of mixtures and the representation of uncertain mixture fields. While the representation of mixture intervals is straight forward, it is only applicable to two-component mixtures. The concept of intervals is therefore generalized for n-component mixtures in the form of "mixture balls". Mixture balls are used by uncertain mixture fields in the way that single mixtures are used by certain mixture fields.

In Euclidean geometry, the higher-dimensional equivalent to a two-dimensional closed interval is a closed ball (see, for example, [Croom, 1989]). It is represented by a center point and a radius, and contains all points whose distance to the center point is less than or equal to the radius. The concept of such balls is obviously applicable to spaces of arbitrary dimension.

Section 4.4.2. showed that mixtures can be represented as points in a plane of an n-dimensional space; and section 4.5.2. defined a distance function for mixtures.

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51 In the general case, the mixture interval for a two-component mixture becomes a set of mixtures for an n-component mixture.

The ball concept can therefore be applied to mixtures: If m_i are mixtures of n-components, m_c is the mixture corresponding to the center point of the ball, and r is the mixture-radius expressed as a mixture distance, then a mixture ball mb is defined as follows:

$mb = \{m_i \mid \text{mixture-distance}(m_c, m_i) \bullet r\}$

Figure 5.4 illustrates a mixture ball for the case of three components. Note that since mixture-distance is closely related to city block metric and mixtures can only exist in a plane, the ball becomes a hexagon.

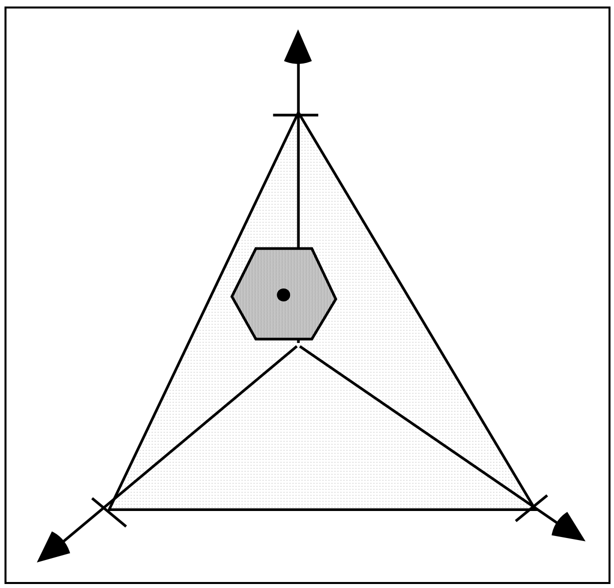

Figure 5.4: A mixture ball for three-component mixtures.

Uncertain mixture fields use mixture balls as a major component of representation. Section 4.3.2 defined certain mixture fields as "a mapping that assigns a mixture (and other observable properties) to every disk of resolution-limited space". An **uncertain mixture field** is then **a mapping that assigns a mixture ball** (and other observable properties) **to every disk of resolution-limited space**.

In classified mixture fields, the additional observable properties are represented as methods rather than property values (see section 4.3.2). The represented mixture uncertainty can therefore be propagated to uncertainty in observable properties which makes a separate representation of property uncertainty unnecessary. In the case of unclassified fields, the property uncertainty can be represented by intervals of property values[52]. Since unclassified fields are only of marginal importance in this thesis, such representation is not further investigated.

5.1.3 Finite Representations as Special Cases of Uncertain Mixture Fields

Finite approximations of mixture fields will be discussed in chapter six. Since approximation introduces uncertainty, such finite approximations are uncertain mixture fields, namely those that can be represented by a finite number of parameters. Figure 5.5 uses the visualization of mixture fields proposed in section 5.1.1[53] to illustrate this point. The certain mixture field (a) is finitely approximated in (b) and (c) by different finite parametrizations of mixture fields. They both capture approximation uncertainty as part of their representation in the form of mixture intervals. It is obvious in the figure that both (b) and (c) are special cases of uncertain mixture fields.

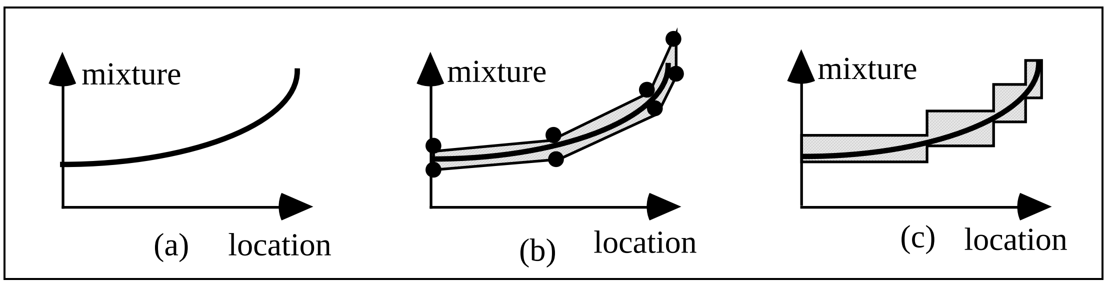

Figure 5.5: Different finite approximations (b,c) of a certain mixture field (a) are special cases of uncertain mixture fields.

5.2 Uncertain Feature Partitions

This section describes the derivation and representation of uncertain feature partitions. The first section points out that feature partitions can only be derived from mixture fields with a limited mixture uncertainty. The next section describes the propagation of mixture uncertainty in fields to geometric uncertainty in feature partitions. The third section reasons that the approximation of certain feature partitions results in the same kind of uncertain feature partition that was developed for handling propagated mixture field uncertainty. The final section discusses the philosophy that underlies the proposed

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52 Under the assumption that observable property values are of ratio scale.

53 This visualization uses a two-component mixture and a cross section through space.

uncertainty representation. It is important for understanding the difference between partitioning and stand-alone features.

5.2.1 Feature Partitions Require Limited Uncertainty

Feature partitions can only be derived from mixture fields with limited uncertainty, since otherwise, the mixture classes that define features can be confused. Figure 5.6 illustrates the case where mixture field uncertainty is too large for the derivation of a feature partition. A single mixture ball represents the uncertain mixture knowledge in a certain location of the mixture field. Obviously, the excessive level of uncertainty makes the separation of mixture classes (and thus features) impossible. This thesis therefore assumes that mixture field uncertainty is relatively small compared to the separation of mixture classes.

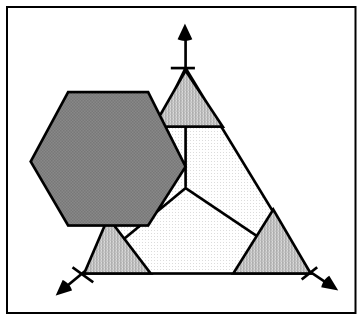

Figure 5.6: If mixture field uncertainty is large compared to the separation of mixture classes, different features become confused and indistinguishable.

5.2.2 Propagation of Mixture Field Uncertainty to Feature Partitions

When feature partitions are derived from uncertain mixture fields, mixture field uncertainty propagates to the feature domain. This section therefore discusses the effect of mixture field uncertainty on feature partitions. For an efficient representation of uncertain feature partitions, the feature abstraction proposed in section 4.4 has to be modified to accommodate the characteristics of uncertain representations. The modified abstraction results in the representation of uncertain feature partitions.

The original feature abstraction is designed for certain representations (see section 4.4) and assigns every disk of space to a feature based on the mixture that the underlying field associates with the disk. Figure 5.7 compares the feature abstraction from a certain mixture field (a) with that from an uncertain one (b). The mixtures m1 and m2 are the attributes of two disks of a certain mixture field. Since they completely fall into a single mixture class, the associated disks can easily be assigned to a feature. The situation becomes more complex in the uncertain case (b) because mixture balls instead of single mixtures are associated with disks. Such mixture balls can either be completely contained by a mixture class (b1 and b3), or intersect with two mixture classes (b2)[54]. The latter case prevents a straight forward assignment of disks to features.

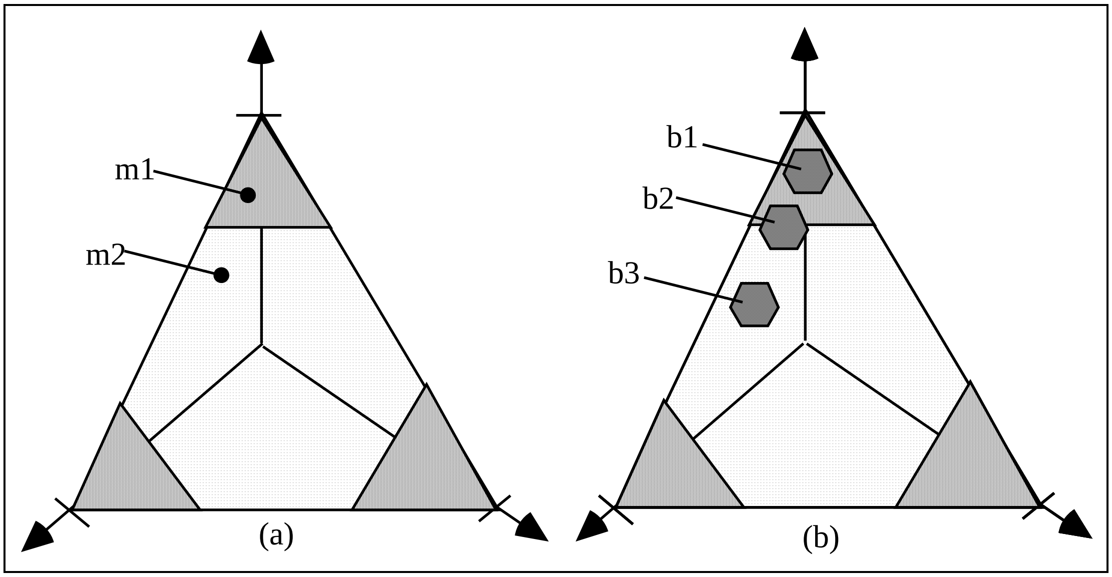

Figure 5.7: Feature abstraction based on a certain (a) and uncertain mixture field (b).

Figure 5.8 illustrates feature abstraction that derives feature partitions from mixture fields. Parts (a) and (b) apply the feature abstraction of chapter four to a certain (a) and uncertain mixture field (b), respectively. Part (c) illustrates the modified feature abstraction that is proposed later in this section. All three parts show a cross section through space along which mixtures represented by mixture fields gradually change from the mixture class "land" to that of "water". The mixtures of the certain mixture fields in (a) are illustrated by a single thick line. The uncertain mixture field in (b) and (c) uses mixture balls (here intervals) rather than mixtures. It is therefore visualized by two lines that mark the upper and lower bound of possible mixtures in the mixture balls. Gray bars mark the possible mixture classes that can be found in their associated regions.

For reference, (a) illustrates the classification of certain mixture fields. The mixture axis shows the mixture classes of land, water, and transition zone. Every disk, identified by its location, is assigned to one of these classes. The shaded bars a1, a2, and a3 illustrate how the classification relates regions (i.e., intervals

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54 Note that mixture balls cannot intersect with more than 2 mixture classes if field uncertainty is relatively small compared to the separation of mixture classes.

along the horizontal axis) with mixture classes (i.e., intervals along the vertical axis). Every region is then related to only one mixture class.

Part (b) of the figure illustrates the application of the original feature abstraction to uncertain mixture fields. Disks whose mixture balls are totally contained in a single mixture class are related to a single mixture class and thus feature (b1, b3, and b5). Disks whose mixture balls intersect with two mixture classes have to be related to two features, one being the transition zone (b2 and b4). Due to uncertainty, it is not known which of these two features the disk belongs to. The actual feature boundary can therefore lie anywhere inside the regions whose disks intersect two mixture classes (b2 and b4). Since the actual boundary location is unknown, any location chosen for the representation deviates from the actual boundary and therefore introduces approximation uncertainty.

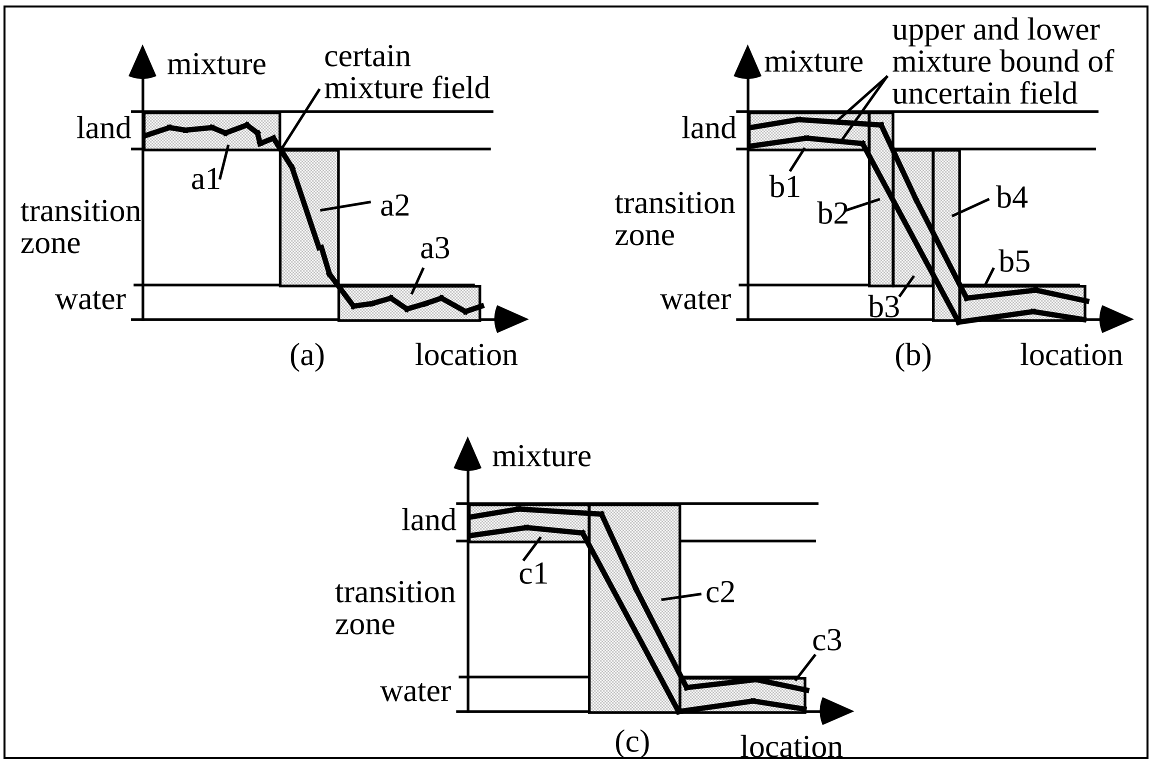

Figure 5.8: Original and modified feature abstraction applied to certain and uncertain mixture fields. The original feature abstraction is applied to a certain (a) and uncertain mixture field(b). The modified feature abstraction is applied to a uncertain mixture field (c).

To manage this uncertainty, a **modified feature abstraction** is proposed here (see figure 5.8.c). It results in **uncertain feature partitions**. Part (b) of the above figure has illustrated that the represented geometry of features always deviates from their actual geometry. According to Sinton, such a change in geometry is inseparable from a change in attribute (see section 3.7).

Following this philosophy, the modified feature abstraction defines the **geometry of all features** other than the transition zone as the **region of disks that carry the feature attribute for certain**. In part (b) of the above figure, these are the regions b1 and b5. The **geometries of these features are thus reduced in size** by the amount of locational uncertainty, while their **attributes stay unaffected**. This change in feature geometries has to go along with a **widening of the transition zone** such that features and transition zone again partition space. The widened transition zone is illustrated by the width of box c2 in part (c) of the above figure. Obviously the new transition zone now includes some disks with attributes of adjacent features. This must result in a change of transition zone attribute. Since the mixtures of absorbed "foreign" disks can fall in any mixture class, any mixture can now be found in the modified transition zone. In other words, **all knowledge of mixtures inside the transition zone is lost** as a consequence of the geometric approximation. This change of the transition zone attribute is visualized by the height of box c2 in part (c) of the figure.

Since in this modified feature abstraction, the transition zone is conceptually different from other features, **the term feature used in the remainder of this thesis will not include the transition zone**.

The above modified feature abstraction incorporates an **approximation of feature geometries** where all positional uncertainty is absorbed in the transition zone. The resulting **uncertain feature partitions** are still composed of features and the transition zone. The major difference to certain feature partitions is the attribute of the transition zone. It now allows arbitrary mixtures. Uncertainty decreases the size of features while widening the transition zone. In other words, it increases the region for which no knowledge about mixtures (and thus geographic objects) is available.

5.2.3 Uncertainty introduced by Finite Approximation of Regions

Feature partitions are subject to two kinds of uncertainty, (i) the one propagated from the underlying mixture field and (ii) additional uncertainty that is introduced during the finite representation of the feature partition. This section argues that both kinds of uncertainty can be combined and represented in the same format that was proposed in the previous section.

In feature representations, uncertainty is introduced by the approximation of feature geometries by regions that can be described with a finite number of parameters (such as polygons or raster zones[55]). Since finite approximations of regions deviate from the original region, they locally **include** disks from outside the actual region and/or **exclude** disks from its inside (see figure 5.9).

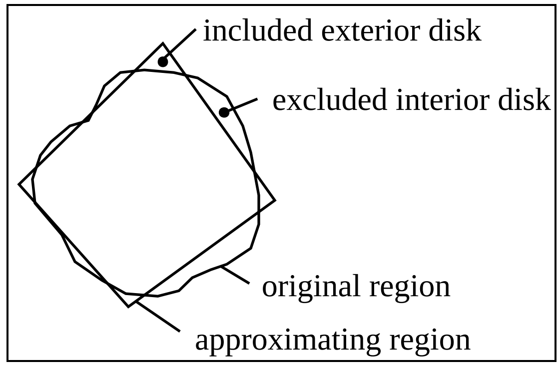

Figure 5.9: Approximation of a region.

Geometry and attributes cannot be separated [Csillag, 1991]. It is therefore necessary to study the effect of an approximation in the geometric domain on the attribute domain. Attributes of features are represented by mixture classes. The following investigates whether the approximating region can be described by the same mixture class (i.e., attribute) as the original region: While the exclusion of disks does not affect the mixture class that is associated with the approximating region, the inclusion of exterior disks adds mixtures from outside the original mixture class.

In order to use the original mixture class as attribute of the approximating region, this region has to be completely contained in the original region (see figure 5.10). The disks that are excluded during approximation of a feature geometry can be absorbed by the transition zone that is already associated with the whole range of possible mixtures. Again, uncertainty increases the size of the transition zone.

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55 In this context, polygons and zones are concepts of resolution-limited space and are therefore regions rather than point sets.

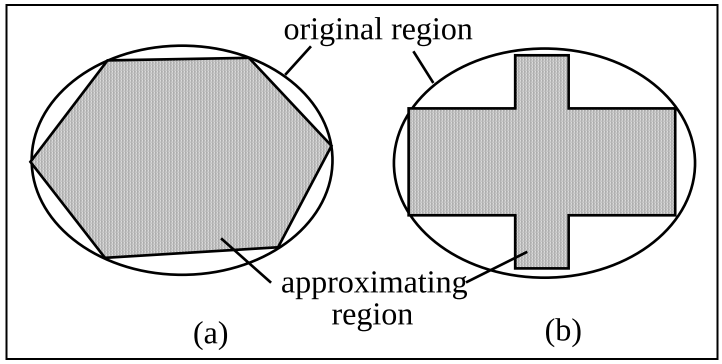

Figure 5.10: Finite kernel approximation of a region with a polygon (a) and a raster zone (b).

The above discussion shows that uncertainty propagated from uncertain mixture fields can be represented the same way as that originating from approximation in the geometric domain. The two kinds of uncertainty can be combined by letting any additional uncertainty further widen the transition zone.

Note that the proposed concept of uncertainty representation is not comparable to locational uncertainty [Chrisman, 1991]. The only similarity is that transition zones and, for example, probability contours [Dutton, 1992] that represent positional uncertainty "look" similar in their visualizations. Conceptually, however, they are different. For example, a probability contour that represents positional uncertainty contains the true boundary[56] with a certain probability. This is a geometric statement that relies on the existence of some kind of "spatial objects" with sharp boundaries and a known topological configuration (see section 2.2.5 for detail). In contrast, a transition zone is a region that is characterized by its lack of attribute knowledge[57]. This is not a geometric statement but rather documents that the relationship between geometry and attributes has been strictly preserved.

5.2.4 Philosophy of Uncertainty Representation

This section discusses the philosophy that underlies the proposed representation of uncertain feature partitions. It justifies the choice of representation and puts it

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56 The problem that the concept of "true boundary" is not sufficiently defined was discussed in section 2.2.5.

57 Note that the transition zone has an areal character even in the absence of uncertainty introduced by approximation (see chapter four). In this thesis, the effects of resolution-limitation are not treated as uncertainty. The width of the transition zone in the certain case could only be considered a "positional uncertainty" if resolution-limitation was considered an uncertainty.

in the context of hierarchical reasoning. Further, an explicit statement of the philosophy explains the differences in the uncertain representations of partitioning and stand-alone features.

The previous section argued that feature partitions can be subject to two kinds of uncertainty, namely (i) that which propagates from uncertain mixture fields and (ii) that introduced by the geometric approximation of regions. Both kinds of uncertainty are treated by a **worst-case scenario**; i.e., the representation captures only knowledge that is certain.

(i) In feature abstraction of the uncertain case, uncertainty allows the mixtures of disks to vary within the limits of a mixture ball. Since features are defined by a predominant geographic object and a level of homogeneity, the best case is represented by the mixture in the mixture ball that maximizes the percentage of the predominant geographic object; Similarly, the worst case minimizes the percentage of the predominant geographic object. Since uncertain feature partitions are based on a worst-case scenario, a disk is only part of a feature if, even in the worst case, its mixture still falls in the associated mixture class.

(ii) In the geometric domain, approximation leads to an uncertainty in the location of feature boundaries. Both, the representation of the approximate feature boundary and its uncertainty must be representable with a finite number of parameters. Figure 5.11 shows a possibility of such a representation that is motivated by Peuker's theory of the cartographic line [Peucker, 1975]. It is known that the original feature boundary lies anywhere inside the represented strips. In the best-case scenario, the feature is located such that the feature's area is maximized; in the worst-case, the feature's area is minimized. The absorption of uncertainty in the transition zone at the cost of feature area thus corresponds to worst-case scenario.

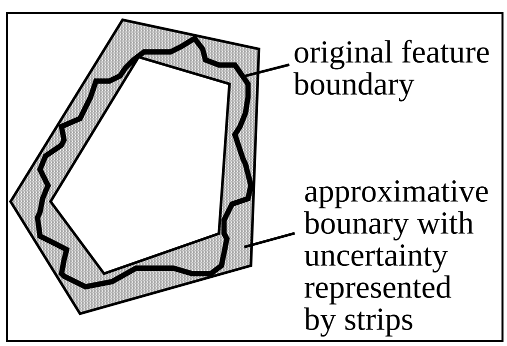

Figure 5.11: Uncertainty in the approximation of a feature geometry.

In computational geometry, such worst-case approximations are known as "**kernels**" [Nievergelt, 1989]. The concept is for example used in quadtrees [Samet, 1983] where the set of quad cells that are totally contained by a feature compose a kernel of this feature (see figure 5.12).

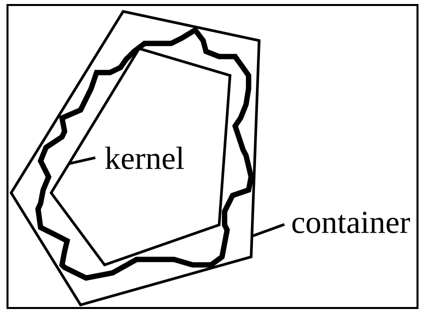

Figure 5.12: A kernel and container of a region.

While the proposed representation is based on the worst-case approach, it also implies a best-case approach where features and transition zone are united. In the best case, the feature attribute can then be found anywhere inside the feature itself or the transition zone. Best-case approximations of geometry are known as "containers" in computational geometry [Nievergelt, 1989] (see figure 5.12). Examples of containers are the strips used in strip trees [Ballard, 1981] where certain curve sections are totally contained by approximating strips.

Kernels and containers (and thus worst- and best-case approximations) are central to many cases of hierarchical reasoning. For example, in the case of strip trees, containers are used in the hierarchical search for curve intersections. The basic idea is to drastically reduce the search space in a coarse approximation in order to save a lot of computing in a more detailed approximation.

I believe that the representation of spatial knowledge at different levels of resolution should ultimately be used to support hierarchical reasoning processes (see also chapter eight). Representations based on a best- or worst-case approach are ideal for this purpose. In contrast, most currently used GIS representations use an average- or most-probable-case approach. For example, finite vector-format approximations of feature geometries attempt to minimize deviations from the actual geometry on average.

Current practice makes use of representations for hierarchical reasoning very difficult: It is known that average-case approximations are close to the actual world but the amount of error introduced by the approximation is usually unknown. This makes it difficult to relate coarser approximations to more detailed ones, and the world itself. Such relations are necessary, however, for hierarchical reasoning--as examples from computational geometry illustrate. The use of best- and worst-case approximations allow easy relation of coarser and finer approximations and are therefore well suited for hierarchical reasoning (see chapter eight).

5.3 Uncertain Stand-Alone Features

In the case of feature partitions, uncertainty was managed by reducing the size of features and altering the attribute of the transition zone. The same concept is not applicable to stand-alone features since they are endangered to be lost due to their relatively small size already. This section therefore proposes an alternative concept where the geometry of the stand-alone feature is widened while at the same time, its attribute is modified. Much of the following discussion focuses on the modified attribute that expresses what can be found in the approximate geometry of the uncertain stand-alone feature.

In the certain case, stand-alone features were special cases of partitioning features and therefore derived from mixture fields. Their disks were defined to contain an arbitrarily small amount of a certain geographic object (see section 4.4). This definition of stand-alone features is not practical in the uncertain case and is therefore modified in this section.

The problem of the certain definition is evident when looking at the component disks of stand-alone features. Since every component in a disk's mixture can now vary within the limits of uncertainty, every disk can potentially contain the required arbitrarily small percentage of the characteristic geographic object. In a best-case approach, all the disks in space would therefore be part of the stand-alone feature; if the mixture percentage was required to be above a certain threshold, stand-alone features would partly or completely disappear. The certain definition of stand-alone features is thus not applicable in an uncertain environment.

A solution for this problem is to express the mixture requirements at a higher resolution where the characteristic geographic object is relatively large and can more easily be detected with certainty. This allows minimal mixture requirements that allow detection but avoid the problem of eliminating major parts of the stand-alone feature. In the modified definition, **a disk is part of an uncertain stand-alone feature if it completely contains at least one smaller disk**[58] **that contains a detectable amount of the characteristic geographic object**.

In most practical cases, this detectable amount will be high enough to make the characteristic geographic object predominant in the small disk's mixture. The above definition is then equivalent to the following one: **A disk is part of an uncertain stand-alone feature if it contains at least one smaller disk that is part of the related feature in a feature partition of higher resolution.**

The higher resolved feature partition that underlies a stand-alone feature is based on a worst-case approach (see section 5.2.4). The weak requirement that at least one smaller

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[58] Since disks of different diameters are compared, the comparison must take place in Euclidean space of geographic reality.

disk has a predominant (or detectable) percentage of the characteristic geographic object in its mixture is compatible with a best-case approach: the geometry of the stand-alone feature can be seen as a container of the underlying feature. It is therefore natural to also deal with the uncertainty introduced by the finite approximation of the stand-alone feature's geometry with a best-case, container approach. In summary, the modified definition of stand-alone features uses a worst-case approach at high resolution to guarantee detectability in an uncertain environment and then uses a best-case approach to preserve the relevant knowledge at the coarser resolution of the stand-alone feature.

Chapter eight defines a transformation that maps from a feature partition to a stand-alone feature of equal resolution. This transformation does nothing but extract a single feature from the feature partition. Another transformation of chapter eight maps between two stand-alone features of different resolution. This transformation implements the best-case approach typical for stand-alone features. A combination of these two transformations allows derivation of a low-resolution stand-alone feature from a feature partition of higher resolution as specified in the above definition.

6 Finite Representation of Spatial Knowledge

The previous chapter has discussed approximations of mixture fields and feature partitions at a general level. Such approximations can be represented in the form of uncertain mixture fields and uncertain feature partitions. This chapter proposes special cases of uncertain mixture fields and feature partitions that can be (physically) represented with a finite number of parameters. They are special cases of the general uncertain representations in the same way as polygons are special cases of general point sets.

The goal of this chapter is to show that the concepts of uncertain mixture fields and feature partitions are physically representable on finite computers. To prove this point, the chapter discusses **only some**, not all possible finite parametrizations of uncertain resolution-limited representations. For example, the discussed finite parametrizations of mixture fields are limited to using either nearest-neighbor and linear interpolation. Additional finite parametrizations based on other interpolation methods are obviously possible but not discussed in this chapter.

In spite of the limited scope of this chapter, it clearly demonstrates how finite parametrization introduces format. This is most evident in the domain of feature partitions where the general shape of features is either approximated with **straight line segments** (i.e., a **vector format**) or **raster cells** (i.e., a **raster format**). Not surprisingly, the proposed finite parametrizations of uncertain mixture fields and feature partitions are therefore very similar to conventional vector and raster representations.

The major differences between conventional and the proposed representations are (i) their attributes, (ii) the incorporation of uncertainty representation, (iii) the use of an areal transition zone instead of a sharp boundary (only for feature partitions), and (vi) the use of resolution-limited rather than Euclidean space. The difference in attributes (i) is for example evident in the use of mixtures and mixture classes instead of a modal nominal attribute value (see also section 2.2.1). The incorporation of an uncertainty model in the proposed representations (ii) leads to the representation of mixture balls and the widening of the transition zone[59]. Neither mixture balls nor the transition zone have direct equivalents in conventional representations (see section 5.2.3 for a distinction of transition zone and positional uncertainty). A major difference is that the geometric entities of conventional representations are defined in Euclidean space while the proposed representations use resolution-limited space (iv). A raster cell in the proposed raster representation is then a special set of disks rather than a point set. Similarly, a resolution-limited polygon is a set of disks rather than points. Since both, Euclidean points and resolution-limited disks are identified by a coordinate pair, this difference does not show up at the level of physical representation in the form of data models or data structures. It is crucial for the conceptual understanding of the proposed representations, however.

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[59] Note that the transition zone is also areal in the absence of uncertainty introduced by approximation (see chapter 4).

The chapter is composed of two major parts. The first discusses finite parametrizations of uncertain mixture fields. While feature partitions and stand-alone features differ in their uncertainty model, they can be treated very similarly for finite representation. The second part therefore discusses the finite representation of uncertain feature partitions and stand-alone features together.

6.1 Finite Representation of Uncertain Mixture Fields

This section describes some finite parametrizations of uncertain mixture fields. The first section describes the concept of parametrization, namely sampling and interpolation. The second and third sections describe the cases of nearest-neighbor and linear interpolation. Finally, the fourth section discusses how conventional raster and TIN data structures can be modified to store such finite representations.

6.1.1 Concept of Finite Parametrization

The finite parametrizations described in this chapter are based on **sampling** and **interpolation**. The data acquisition from geographic reality is always based on sampling since the observation effort must be limited. Since sampling and interpolation can only yield an approximation of the actual (certain) mixture field, the resulting uncertain mixture field contains approximation uncertainty. This section discusses how such uncertainty is limited by limited resolution, sampling density, and interpolation method.

At infinite resolution, mixtures can abruptly change with location. For example, it can change from 100% land to 100% water. **Limited resolution** smoothes these abrupt transitions by limiting the **maximal rate of mixture change** that can be observed in IFOVs (see section 4.5.3). This maximal rate of change guarantees that the interpolation of mixtures between sampling location stays close to the actual mixture in this location. More precisely, the **maximal possible deviation between actual and interpolated mixture is then a function of the resolution**, **sampling density**, and **interpolation method**[60].

Since at limited resolution, mixtures change continuously with location, the approximation quality depends on how well an interpolation method reflects this

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[60] The maximal approximation error discussed in this section does not limit the possible configurations of geographic reality. It may be possible to further bound the maximal error if limitations in terms of size or shape of possible geographic objects are imposed. Like the model of geographic reality itself, such limitations are very application dependent and are therefore not explored in this thesis.

continuous character. For example, nearest-neighbor interpolation leads to a coarser approximation than a bi-cubic interpolation. In other words, the better the interpolation method, the lower the sampling density can be to achieve an equivalent approximation quality.

Figure 6.1 illustrates how approximation quality, and thus mixture uncertainty, depends on sampling density and resolution. Sampling density is directly visualized in the figure; resolution is visualized indirectly by the maximal rate of mixture change. The higher the resolution, the larger the mixture change over a given (spatial) distance can be (see section 4.5.3). The figure shows two situations of linear interpolation of mixtures between pairs of sampling disks (shown as points). The maximal rate of mixture change is visualized by curves. The interpolated mixture falls on the straight line between the points. The actual mixture can fall anywhere between the curves of maximal change. The maximal possible error therefore increases with the distance from the sampling location. In both situations (a and b), the maximal possible mixture uncertainty occurs in the middle between the sampling locations. The maximal uncertainty is marked with arrows. A comparison of situation (a) and (b) shows that in the case of linear interpolation, the maximal possible uncertainty occurs in the case where the mixtures in the sampling locations are equal (a). When the observed mixtures in the sampling disks differ considerably, the maximal possible approximation error is smaller.

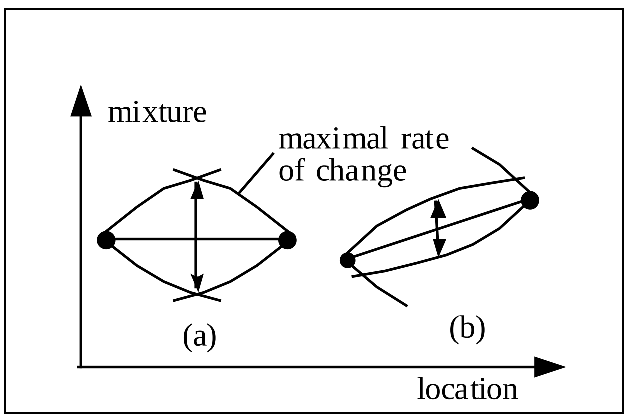

Figure 6.1: Interpolation error is limited by the maximal rate of mixture change and sampling density.

In uncertain mixture fields, the uncertainty introduced by approximation based on sampling and interpolation is represented by mixture balls (see section 5.1). In every location, the mixture ball must be large enough to contain the actual mixture for certain. The maximal rate of mixture change then determines the necessary radius of such mixture balls.

Since the maximal rate of change is not easily representable with few parameters, the radii of mixture balls can be represented by simpler functions that are easier to represent. For example, the representation described in the following section simply uses a constant mixture radius in the whole neighborhood of a sampling location. To capture uncertainty truthfully, the represented mixture radii must always be equal to or larger than the ones determined by the maximal rate of mixture change.

6.1.2 Nearest-Neighbor Interpolation

The simplest finite parametrization of uncertain mixture fields is based on sampling and nearest-neighbor interpolation. In nearest-neighbor interpolation, all disks that are closest to the same sampling location are described by the same mixture ball. Its radius must therefore be large enough such that the actual mixtures in any location inside the neighborhood fall inside the mixture ball.

Figure 6.2 illustrates this finite representation. It uses a cross section through space and a two-component mixture for easy visualization. The points on the location axes mark the sampling disks (or locations). The dashed vertical lines mark the neighborhood boundaries used by the nearest-neighbor interpolation. The black curve shows the mixtures as they are actually observable in geographic reality (see section 4.3.1). The gray points on this curve represent the observed mixtures of the sampling disks. These mixtures become the center points of mixture balls that represent the maximal possible uncertainty introduced by the finite parametrization within the neighborhood. In two-component mixtures, mixture balls degenerate to mixture intervals. An example of such an interval is shown as vertical line segments (b) in the figure. In nearest-neighbor interpolation, all locations within the neighborhood are described by the same mixture interval. The entireties of all center points of mixture intervals are shown as thin horizontal line segments in every neighborhood; the entirety of all intervals are visualized as gray rectangles. Mixture uncertainty (i.e., the height of the gray rectangles in the figure) is derived from the maximal rate of mixture change. It depends on the distance between sampling points and the mixture difference observed in the sampling points (see figure 6.1 above).

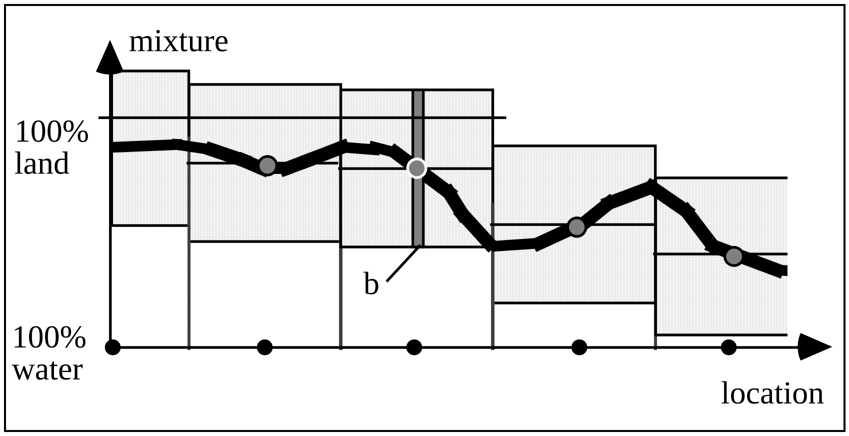

Figure 6.2: Finite representation of an uncertain mixture field based on regular sampling and nearest neighbor interpolation (cross section through space and two-component mixture).

Figure 6.2 shows regular sampling. The concept is equally valid for irregular sampling, however. In the case of regular sampling of resolution-limited space, neighborhoods are regions that are comparable to Euclidean raster cells; in the case of irregular sampling, they are regions comparable to Thiessen or Voronoi polygons [Gold, 1990].

6.1.3 Linear Interpolation

A refined finite representation of uncertain mixture fields results from linear interpolation of mixture balls between sampling locations. The linear interpolation is then based on a triangulation[61] of resolution-limited space where sampling disks are the vertices of triangles. Within each triangle, linear interpolation is used to determine the center points of mixture balls. Their radii are determined by a simple linear function of the distance from the closest sampling disk.

Figure 6.3 illustrates this concept. The upper half of the figure shows the triangulation of resolution-limited space. For easy visualization, disks are projected onto their center points. The lower half of the figure shows a two-component mixture along the marked cross section. The thin line segments between the gray points illustrate the **linear interpolation of the center points of**

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61 Linear interpolation requires a triangulation. This is evident for example in an elevation model where elevation is interpolated between sampling points. Linear interpolation then models the elevations between sampling points by segments of planes. Since a plane in the three-dimensional space defined by x, y, and elevation is determined by three points, a single plane segment is always associated with three points and is therefore triangular.

mixture balls; the height of the gray polygons again visualizes the local mixture-radii. The radii are determined by a linear function that approximates[62] the maximal rate of mixture change.

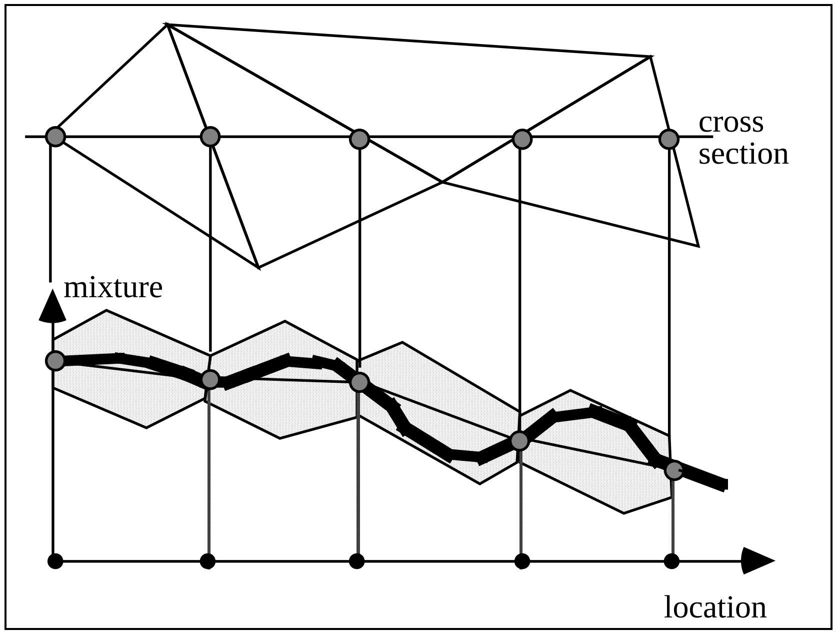

Figure 6.3: Finite representation of an uncertain mixture field based on sampling and linear interpolation. The upper half of the figure shows a triangulation of resolution-limited space; the lower half a two-component mixture along the marked cross section.

Saalfeld [1985] describes a method of linear interpolation for triangulations that is based on simplicial coordinates. It is directly applicable to mixture radii. It can also be applied to each mixture component separately while preserving the consistency of interpolated mixtures (i.e., their property that percentages add up to 100%).

6.1.4 Raster and TIN Storage of Uncertain Mixture Fields

The two representations proposed above can be stored in modified versions of **raster** and **triangular irregular networks** (TINs) data models. The choice of data model depends of the sampling strategy, rather than the interpolation method. [Laurini, 1992]. The modifications of the conventional data models are discussed in this section.

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62 and is always greater than.

Raster data models represent a single value per raster cell. Depending on the GIS, different types of cell values are possible. For example, Idrisi supports values of the type integer and real. To store the proposed representations in a raster model, **cell values of type mixture ball** must be supported.

Cells in conventional raster models are usually understood as special Euclidean point sets. For the storage of the proposed representations, **cells are resolution-limited regions** rather than point sets. Due to the structural equivalence between Euclidean and resolution-limited space (see section 4.2.2), the physical storage structure is the same for cells as point sets or cells as regions. The structural equivalence is most evident in the coordinate pair that identifies both, Euclidean points and resolution-limited disks.

Each cell of the proposed raster representations represents a sampling location, i.e., a disk. The cell's **mixture ball** (or attribute) thus **describes the cell's center disk**. In the case of nearest-neighbor interpolation, the mixture ball also describes all disks inside the cell.

In raster representations, the **cell size** (also called "raster resolution") specifies the **sampling density**. While this sampling density is often understood as a measure for resolution, it is **independent of the resolution concept proposed in this thesis** which is determined by the IFOV of the sensor. Instead, sampling density is a **measure of the approximation quality**: The more densely a mixture field is sampled, the smaller the uncertainty introduced by such finitization becomes (see section 6.1.1 above).

In the case of **irregular sampling** with an **interpolation method that is based on a triangulation**, uncertain mixture fields can be stored by **triangular irregular networks** (TINs) [Laurini, 1992]. TINs are usually associated with vector GIS. While conventional TINs are usually designed for ratio valued attributes (such as elevation), they have to be modified to allow mixture balls as attributes.

The **finite format** of uncertain mixture field representations must be captured in the form of **meta data**. These meta data describe the sampling strategy and interpolation method. The above **raster representation** is then specified by the **grid** used for regular sampling and its **interpolation method** (nearest-neighbor or linear). The TIN is specified by the **triangulation**[63] and **interpolation method**[64].

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63 Different triangulations of the same sampling points lead to different interpolated mixture values. A specific triangulation can be identified by its algorithm (such as Delauny triangulation) and a starting point to make the specification unique.

64 Besides linear interpolation, also bi-cubic interpolation would be possible in a triangulation.

Approximation uncertainty is not part of the meta data that describe whole representations, but an integral part of the representation in the form of mixture balls (that are used as cell values). Approximation uncertainty is affected by the finite format parameters, however. For example, at the same sampling density, a more sophisticated interpolation method (such as bi-cubic or liner) causes a smaller approximation error than nearest-neighbor interpolation.

6.2 Finite Representation of Uncertain Feature Partitions and Stand-Alone Features

This section discusses different possibilities of finite parametrization of uncertain feature partitions and stand-alone features. Since these two kinds of representations are very similar from a representation point of view, every parametrization alternative is applied to both of them.

The proposed finite parametrizations are very similar to conventional raster and vector data models [Laurini, 1992]. A major difference is the **areal** character of the **transition zone** that compares to sharp boundaries in most conventional representations. Another major difference is again the use of **resolution-limited space rather than Euclidean space**. The geometry of features is represented by resolution-limited regions rather than point sets. As in conventional raster and vector representations, regions that can be described by a finite number of parameters are used. Namely, these regions are **resolution-limited raster cells and zones** (see [Tomlin, 1983] for Euclidean raster zones), and **resolution-limited polygons**. While conceptually, resolution-limited regions are different from point sets, they can be physically represented in the same data structures.

In feature partitions, **feature attributes** are mixture classes. The special case of mixture classes used in this thesis can be represented by the identifier of the predominant geographic object and the level of homogeneity. Stand-alone features have their characteristic geographic features as attribute. The **major issues** of finitely representing uncertain feature partitions and stand-alone features **are of a geometric nature** and are therefore the focus of the following section.

In the case of mixture fields, uncertainty was introduced by sampling during data acquisition. The resulting uncertain mixture field is then the most detailed knowledge about geographic reality that is available. The approximation uncertainty is therefore not determined by comparison with more detailed knowledge but has to be derived theoretically. In the case of uncertain feature partitions and stand-alone features, more detailed finite representations are always available in the form of uncertain mixture fields. The approximation error is therefore determined by propagation of mixture field uncertainty. The origin of uncertainty is therefore not discussed in this section but will be explored in chapter eight which focuses on transformations from (uncertain) mixture fields to feature partitions and stand-alone features.

The remainder of section 6.2 discusses the geometric aspects of three different finite parametrizations of uncertain feature partitions and stand-alone features. The first uses a conventional raster representation. The second parametrization uses a conventional topologically structured vector model. Its major shortcoming is a lack of support for "cognitive topology" that is defined in section 6.2.3. The third parametrization uses a Voronoi-based vector representation [Gold, 1992] that avoids this shortcoming.

6.2.1 Raster Approximation

Regular tessellations [Peuquet, 1984] of resolution-limited space define **cells**, i.e., special kinds of regions. Square raster cells are a prominent example. Such regions are uniquely identified by their tessellation index, such as a pair of row and column number in a raster grid. Cells can thus be represented by a **single parameter**, namely their tessellation index. The second kind of special regions defined by tessellations are **zones** [Tomlin, 1983], i.e., sets of cells. They can be represented by a **finite set of** cell indices or **parameters**. Finite parametrizations of feature partitions and stand-alone features use such zones to approximate the actual feature and transition zone geometries.

Chapter five has shown how the uncertainty introduced by finite approximation of the geometry can be represented in uncertain feature partitions and stand-alone features. For **feature partitions**, a **worst-case approach** was proposed where the **approximating geometry is a kernel to the actual feature geometry**. The differences between approximating and actual geometry are then absorbed in the transition zone that separates features (see figure 6.4). The attribute of the transition zone was modified to preserve the relationship between geometry and attributes.

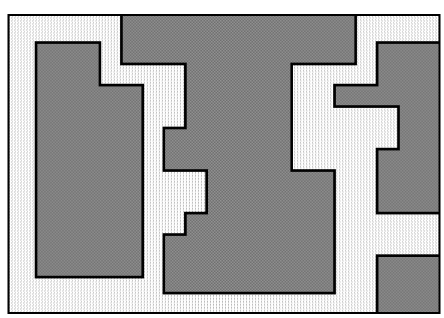

Figure 6.4: Raster approximation of a feature partition: features (dark gray) are separated by the transition zone (light gray).

The parts (a) and (b) of figure 6.5 show the effects of kernel approximation of feature geometries on feature partitions. Part (a) shows a feature with its actual geometry and raster kernel. Part (b) illustrates that it is not possible to find raster kernels for features that are relatively small compared to raster cells. In this case, the feature cannot be represented and the cells that intersect such features belong to the transition zone.

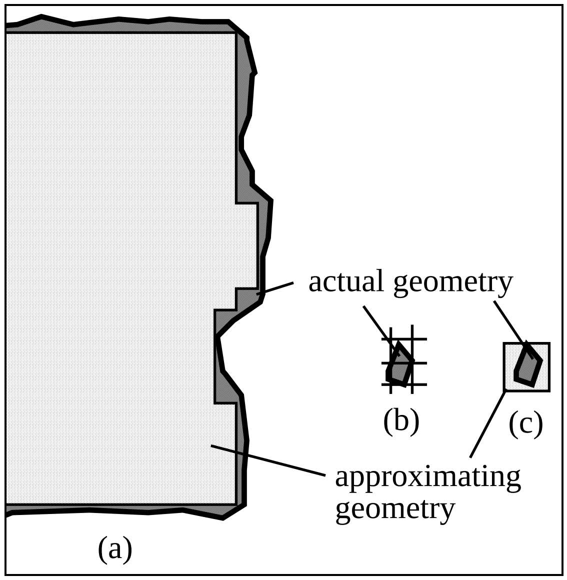

Figure 6.5: Effects of finite raster approximation of features in feature partitions (a and b) and of stand-alone features (c).

Stand-alone features are approximated with **containers** according to a **best-case approach** (see chapter five). Their attribute is chosen accordingly in order to preserve the relationship between geometry and attribute. Part (c) of the figure 6.5 above shows such a container approximation of a small feature. A comparison to (b) shows that the best-case approach preserves small features that are dissolved in the transition zone in the worst-case approach of feature partitions.

6.2.2 Vector Polygon Approximation

Another finite parametrization of general regions is defined by vector format. In resolution-limited space, resolution-limited vector **polygons** are finite parametrizations of general regions. Polygon boundaries are then chains of (resolution-limited) straight line segments. Polygons and boundaries can be represented by a finite number of **parameters**, namely the **locations** (or coordinate pairs) **of their vertices**. Figure 6.6 illustrates a polygonal kernel approximation of an actual feature geometry, as it is used in feature partitions.

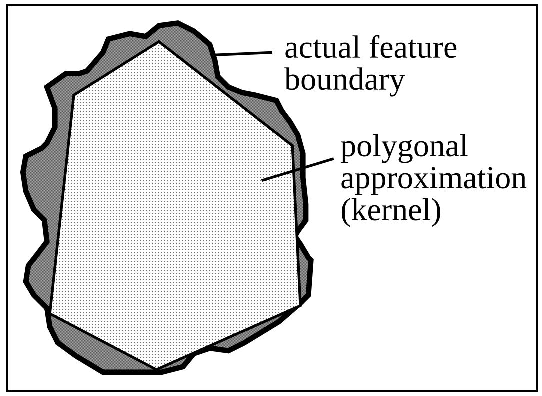

Figure 6.6: Polygonal approximation of feature geometry.

In partitions of space, **boundaries are shared** by adjacent regions. In feature partitions, the representation of **feature boundaries** is therefore **equivalent** to the representation of the **transition zone boundary**. More precisely, the transition zone boundary consists of all feature boundaries (see figure 6.7).

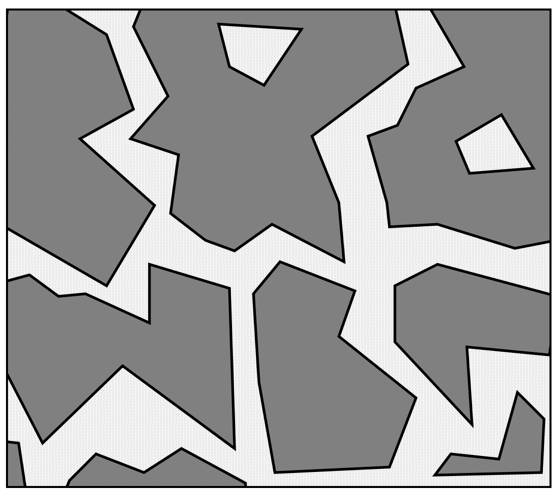

Figure 6.7: The transition zone boundary consists of all feature boundaries. Features are shown in dark, the transition zone in light gray.

The discussion above shows that conventional vector data models are well suited for the representation of the geometric aspects of resolution-limited features. This is particularly true for stand-alone objects, since they are represented separately from other objects. In feature partitions, the representation contains many features and therefore has to model their relationships. In conventional vector models, such relationships are captured by topological data models. The remainder discusses the shortcomings of topologic data models in the context of resolution-limited feature partitions.

In current vector GISs, topological relationships are captured by a data structuring that is based on **mathematical topology** [Broome, 1990] [Herring, 1987]. The topological structure relates regions to their boundaries and captures the neighborhood relationships between regions that share boundaries. **In mathematical topology, boundaries are therefore sharp and separate features from the transition zone**. While the transition zone is neighbor to all features, features cannot be neighbors since they are separated by the transition zone and cannot share boundary sections.

In contrast, in **human cognition**, **features can be neighbors** and the major purpose of the transition zone is to separate features. This cognitive understanding of (non-metric) relationships between features and transition zone is called "**cognitive topology"** in this thesis. While in mathematical topology, the **transition zone** is a region with the same status as a feature, it constitutes the **"boundary" between features** in cognitive topology. Features that "share" a section of the transition zone are then considered neighbors.

Since **GISs support human spatial reasoning, they must support cognitive topology** in order to be user friendly. In **conventional vector representations** that use sharp feature boundaries rather than an areal transition zone, this support comes automatically with the topological structuring that relates polygons to their boundaries. The relevance of cognitive topology in current GIS is, for example, documented by work on topologic relations and the incorporation of such relations into query languages [Egenhofer, 1993].

In resolution-limited representations, cognitive topology must be modeled explicitly. The following section proposes such a model that explains cognitive topology in terms of mathematical geometry. While the above vector representation for feature partitions lacks support of cognitive topology, section 6.2.4 reviews a Voronoi-based vector representation [Gold, 1992] that avoids this shortcoming.

6.2.3 "Cognitive Topology" in Resolution-Limited Feature Partitions

This section models cognitive topology in terms of mathematical geometry applied to resolution-limited space. In particular, it defines entities that correspond to "boundary sections" of cognitive topology that can be shared by features. This thesis uses the term "cognitive topology" to express that the human understanding of relations between features is modeled and to contrast the concepts with mathematical topology.

The basic concept used for the definition of such "boundary sections" is a **modified medial axis transform** of the transition zone. The original medial axis transform is defined, for example, in [Castleman, 1979, page 329]. An efficient algorithm for the computation of the medial axes of polygons was proposed by Lee [1982]. A definition of the medial axis is also found in appendix D.

For the purpose of defining cognitive topology, the original medial axis transform is modified in two respects: (i) only a **subset of the original medial axis** is used and (ii) the **maximal distance between the axis and boundary** of the transition zone is **limited**.

(i) Every point of the (original) medial axis of a point set is related to a disk that intersects the boundary of the point set in at least two points (see definition in appendix D for detail). Figure 6.8 illustrates this. It shows a section of the transition zone between features A and B and its medial axis. One point of the medial axis is shown with its related disk that intersects the transition zone boundary in two points. One of the shown points is in the boundary of feature A, the other in that of feature B. The modification of the medial axis **eliminates all axis points that are related to disks that intersect the boundary of only a single feature boundary**.

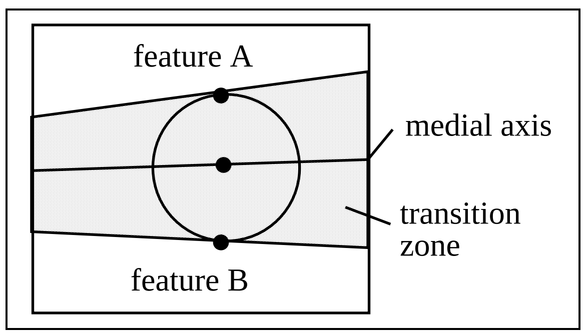

Figure 6.8: Disk related to a point of medial axis.

Figure 6.9 shows two examples for axis points that are eliminated in the modification. In (a), the eliminated point is inside a section of the transition zone inside a feature. In (b), the eliminated point is on a "free branch" of the original medial axis.

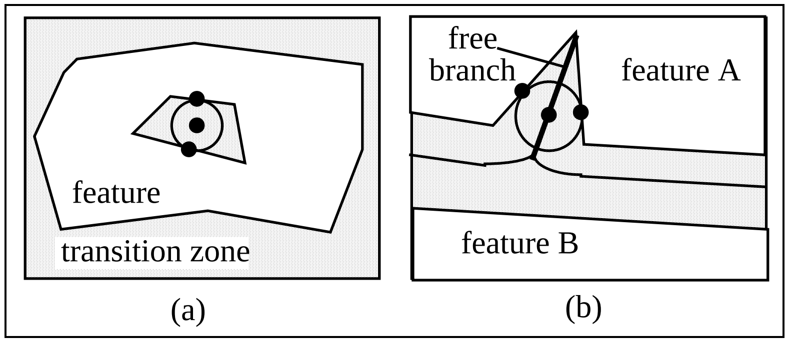

Figure 6.9: Examples for medial axis points that are eliminated in the modification.

(ii) In the original medial axis transform, the maximal possible distance between axis points and boundary of the transition zone is unlimited. In other words, the disks related with axis points can become very large in wide sections of the transition zone. In contrast, the modified medial axis transform **limits the maximal possible distance between axis points and boundary**. Where this distance is exceeded by the original transform, the modified transform creates a **transition feature**. The concept is illustrated in figure 6.10. The maximal allowable distance **d** is shown in a location marked by arrows. Where the original medial axis would exceed **d**, the **modified transform "splits" the axis into two sections that bound the transition feature and follow the feature boundaries at the distance of d**.

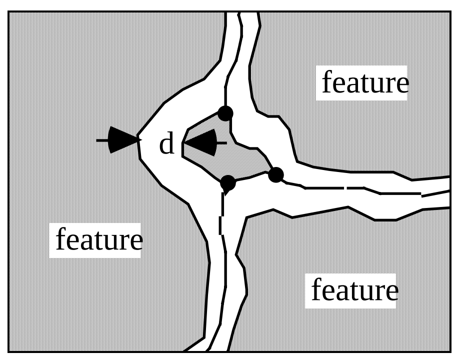

Figure 6.10: Example of a transition feature (shown as hatched area)

Cognitive topology can now be formalized by applying the **modified medial axis transform** to the transition zone. This transform then defines a "sharp boundary" of "neighboring" features and is therefore comparable to the sharp boundaries of conventional vector representations. The same topological structuring of conventional representations [Broome, 1990] [Herring, 1987] [Egenhofer, 1989b] can therefore be applied to the modified medial axis of the transition zone. Figure 6.11 shows how the medial axis can be subdivided into **nodes** and **edges** that are the entities of the conventional topological data structures.

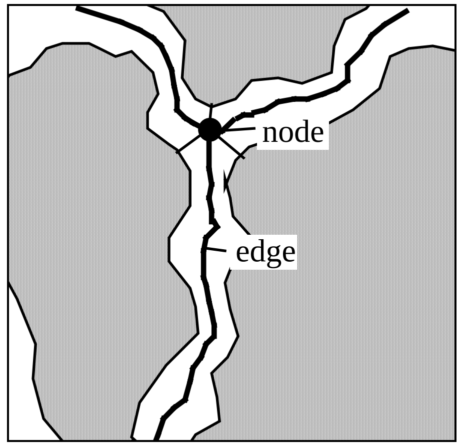

Figure 6.11: Medial axis of the transition zone and its topological structure that is defined by edges and nodes.

The subdivision of the modified medial axis into discrete topological entities (i.e., nodes and edges) can be propagated to a separation of the transition zone into discrete entities: every edge defines a subregion of the transition zone that contains all locations that are closest to this edge. This region is called the **edge neighborhood** and its visualized in figure 6.12.a. Similarly, a **node neighborhood** (see figure 6.12.b) is constructed from a node of the medial axis and the locations in the boundary of the edge neighborhood that are equally close to two edges.

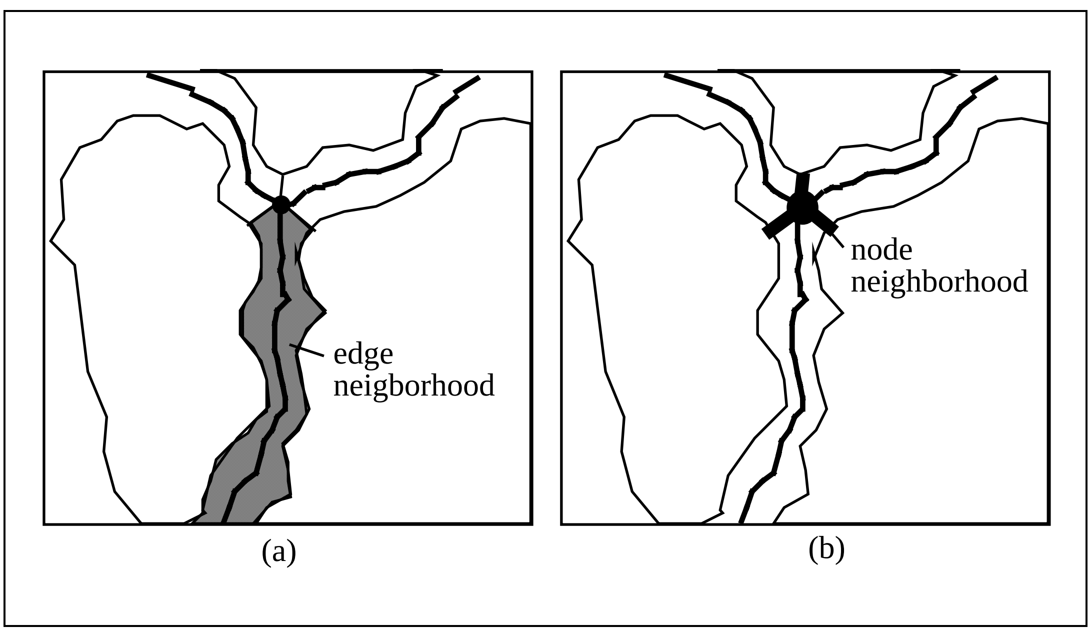

Figure 6.12: Edge (a) and node neighborhood (b).

Cognitive topology can now be **defined in relation to mathematical topology**[65]. Edge and node neighborhoods are comparable to topologic edges and nodes. While in mathematical topology, feature boundaries are composed of edges and nodes, they are composed of edge and node neighborhoods in cognitive topology. In mathematical topology, two features are neighbors if their boundaries share edges and/or nodes [Egenhofer, 1993]; in cognitive topology, two features are neighbors if their cognitive boundaries share edge and node neighborhoods.

While relatively thin sections of the transition zone are treated as cognitive boundaries, wide sections become **transition features** and take on the role of features. Their attribute is that of the transition zone, namely a total lack of knowledge about which geographic objects are found in its location. A cognitive neighborhood of features is based on spatial proximity since closeness of features usually implies some form of interaction. When two features are separated by a transition feature, they are not close enough for such interactions and are therefore not considered to be neighbors.

6.2.4 Voronoi-based Vector Representation

This section discusses the use of Voronoi-based [Gold, 1990] vector models for the representation of feature partitions. Their major advantage as compared to conventional (topological) vector models is their ability to **support cognitive topology** in resolution-limited feature partitions.

The Voronoi-based data model developed by Gold [1992] provides all the entities introduced in the above discussion of cognitive topology. Namely, these entities include edges and nodes of the transition zone's medial axis, as well as edge and node neighborhoods. This is evidence for a strong similarity of concepts used in Gold's Voronoi-based data model and those of cognitive topology that are defined in terms of the medial axis of the transition zone. Considering, that the medial axis of a polygon is a subset of the Voronoi diagram defined by the straight line segments and points of the polygon boundary [Lee, 1982] this similarity is not surprising. The cognitive neighborhood concept is actually equivalent to "spatial adjacency" proposed by Gold [1989]. Since Gold's data model promises to support cognitive topology in the context of an areal transition zone, it seems to be an ideal vector representation for resolution-limited feature partitions.

Gold's data model allows the representation of feature partitions with the following properties:

- **features** can be retrieved as polygons,

65 In particular, combinatorial and algebraic topology.

- the **transition zone** can be retrieved as a polygon,
- **edge neighborhoods** of the transition zone can be retrieved as polygons,
- **edges of the modified medial axis** of the transition zone can be retrieved as polylines[66],
- the **cognitive boundary** of a feature can be retrieved as sets of edges or edge neighborhoods.
- **transition features** can be retrieved as polygons,

Such a representation supports **two views of the transition zone**, namely it can be treated as an **areal region** or as a **cognitive boundary of features**. The former view is important, for example, during transformations to coarser levels of resolution (see chapter eight). The latter view supports reasoning about the relations between features. It is also used for visualization purposes (see chapter seven). Here, the transition zone is accessed in the form of edges of the modified medial axis for thin sections, and transition features for wide sections. The medial axis segments can then be visualized by graphical lines of a certain width; and transition features can be shown as polygons.

66 The medial axis of polygons consists of straight line segments and parabolic sections [Lee, 1982]. The parabolic sections would have to be approximated by polylines.

7 Visualization of Resolution-Limited Knowledge

While the main concern of this thesis is **representation** of resolution-limited spatial knowledge, the proposed representations would be of little use if they were not displayable. The proposed representations differ in two major aspects from conventional ones. Namely the proposed representations are based on **resolution-limited space** rather than Euclidean space, and they use **mixtures** that are uncommon in conventional representations. These differences prevent a straight forward application of conventional visualization methods as they are used in commercial GIS [Beard, 1991b] and cartography [Robinson, 1984]. A particular problem is that **resolution-limited space is not visually inspectable** like Euclidean space. The visualization methods proposed in this chapter therefore investigate possible **projections** of resolution-limited space to Euclidean space. Also, the possibilities of visualizing multi-component mixtures are discussed. This chapter thus focuses on the **particular needs of visualization of the proposed resolution-limited representations**.

In addition to proposing visualization techniques for resolution-limited representations, this chapter investigates the **relationship between the proposed resolution concept and the limitations of graphic media**. Namely, section 7.2.5 shows that the resolution limitation in feature partitions guarantees displayability within the limitations of graphic media. Consequently, the transformations to coarser levels of resolution that are part of the proposed spatial theory (see chapter eight) can be used in conjunction with a visualization method to yield coarser visual representations of the available spatial knowledge. The mentioned components of the proposed spatial theory could thus serve a purpose similar to that of line generalization methods [Buttenfield, 1985].

The chapter is structured in three sections that discuss the visualization of mixture fields, feature partitions, and stand-alone features, respectively.

7.1 Visualization of Mixture Fields

Mixture fields are continuous models and their visualization is therefore related to that of ratio valued fields such as elevation models. Instead of using ratio values, uncertain mixture fields use **mixture balls**. This section discusses how to reduce the complexity of mixture balls such that mixture fields can be visualized with methods adapted from ratio valued fields. The section is structured in two parts. The first discusses the visualization of mixtures in the center of mixture balls; the second the visualization of the uncertainty expressed by mixture ball radii. The separation of mixture ball center points and radii reduces the complexity of uncertain mixture fields to that of mixture valued fields.

7.1.1 Visualization of Mixture Valued Fields

The visualization of multi-component mixtures cannot be treated with visualization methods of ratio valued fields. For example, a mixture field does not directly define contours, and mixture values cannot be directly mapped to gray tones of pixels. Instead, the following three visualizations methods are possible: (i) mixture pie charts based on coarse sampling[67] of the mixture field, (ii) visualization of a single mixture component at a time, and (iii) visualization of the mixture field trends in the form of a feature partition. These possibilities are discussed in the following:

(i) Mixtures with a small number of components can easily be mapped to color coded pie charts (see figure 7.1). Showing such pie charts only at certain sampling locations avoids the problem of overlaps. In most cases, only a part of the mixture field will be displayable at a time. Panning and zooming are therefore crucial operations. Zooming should automatically change the density of the sampling such that the density of pie charts remains constant in the visualization.

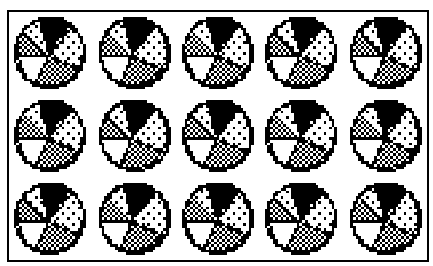

Figure 7.1: Pie chart visualization of mixture fields.

(ii) Looking at only a single mixture component at one time makes it possible to use the visualization methods for ratio valued fields. These visualization methods include contouring and using pixels whose gray value reflects the percentage of the visualized mixture component. A graphical user interface that supports multiple windows would allow views of several or all mixture components simultaneously. The following combination with the pie chart method would be possible in interactive systems: A pointing device can be used to probe single locations of single component visualization. The mixture at this location could then be displayed as a pie chart or numeric table.

(iii) A third visualization method could reduce the multi-component mixture to a single predominant mixture component [Leung, 1992]. This is equivalent to what

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67 The sampling density used for visualization does not have to be the same as that used for finite representation. For example, the representation sampling density can be encapsulated in a "virtual representation" [Stephan, 1993] such that it is invisible whether a mixture is a actually sampled or interpolated. The visualization module is then absolutely free in the choice of its sampling locations.

happens in the construction of a feature partition from a mixture field (see section 4.4). The visualization of feature partitions is thus a possible visualization of the trends of a mixture field. Rather than using only a single static visualization, an interactive system should support easy editing of the definition of mixture classes (see section 4.4) and give instantaneous feedback by updating the visualization of the feature partition. This would allow users to explore the mixture field and get a feel for the distribution of mixtures.

7.1.2 Visualization of Uncertainty

The above discussion of visualization considered only the center points of mixture balls. Several possibilities exist to visualize mixture field uncertainty: (i) spatial cross-sections that show upper and lower bounds of a single mixture component, (ii) toggling between extreme cases, and (iii) cursor controlled point probes.

(i) The first method shows a single mixture component along a user defined cross section. In the projection defined by the cross section and use of a single mixture component, mixture balls become vertical intervals. This makes it possible to show an upper and lower bound of the shown mixture component along the cross section (see figure 7.2).

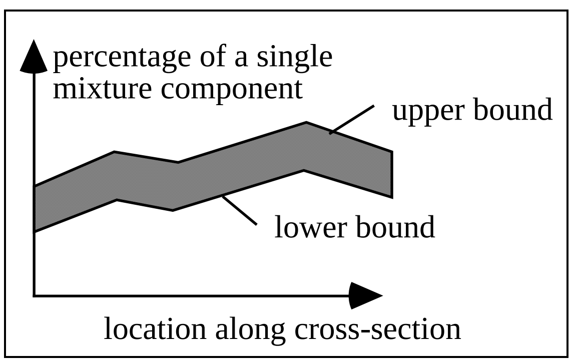

Figure 7.2: Visualization of mixture field uncertainty by upper and lower bounds of a single mixture component along cross-section.

(ii) Another possibility is to toggle between extreme cases of possible mixtures. In the former case, extremes are mixtures that maximize or minimize one of the mixture components. The toggling can be user controlled or automated as a slow movie.

(iii) A third possibility to visualize uncertainty is to use a pointing device to sample point probes. Several visualization options exist for the uncertainty in the probed location: An example is given in figure 7.3. Here, bars visualize the amount of uncertainty in each mixture component. This simple visualization could be shown in real-time while the cursor is moved continuously over space.

Another possibility would show all extreme mixtures for the probed location simultaneously.

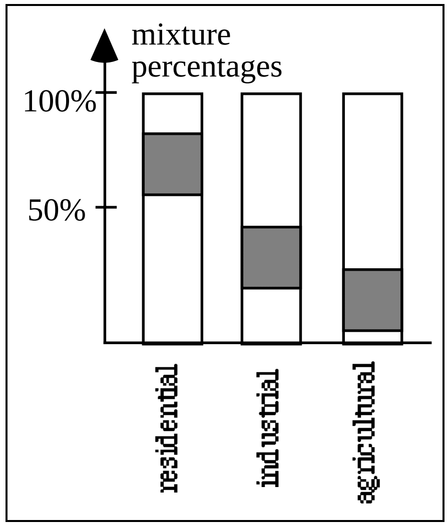

Figure 7.3: Visualization of mixture uncertainty in probed locations.

7.2 Visualization of Feature Partitions

Currently used data models for discrete views of the world are closely related to the commands of graphic output devices such as monitors, raster printers and vector plotters. The visualization of knowledge represented in such models is consequently straight forward. In contrast, resolution-limited object representations require more sophisticated means of visualization due to the greater gap between representation and graphics. This gap is most evident in the overlapping disks that cannot be visualized without first projecting them into Euclidean space. This section is concerned with such projections. One of the discussed projections shows similarities to Perkal's work on cartographic generalization [Perkal, 1966]. The relationship between limited resolution of representations and to resolution-limitations of (carto-) graphic displays is also discussed in this section.

7.2.1 Disk_to_Point Projection

The simplest projection that maps resolution-limited space to Euclidean space reduces resolution-limited disks to their center points (see figure 7.4). This projection is well suited for the visualization of the concepts that underlie the representation of resolution-limited features. It has consequently been used in the figures of previous chapters. This projection is not well suited for the visualization of the represented spatial knowledge in GISs, however. The major problem is that disk_to_point projections are incompatible with the resolution limitations of graphic media. For example, very small features would be projected into point sets that are too small for display; no minimal dimensions can be guaranteed with this visualization approach.

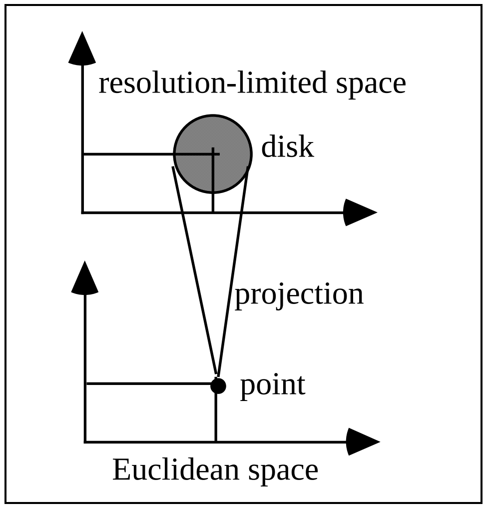

Figure 7.4: Disk_to_point projection.

7.2.2 Review of Perkal's Work on Cartographic Generalization

In an attempt to formalize the problem of cartographic generalization, Julian Perkal [1966] proposed a well-defined procedure to transform a map from one scale to another. This procedure was already discussed in section 2.2.3. This section interprets Perkal's work in the framework of the proposed spatial theory.

Perkal's maps consist of a family of point sets that partition Euclidean space. For example, a map could partition space into "land" and "water". His generalization procedure consists of moving a disk over the whole space. The disk radius is a measure of resolution. If a disk contains only points of a single attribute such as "water" or "land", all contained points are assigned the same attribute in the generalized map. If a disk contains a mixture of attributes (e.g., "land" and "water"), no attribute is assigned to the contained points. Points without attribute assignments form the generalized boundary (see [Perkal, 1966], page 4).

Figure 7.5 illustrates Perkal's generalization procedure. (a) shows the original map with the two point sets "land" and "water". The moving disk is shown in critical locations where the attribute changes from pure to a mixture in (b). The generalized map is shown in (c). Note that the boundary has become an area in certain parts (hatched section).

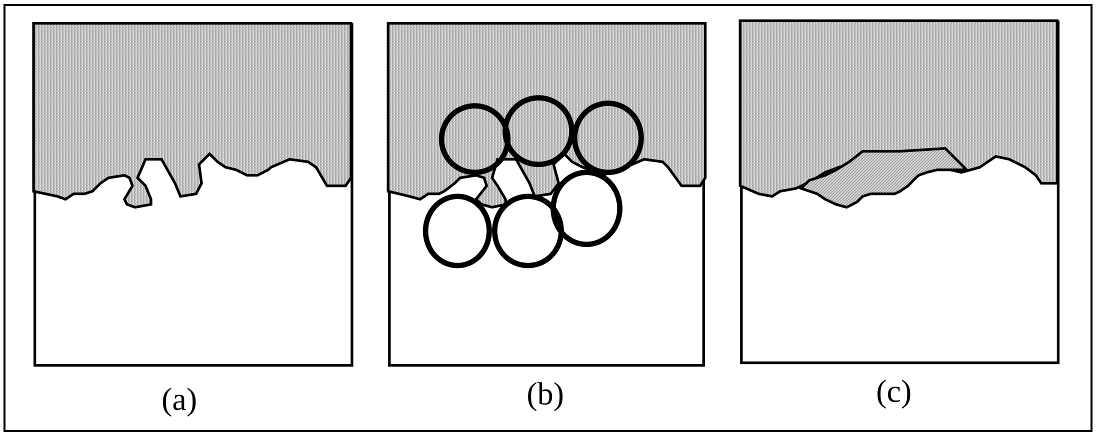

Figure 7.5: Illustration of Perkal's generalization method.

For the purpose of this discussion, Perkal's procedure is interpreted to consist of two steps: (1) the classification of resolution-limited disks and (2) a projection of the resulting resolution-limited feature partition to Euclidean space. The classification of disks (1) is equivalent to the feature abstraction proposed in chapter four. Every disk contains a mixture of the geographic objects land and water. Perkal uses the mixture partition that is defined by a level of homogeneity of 100%. Obviously, the result is a special case of a feature partition. All disks with a 100% pure mixture belong to the features "land" or "water" and all other disks to the transition zone.

The second step (2) now projects the resolution-limited feature geometries to Euclidean space. This projection is visualized in figure 7.6. The figure shows resolution limited space as a three-dimensional box and two of its disks. The light filling marks the left disk as an inhomogeneous disk, and the dark filling represents a disk that contains purely "water". While these disks can coexist with different attributes, their projection to Euclidean space causes a potential problem, namely what attribute to assign to their intersection area.

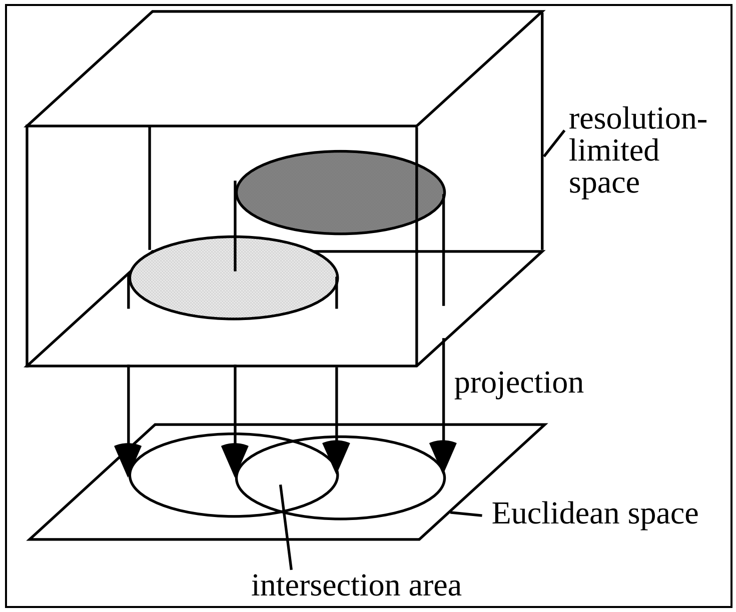

Figure 7.6: Projection from resolution-limited to Euclidean space.

Figure 7.7 shows two possible solutions to the overlap problem. In (a), the inhomogeneous disk is put in the foreground, while the water disk is part of the background. The reverse is shown in (b).

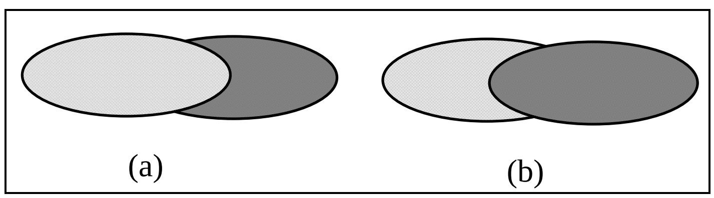

Figure 7.7: Foreground/Background projection of disks.

While the above figure shows only two disks at a time, figure 7.8 illustrates what happens if the foreground is constructed as the union of all water disks, while the background is formed by the union of all inhomogeneous disks.

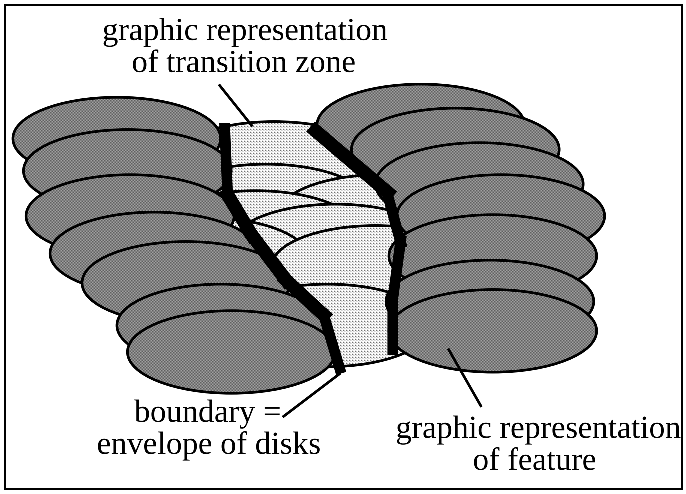

Figure 7.8: Foreground/Background projection of disks defines the boundary between the graphic representation of the transition zone and features.

In the case Perkal's generalization procedure, all **feature disks** are projected to the **foreground** while all **inhomogeneous disks** form the **background**. Note that the 100% level of homogeneity guarantees that disks of different features (such as land and water) do not overlap. If the projected disks are open, features are always separated by a boundary line or boundary area that is formed by what is visible of the background through the "gaps" and "holes" in the foreground.

In summary, this section has interpreted Perkal's generalization procedure in the framework of the proposed spatial theory. Perkal's original procedure is equivalent to the above two-step interpretation that first creates a special case of a feature partition and then projects it to Euclidean space.

7.2.3 Precise Foreground/Background Projection

This section generalizes the projection that Perkal uses in the two-step interpretation to the case of limited level of homogeneity. Since Perkal uses a 100% level of homogeneity, disks of different features (such as "water" and "land") are always disjoint; they are either separated by a sharp line or a boundary area. If the level of homogeneity is less than 100%, disks of different features can overlap in their projection to Euclidean space. Figure 7.9 illustrates such a situation: The class associations "land" and "water" in geographic reality are shown as white and gray areas, respectively. Two disks are superimposed. Both contain 80% of the class association "land" or "water", respectively. At level of homogeneity below 100%, features can thus obviously overlap in their projection to Euclidean space.

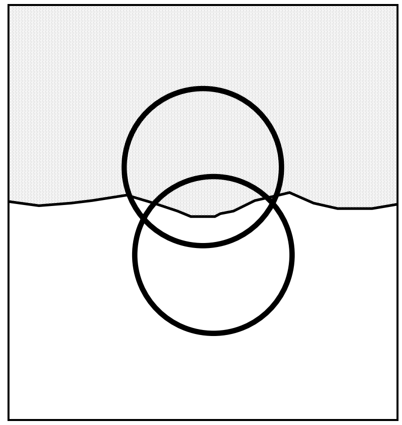

Figure 7.9: The class associations "land" and "water" in geographic reality and two disks that both contain 80% of one class association and 20% of the other.

To avoid the problem, the above projection can be generalized. Figure 7.10 shows such a generalization. Resolution-limited disks are now projected to graphic disks of a reduced size using a **disk_to_disk projection**. Overlaps are still resolved by a **foreground/background solution**. In this thesis, the combination of a disk_to_disk projection and a foreground/background overlap solution is referred to as a **precise foreground/background solution.**

The maximal radius of the graphic disks used in the disk_to_disk projection can be derived from the level of homogeneity using the maximal rate of mixture change (see section 4.5.3). For example, the transition from 80% "water" to 80% "land" corresponds to a minimal mixture distance of 60% . This is evident since 80% or more of "water" can include a maximum of 20% "land"; thus the mixture changes from 20% or less "land" to 80% or more "land"; this changes the mixture by 60% (i.e., 80%-20%) or more. The maximal rate of change relates a mixture distance of 60% to a minimal spatial distance of one disk radius (see table 4.3 in section 4.5.3). If the disk center points are at least one disk radius apart, these resolution-limited disks can be visualized as graphic disks of half the original radius; this guarantees that in Euclidean space, disks of different features are always disjoint.

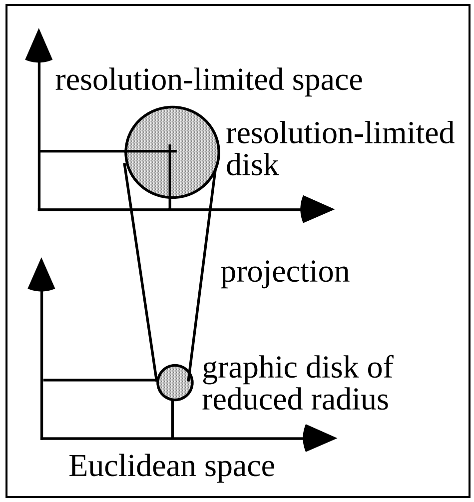

Figure 7.10: Disk_to_disk projection used as part of the general foreground/background projection.

7.2.4 Approximate Foreground/Background Projection

While the above projection resolves the overlap problem and thus makes display possible, it can be computationally expensive. Namely, the boundary between the graphic representations of features and the transition zone must be computed as the envelope[68] of all visualization disks of the foreground (see figure 7.8 above). The computation of the envelope is comparable to the construction of a buffer around a disk_to_point visualization of features. Buffering is an expensive operation in vector representations. This section therefore proposes an approximative foregound/background projection that is much faster and adequate for most purposes.

While in the precise foreground/background projection, the transition zone's graphic representation is usually a thin band, it varies in width. As a simplification of the above foregound/background projection, this section proposes to **graphically represent the transition zone by a curve of constant width**. This is obviously an **approximation** compared to the precise foreground/background projection. The simplified projection is therefore called **approximate foreground/background projection**.

The constant width curve that represents the **transition zone** follows the **modified medial axis** (see section 6.2.3). In the graphic representation, the transition zone

68 the envelope is the boundary of the union of disks

curve forms the (cognitive) boundary of **features**. Individual features are thus bound by the edges of the transition zone curve. Wide sections of the transition zone, i.e., transition features, are visualized like normal features. From a graphical point of view, features are represented by a **fill** inside cycles of the transition zone curve.

Figure 7.11 illustrates this by overlaying the following three layers: (i) a disk_to_point projection of a feature partition, (ii) its graphic representation produced by an approximate foreground/background projection, and (iii) its precise foreground/background projection.

(i) In their disk_to_point projection, features are shown as gray polygons, the transition zone is left white. (ii) The approximate foreground/background visualization is illustrated by the constant width curve that is the graphic representation of the transition zone and follows the modified medial axis. (iii) The precise foreground/background visualization is sketched by showing some graphic foreground disks whose envelope forms the boundary between the transition zone and features in their precise graphic representation. The graphic foreground (feature) disks are visualized by circles. A transition feature is represented by the polygon **c**.

The differences between precise and approximate foreground/background visualization can be seen, for example, in the locations **a** and **b**. The precise boundary of the graphic feature representation is given by the envelope of all graphic disks in the foreground. The figure shows some of these disks that are projected from resolution-limited disks (shown as points in the center of circles) along the feature boundary. Since in location **a**, the envelope of circles clearly intersect with the constant width curve of the approximate visualization, the approximation is locally too wide. In contrast, in the location **b**, the envelope used in the precise graphic representation is well apart from the constant width curve. The approximate graphic representation is therefore locally too narrow. The approximation error is limited by the use of transition features. They prevent excessively wide sections of the transition zone to be approximated by a relatively thin curve.

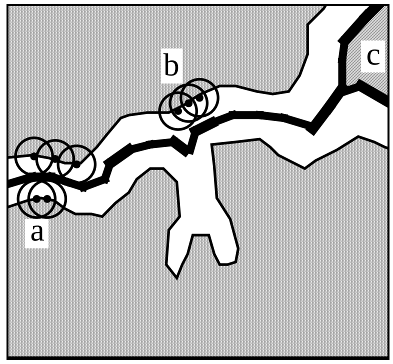

Figure 7.11: Approximate and precise background/foreground approximation compared to a disk_to_point projection of a feature partition.

I believe that this approximate visualization is sufficient for most purposes of human interpretation. The information about data quality that is lost in this approximation can be visualized in better ways (see section 7.2.6). Note that the error introduced by the approximation affects only the visualization while leaving the actual representation unaffected.

7.2.5 Spatial Resolution and Limitations of Graphic Media

This section attempts to prove that approximate foreground/background visualizations are always displayable within the limitations of graphic media. For this purpose, it uses the following two criteria for graphic displayability:

(i) The **transition zone must be visualizable with minimal graphical dimensions** (e.g., a line width) **such that its topology is preserved by its graphic representation**. This requirement prevents **cognitive feature boundaries** from becoming **self-intersecting** as an effect of minimal graphical dimensions.

(ii) Also the **graphic representation of features** shall have minimal dimensions. This can be formalized in terms of a disk_to_disk projection where graphic disks are of a minimal size. It is then required that the **disk_to_disk projection of a feature is totally contained in its graphic representation**. A feature's graphic representation can thus never be narrower than the associated graphic disks.

To illustrate the effect of the requirements, figure 7.12 shows two cases that fail to satisfy these requirements. In (a), the minimal drawing width of the transition zone leads to a self-intersecting cognitive feature boundary (see arrow). Obviously, the transition zone and its graphic representation are not topologically equivalent. In (b), the transition zone topology is preserved in the visualization. However, the feature inside the cycling section of the transition zone has smaller than minimal graphical dimensions. This is evident when comparing the feature's graphic representation to the line width of the graphic transition zone.

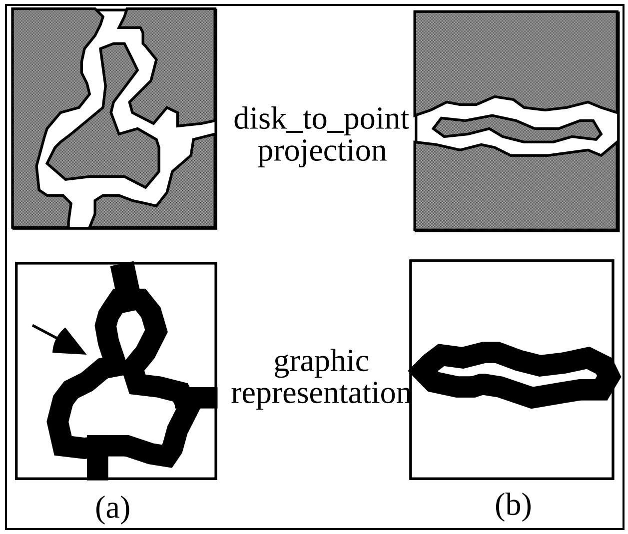

Figure 7.12: Two cases where the requirements for displayability are violated.

To prove that the approximate foreground/background visualization satisfies this first criterion of displayability, we have to show that the modified medial axis can be **drawn** in a certain **line width** $\mathbf{w_l}$ without being self-intersecting. For simplicity, the following argument and figures use the context of a disk_to_point projection of the feature partition. The proof uses the **minimal width of the transition zone** $\mathbf{w_{tz}}$ that can be derived from the maximal rate of mixture change (see section 7.2.3) and is thus **a function of the resolution**. Note that the minimal width is only guaranteed for the modified medial axis (as opposed to free branches and other parts of the original medial axis) that separates different features (rather than separate components of the same feature).

The minimal width of the transition zone $\mathbf{w_{tz}}$ guarantees that all disks of diameter of $\mathbf{d} \bullet \mathbf{w_{tz}}$ that are centered on arbitrary points of the transition zone's modified medial axis are totally contained in the transition zone. Figure 7.13 illustrates this. The large disk in location **a** marks a section of the transition zone that has the minimal width of $\mathbf{w_{tz}}$. The modified medial axis by definition goes through the center point of this disk. A smaller open disk of diameter **d** is contained in the larger disk. It is obvious that such open disks are always completely contained in the transition zone.

The approximate foreground/background visualization of the transition zone as a curve of constant width can be seen as the union of infinitely many closed disks whose diameter is equal to the drawing line width $\mathbf{w_l}$ (see location **b** in figure 7.13). If $\mathbf{w_l}$ is smaller than $\mathbf{w_{tz}}$[69], all these disks, and thus the transition zone's graphic representation, are always contained in the transition zone. Since the transition zone's graphic representation (of an adequate line width) never goes outside the boundaries of the transition zone, it cannot be self-intersecting.

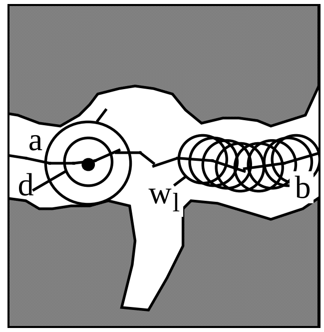

Figure 7.13: The minimal width of the transition zone compared to the width of the curve that visualizes the transition zone.

The second criterion requires that a disk_to_disk projection of a feature is totally contained in the feature's graphic representation. In the approximate foreground/background visualization, the feature's graphic representation is the fill inside a cycle of the curve that represents the transition zone. To prove that the second criterion of displayability is always satisfied for this kind of visualization, it has to be shown that the disk_to_disk projection of an arbitrary feature disk is always disjoint from the transition zone curve. The following argument assumes that the graphic disks used in the disk_to_disk projection are open.

Figure 7.14 shows that every disk of a feature can always be projected to a graphic disk whose diameter is $\mathbf{w_f} = \mathbf{w_{tz}} - \mathbf{w_l}$. In the worst-case scenario, the transition zone reaches its minimal possible width $\mathbf{w_{tz}}$. This is illustrated by the larger disk around point **M**. The small disk around **M** illustrates the drawing width $\mathbf{w_l}$ of the transition zone's graphic representation. Point **P** represents a resolution-limited disk at the feature boundary (in its disk_to_point projection). Its disk_to_disk projection results in the illustrated disk of diameter $\mathbf{w_f}$. Considering that the transition zone disks are closed, the graphic feature disks open, and the diameters are related by $\mathbf{w_f} = \mathbf{w_{tz}} - \mathbf{w_l}$, the disk_to_disk projections of arbitrary feature disks are always disjoint from the transition zone curve.

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69 Note that since these disks are closed, their diameter cannot be equal to the minimal width but has to be smaller.

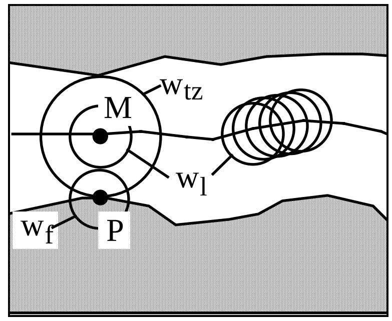

Figure 7.14: Relations between minimal transition zone width, width of the transition zone's graphic representation, and minimal size of a feature's graphic representation.

7.2.6 Visualization of Data Quality

In the context of feature partitions, two measures are suited for the expression of data quality. Namely, these are the (i) **transition zone width** and (ii) **the level of homogeneity in conjunction with resolution**. Both these aspects are discussed in this section.

(i) Visualizing features in the foreground as is done in foreground/background projections, gives a rather optimistic view of the available spatial knowledge since the graphic representation reduces the size of the transition zone in favor of features. Further, the constant width curve used in the approximate foreground/background projection suggests a crisp boundary between features. To visualize resolution effects on feature boundaries, the transition zone can be put in the foreground. The possibility to toggle back and forth between putting features or the transition zone in the foreground may give the best understanding of the spatial knowledge and its uncertainty.

Putting transition zone disks in the foreground requires the computation of the envelope of disks. This envelope can be achieved graphically by drawing the disk_to_point projected transition zone boundary with an appropriate line width. If the same drawing color is used for this transition boundary and the fill, problems of undisplayably small sections of the fill will not occur. The visualization of uncertainty should thus be adequately fast.

(ii) The second kind of data quality is expressed by the level of homogeneity in conjunction with resolution. Together, they determine how large and dense inhomogeneitites in features can be. The allowed inhomogeneity of features can be visualized with a concept proposed by Leung et al [1992]: A random process creates different possible configurations of inhomogeneities in the otherwise pure feature. Such a process could for example create small islands of "land" in a "water" feature, such that every disk still contains at least 80% of "water".

7.3 Visualization of Stand-Alone Features

From a representation point of view, stand-alone features preserve relevant knowledge that would get lost in a feature partition of the same resolution; from a visualization point of view, the displayability of this preserved information is of concern. This section addresses this issue first for a single stand-alone feature and then for one or several stand-alone features displayed on top of a feature partition.

7.3.1 Visualization of Single Stand-Alone Features

A first concern of visualization is that a single stand-alone feature can be visualized within the limitations of graphic media. For this purpose, two cases are distinguished: visualization of the feature by (i) its container, and (ii) by an arbitrary symbol.

(i) Stand-alone features are represented by containers (see section 5.3). They can be visualized by directly displaying these containers. An adequate visualization for this purpose is the disk_to_point projection (see section 7.2.1). Due to the definition of stand-alone features, this projection guarantees minimal dimensions of the graphic representation: In the underlying higher resolution feature partition used in the definition of the stand-alone feature (see section 5.3), the stand-alone feature shows up as at least one high resolution disk. This guarantees the minimal dimension of the container in its disk_to_point visualization. This is evident from figure 7.15 which shows two of the resolution-limited disks that make up the container. The figure assumes that the geographic object shows up only as a single high resolution disk that is shown in gray. This is the worst-case scenario in respect of minimal dimensions of the graphic representation. The graphic representation of the stand-alone feature is given by the disk_to_point projection of all disks in its resolution-limited container. The points projected from the two shown disks are visualized in the figure. The boundary of the complete graphic representation is illustrated by a circle through these two points. It is evident that the smallest possible graphic representation of a stand-alone feature is a disk whose radius is larger than the resolution (i.e., diameter of the resolution-limited disks).

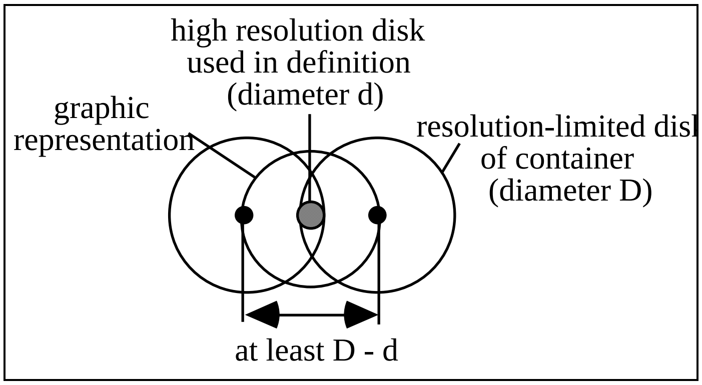

Figure 7.15: Containers of stand-alone features guarantee minimal dimensions of graphic representations based on a disk_to_point projection.

(ii) Stand-alone features can alternatively be visualized by arbitrary symbols that are centered on the feature's container. Examples of such symbols are disks that represent cities or double lines that represent roads. Some symbols can be constructed as exaggerated versions of a higher resolution appearance of the feature: for example, a fjord that would not be identifiable by its shape at the resolution at hand can be symbolized by its lowest-resolution appearance that is still recognizable. Since the choice of symbol is arbitrary, it is always possible to make symbols satisfy display limitations.

7.3.2 Visual Integration of Stand-Alone Features with a Feature Partition

Displaying a single stand-alone feature is rather uncommon; and it is usual to display several stand-alone features on top of a (single) feature partition. While the graphic representation of feature partitions partition Euclidean space, the graphic representations of stand-alone features can overlap with that of other stand-alone or partitioning features when displayed simultaneously.

Stand-alone features are usually put in the foreground over feature partitions. They thus mask part of the feature partition. Also, different stand-alone features can compete for the same space. Depending on the configuration, these masking and competition problems may eliminate some relevant information from the visualization. A straight forward solution of this problem is the support of interactive toggling that includes or removes individual stand-alone features from the visualization. Alternatively, the overlap areas could be marked as "conflict regions"; and fast zooming to a higher resolution would allow visualization of all involved geographic objects as in a higher resolved feature partition.

In the case of non-interactive systems or hard copy visualization, these simple methods of conflict resolution are inapplicable. A suitable solution to the conflict problem would then be to locally deform space. Since at infinite resolution, no conflicts between geographic objects occur, it will be possible to use deformations to make more space in densely populated regions at the cost of sparsely populated ones. The location occupied by a very small region can then be locally deformed such that it becomes large enough to accommodate the geometry of its graphic representation in the form of a stand-alone feature. Such a local deformation then automatically displaces features in its proximity. Note that name labels cause very similar visualization problems as overlapping stand-alone features since both compete for space already used by feature partitions.

A possible implementation of local deformations of space uses rubber sheeting based on a triangulation [Saalfeld, 1985]. Topological constraints could be used such that relevant topologic relations of geographic objects have their equivalents in the related stand-alone objects [Kainz, 1994].

8 Meta Data and Transformations between Resolution-Limited Representations

The previous chapters have proposed resolution-limited representations that are part of the consistent spatial theory. Namely, these representations are uncertain mixture fields, feature-partitions, and stand-alone features. Depending on type and parameters of the representations, they model different kinds of spatial knowledge at different levels of detail. Only uncertain resolution-limited representations are considered since only transformations between finitely approximated, and thus uncertain, representations are of interest. This chapter defines **meta data** that captures the knowledge content of representations and **transformations between the proposed representations**.

Figure 8.1 illustrates the concept that underlies the definition of meta data and transformations. R1 and R2 are resolution-limited representations that are derived from the model of geographic reality by one of the abstraction processes described in the chapters four and five. The **meta data** are the **parameters of these abstraction processes**. A representation can therefore always be derived from its meta data and geographic reality. While geographic reality describes the actual state of the world, the meta data describe the **knowledge content of representations**, i.e., the kinds and levels of abstraction of the represented spatial knowledge.

Transformations between representations are the **basic tool for data integration**. A transformation **T** between two representations R1 and R2 is completely determined by the meta data of R1 and R2. This is possible by defining T such that T (A1) = A2, since both, A1 and A2, are completely determined by meta data.

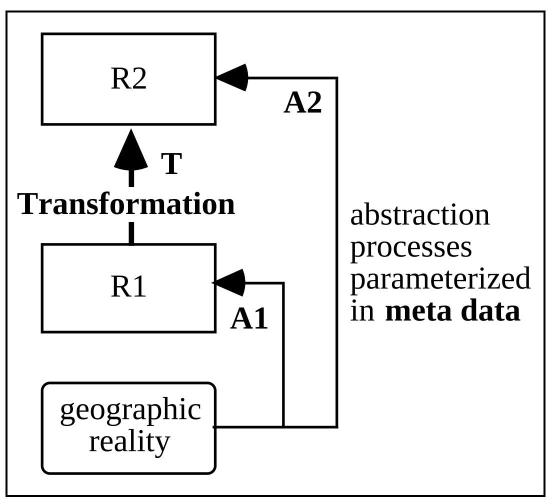

Figure 8.1: Transformations between resolution-limited representations.

The chapter starts with a description of meta data in the first section. Part of the meta data distinguishes between different **representation types**; namely, mixture fields, feature partitions, and stand-alone features. The remaining four sections describe transformations: Sections two through four define **primitive transformations** that can be combined to **general transformations** (see section five). Three kinds of primitive transformations are discussed: transformations within a representation type (section two), across representation types (section three), and across finite formats (section four).

8.1 Meta Data

The **most prominent component of meta data** is the distinction of the following **types of representation**:
(1) unclassified mixture fields
(2) classified mixture fields
(3) feature partitions
(4) stand-alone features
This meta data component can, for example be represented by a string. The representation type determines the **kind of abstraction process** that resulted in the representation. The **parameters of the abstraction process** are captured by **additional meta data components** that vary with representation type. The following reviews the abstraction process and its parameters for every representation type. An overview of meta data components is given in table 8.1 below.

(1) **Unclassified mixture fields** are derived from geographic reality by means of an **imaginary sensor** that observes every location in space. In the case of unclassified mixture fields, this sensor senses **observable properties** but cannot directly observe mixtures. The only parameter of this process is the **resolution** of the sensor. Since sensor resolution is measured by the diameter of the IFOV, this meta data component can be represented by a real number. Since different finite parametrizations of uncertain unclassified mixture fields are possible, the **finite format** has to be specified. In sampling-based parametrizations, the finite format is identified by the **sampling locations** and the **interpolation method**. In the case of regular sampling based on a grid, the sampling locations can be represented by a reference point, a sampling width, and an orientation. In the case of irregular sampling, the coordinate pairs of all sampling locations must be represented.

(2) **Classified mixture fields** are also derived from geographic reality by an imaginary sensor. **Resolution** is therefore one of the parameters of this abstraction process. In contrast to the unclassified case, however, the imaginary sensor can directly sense mixtures. Since mixtures are defined in terms of a set of geographic objects, the **object partition** related to the mixture is another parameter of the abstraction process. The meta data of classified mixture fields are thus composed of a resolution and an object partition. Object partitions can, for example, be represented by a set of strings that identify the contained geographic objects. Alternatively, if a representation of the object hierarchy in the form of a graph is available in a GIS, an object partition can be represented by a set of

pointers to nodes in this graph. The meta data on **finite format** are the same as for unclassified mixture fields.

(3) **Feature partitions** are derived from mixture fields by a feature abstraction that is based on one of Sinton's concepts. The first part of the overall abstraction process from geographic reality to the underlying mixture field is described by **resolution** and an **object partition**. The feature abstraction, i.e., the second part, is determined by the **mixture partition** that was used for theme control. The meta data of feature partitions are thus composed of a resolution, an object partition, and a mixture partition. For the special kind of mixture partition considered in this thesis, the mixture partition is determined by the object partition and a single **level of homogeneity**. This special kind of feature partitions can thus be described by its **resolution**, **object partition**, and **level of homogeneity**. The latter parameter can be represented by a real number. The **finite format** of feature partitions can be either **raster** or **vector**. In the case of raster, the used **grid** has to be specified in the way discussed for unclassified mixture fields. In the case of vector, the **minimal length** of line segments further describes the finite format.

(4) In the uncertain case, **stand-alone features** are derived from feature partitions of higher resolution (see section 5.3). The **resolution**, **object partition**, and **level of homogeneity** of the underlying feature partition are thus meta data components of the meta data. The second part of the abstraction process consisted of finding all disks of a coarser resolved space that contain the characteristic feature at higher resolution. The only parameter of this process is the **final resolution** of the stand-alone feature. In summary, the meta data of stand-alone features are thus composed from the **resolution of the underlying feature partition**, an **object partition**, a **level of homogeneity**, and the **final resolution**. The **finite format** of stand-alone features is equivalent to that of feature partitions.

Table 8.1 gives an overview of the meta data used for different representation types. Fields use an X to mark that a representation type has a certain meta data component. The meta data that identify the finite parametrization apply to all representation types but are excluded from the table.

representation type	**resolution** of imaginatory sensor	**object partition** of mixture	**level of homogeneity** (theme control)	**resolution** of final representation
unclassified mixture field	**X**			
classified mixture field	**X**	**X**		
feature partition	**X**	**X**	**X**	
stand-alone feature	**X**	**X**	**X**	**X**

Table 8.1: Overview of meta data components for different representation types. Fields use an X to mark that a representation type has a certain meta data component.

Chapter five modeled finite approximation as an additional step of the overall abstraction process. The resulting **approximation uncertainty** varies with location. For example, mixture uncertainty increases with increasing distance from sampling locations. Also, a raster approximation of a generally shaped geometry introduces different errors in different locations. Approximation uncertainty is therefore modeled by representation entities such as mixture balls or the transition zone, rather than in the form of meta data that describe a whole representation.

Another kind of meta data that does not relate to individual representations is the **object hierarchy**. It describes how geographic objects are related by abstraction mechanisms (see section 4.1) and is valid for all representations simultaneously. Such object hierarchies can be represented by graphs, partially ordered sets, or lattices [Kainz, 1994].

8.2 Primitive Transformations within a Representation Type

Primitive transformations that map knowledge between two representations of the same type are discussed in this chapter. Namely, these transformation are changes of resolution between two mixture fields (section one), changes of resolution and homogeneity between two feature partitions (section two), changes of resolution between two stand-alone features (section three), and changes of the object partition of the above representation types (section four). A comparison to the previous section shows that these transformations cover all meta data components of a given representation type can be changed by one of these transformations.

The discussion of resolution change heavily relies on appendix B that mathematically describes two methods to estimate the mixtures of coarser mixture fields from higher resolved ones. Transformations to higher levels of resolution are obviously impossible since they would increase the information content.

8.2.1 Resolution Change for Mixture Fields

This section discusses resolution change for mixture fields. It proposes to use linear filters to model resolution change. In classified mixture fields, such filters are applied to each mixture component individually; in unclassified mixture fields, the same kind of filter is applied to a single observable property. This section focuses on the more interesting case of classified mixture fields since the treatment of the unclassified case follows by analogy.

Appendix B describes how the resolution effect of imaginary sensors can be modeled with linear filters. The abstraction process from geographic reality to a mixture field is then determined by a disk shaped linear filter. According to the general definition of transformations, a transformation T that maps from the representation R1 to R2 is defined in terms of the related abstraction processes A1 and A2 as follows: T(A1) = A2. Since A1 and A2 are linear filters, T can be modeled as a linear filter as well. To satisfy the above equation, T becomes $A2(A1^{-1})$ where $A1^{-1}$ is the inverse of A1. T(A1) then becomes $A2(A1^{-1}(A1))$ which is obviously equal to A2. While $A1^{-1}$ does not exist by itself since it would reconstruct information that was eliminated by A1, the combination $A2(A1^{-1})$ can be determined precisely or in a good approximation since A2 eliminates the information that was reconstructed by $A1^{-1}$ (see appendix B for detail).

While appendix B deals with a single certain mixture component, the following discusses how to apply the method to an uncertain mixture field that uses multi-component mixture balls. The generalization of the method to mixture balls consists of three steps:

(i) First it extracts the upper and lower bounds for every mixture component from the field's mixture balls. This results in a ratio valued field for every component and both bounds: For example, a three component mixture field is decomposed into six ratio valued fields, namely those of component1/lower bound, component1/upper bound, ..., component3/upper bound.

(ii) The second step changes all these ratio valued fields to coarser resolution by applying a linear filter.

(iii) The final step reconstructs a single uncertain mixture field from the family of ratio valued fields. The center point of each mixture ball is then determined as the average between upper and lower bound. The mixture ball radius in a given location is computed as half the maximal difference between upper and lower bound out of all mixture components.

The use of upper and lower mixture bounds propagates uncertainty from the source to the target representation. As shown in appendix B, the filtering can introduce some additional mixture uncertainty. This uncertainty increase can be expressed in an increase of mixture ball radii. For example, if the filtering process introduces 2% mixture uncertainty, every mixture radius has to be increased by 2%. This shows that in addition to uncertainty propagation, also the uncertainty introduced by the transformation itself can be captured by the existing uncertainty model.

While it is evident from the appendix that these transformations are uniquely determined by the source and target meta data, the remainder of this section discusses issues of finite implementation.

Linear filters can easily be implemented in the raster domain [Castleman, 1979]. Such implementations are based on a nearest-neighbor interpolation[70]. The implementation of such filters for finite mixture field representations based on linear interpolation and/or irregular sampling are theoretically possible but impractical. It is therefore proposed to always compute resolution changes in the raster domain and, if necessary, use conversions of finite format (see section 8.4.1 below) before and after the transformation. Such conversions can also be used to provide a dense enough raster grid, since the uncertainty introduced by the raster implementation of linear filters depends on the grid density.

8.2.2 Resolution and Homogeneity Change for Feature Partitions

Since feature partitions are abstractions of mixture fields, they extract the major trends of mixture fields. In particular, they inform about the minimal presence of the mixture components that are characteristic for features. This allows the definition of the resolution change for feature partitions in terms of the methods used in mixture fields. While mixture fields are much better suited for the derivation of knowledge at coarser resolution, feature partitions are often the only available information.

The resolution change for feature partition consists of three steps: (1) deriving a certain mixture field that represents the minimal presence of each mixture component, (2) transforming this mixture field to a coarser resolution, and (3) using a new feature abstraction (see section 4.4) to transform this mixture field into a coarser-resolution feature partition.

The mixture field representation of step (1) can be derived by inspection all locations of the feature partition: locations inside a feature inherit the minimal percentage of the characteristic mixture component as specified by the feature's

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[70] This is evident since the numeric approximate computation of the convolution integral uses a constant value in every cell.

level of homogeneity. While the minimal mixture percentages for sampling locations in the transition zone are generally zero, the maximal rate of mixture change (see section 4.5.3) can be applied to gradually decrease the mixture percentage from the level of homogeneity to zero. The inspection of locations of step (1) can be finitely implemented by sampling. Uncertainty introduced by such sampling is captured by decreasing the minimal mixture percentages.

Step (2) was already described in the previous section. The only difference is that only the minimal possible mixture percentages are known. This corresponds to the lower bound derived from mixture balls in the case above. Instead of filtering both upper and lower bound, only the lower one has to be treated and the reconstruction of mixture balls is unnecessary. Since the method already deals with the lower bound of possible mixtures, uncertainty is automatically propagated. The additional uncertainty introduced by the filter further reduces the lower mixture bounds. This second step (2) is obviously determined by the resolution of the source and target feature partition.

The homogeneity meta data of the target feature partition determines the parameters of the feature abstraction that composes step (3). A certain decrease in homogeneity from the source to target representation is necessary to allow the absorption of mixture uncertainty introduced by step (2). A further decrease causes the absorption of additional inhomogeneities[71], i.e., areas that belong to the transition zone or a different feature in the higher resolved feature partition. For example, if the source and target feature partitions have levels of homogeneity of 90% and 80%, respectively, and step (2) introduces 3% mixture uncertainty, the result of step two is a mixture field with maximal mixture percentages of 90% - 3% = 87%. These maximal mixture percentages are reached inside features that are relatively large compared to the target resolution[72]. This means that an additional 7% is available for the absorption of additional inhomogeneities. The finite implementation of step (3) will be discussed in section 8.3.2.

The change of resolution from one feature partition to another again satisfies the general requirement for transformations, i.e., T(A1) = A2 (see section 8.2.1 above). This is evident for step (2) since it used the resolution change designed for mixture fields. Step (1) and (2) are again in the format of $A2(A1^{-1})$. The abstractions A1 and A2 here are feature abstractions applied to mixture fields, rather than overall abstraction processes applied to geographic reality. Step (1) is an inverse of the original feature abstraction. Since the information that was lost in the original abstraction cannot be reproduced, step (1) introduces considerable

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[71] The term inhomogeneity here is not used as defined earlier. I.e., inhomogeneities here are not geographic objects but small sections of transition zone and "foreign" features of the source feature partition.

[72] This can be seen when considering that filters use convolution integrals and that such convolution integrals applied to a constant function yield the same constant function as a result.

uncertainty. As expected, step (2) is the feature abstraction related to the target representation.

8.2.3 Resolution Change for Stand-Alone Features

Stand-alone features are derived from a higher resolution feature partition by an abstraction process. More precisely, the stand-alone feature's geometry consist of all resolution-limited disks that contain[73] at least one higher resolution disk that is part of a characteristic feature in some higher resolution feature partition (see section 5.3). According to the general definition of transformations, a resolution change of stand-alone features must find all target resolution disks that contain the smaller disks of the characteristic feature that were used in the original definition. This is visualized in figure 8.2: One of the original small disks of the characteristic feature is shown in gray. Two disks of the source representation are shown as medium size circles. The thicker and larger circles visualize two disks of target resolution that contain the original small disk.

The figure illustrates that a target resolution disks contains the original small disk if it contains at least one disk of the source stand-alone feature. Further, in their disk_to_point projections, the **target resolution geometry is wider than the source resolution one**. The width difference **w** is equal to the half the resolution difference, i.e., the difference of the disks' radii. This is the basic concept used for the resolution change of stand-alone features.

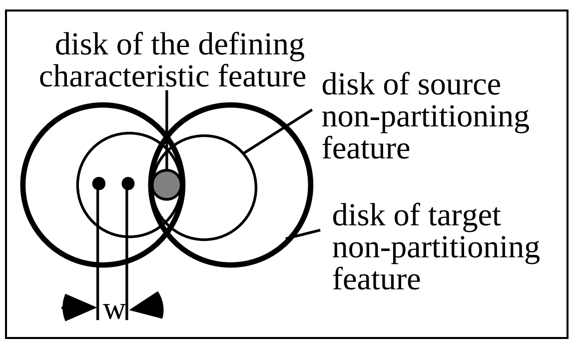

Figure 8.2: Visualization of resolution change for stand-alone features.

In vector representations, the target resolution geometry can be computed by buffering the source geometry in their disk_to_point projection by the distance **w**. Figure 8.3 shows the situation in case of raster format. In the shown

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[73] This containment must be evaluated in Euclidean space, as usually when disks of differently resolved resolution-limited spaces are compared.

disk_to_point projection, the source resolution geometry has to be buffered by half the resolution difference. The buffer is shown in gray[74]. One cell of the target resolution geometry is shown on the left. In accordance with the container philosophy, it belongs to the target geometry since it intersects with the buffer. Note that it is defined in the same grid that is used for the source geometry, since the transformation leaves the finite format unaffected[75]. The figure shows that the resolution change can be achieved by raster buffering.

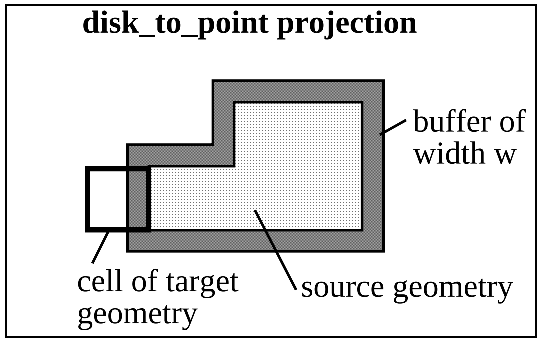

Figure 8.3: Resolution change in raster representation of stand-alone features.

8.2.4 Changes of Object Partitions

Classified mixture fields, feature partitions, and stand-alone features all use object partitions as a meta data component. Representations can obviously differ in the object partitions that underlie their mixtures. A transformation from a source to a target object partition is only possible if every object in the target object partition is hierarchically related[76] to the objects of the source object partition (see figure 8.4).

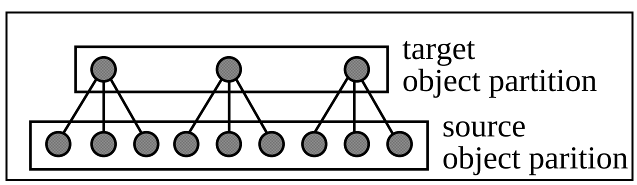

Figure 8.4: Hierarchical relations between source and target object partitions.

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74 Actually, the shown buffer is slightly too large in the corners.

75 This illustrates again that "raster resolution" is not related to the resolution concept proposed in this thesis.

76 More precisely, the source object partition must be a refinement [Gill, 1976] of the target one.

In **mixture fields**, a transformation to a coarser object partition derives the mixtures of the target representations from those of the source representation. More precisely, the target mixture component related to a higher-level geographic object is the sum of the components related to its component objects.

For example, let "pastures", "corn fields", "apple orchards", etc. be geographic objects of the source object partition, and "agricultural area" one of the geographic objects of the target object partition. In this case, there is a hierarchical relationship between the mentioned low-level objects and "agricultural area". If one of the mixtures in the source representation contains 2% of "pasture", 12% of "corn fields", and 6% of "apple orchards", the mixture in the target representation at the same location becomes 2% + 12% + 6% = 20% "agricultural area". A mixture ball can be constructed with this method by applying it to the center point mixture and setting the mixture radius as the product of source mixture radius with the maximal number of components of the higher-level objects of the target object partition. For example, if "industrial area" is an association of four components, and all other higher-level objects have four or less components, then the target mixture radius is four times the source one.

In **feature partitions** and **stand-alone features**, transformations to coarser object partitions trigger a change in feature attribues, i.e., the mixture classes associated with features. More precisely, such a transformation forms a higher-level mixture class related to a higher-level geograpic object as an association of the mixture classes related to its component objects. Since features are always separated by a transition zone, the combination of feature geometries in not possible.

For example, if the same object partitions are used as above, the source features "pasture", "corn field", and "apple orchard" would all become parts of the target feature "agricultural area". Since the source features are all separated by transition zone, the geometry of "agricultural area" is not simpler than those of the source representation. This shows a major difference to the conventional sharp boundary approach.

8.3 Primitive Transformations across Representation Types

This section describes different transformations across representation types. All but the transformation from unclassified to classified mixture fields reduce knowledge content. Inverses for these transformations can thus only exist as coarse approximations that introduce significant uncertainty. An example of such an inverse was used in the transformation of feature partitions to coarser scales (see section 8.2.2).

In the general definition of transformations, here, A1 and A2 are the abstraction processes from geographic reality to the source and target representation, respectively. A2 uses A1 as a first step and adds an additional abstraction step. This additional abstraction step is therefore directly the transformation T. All steps of the abstraction process were already described in the chapters four and five. This section therefore focuses on their finite implementation.

8.3.1 Unclassified to Classified Mixture Fields

The prime example of an explicit transformation from an unclassified to a classified mixture field is image classification in remote sensing. The most commonly used classification methods are not geared towards mixtures but usually use pure "categories" or mixed pixels with unspecified component percentages [Castleman, 1979]. This section briefly discusses how classification can be adapted to yield mixtures. A detailed study is left for future research.

Supervised image classification is based on the distribution of properties of mixture components that is determined by training sets. Figure 8.5 shows a possible distribution in a two-dimensional property space: **A**, **B**, **C**, and **D**, are the point clouds constructed by training; and **p** is the property vector of a pixel that shall be classified.

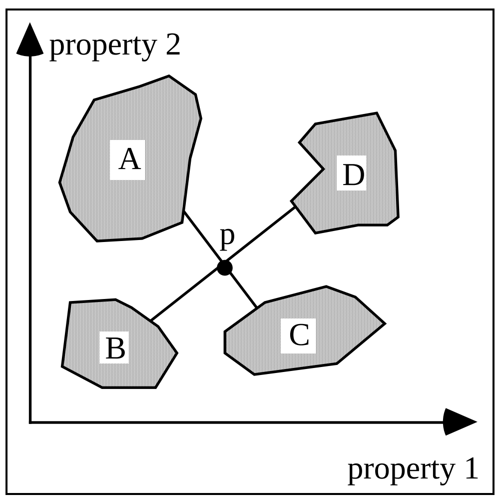

Figure 8.5: Property distribution constructed by training and the property **p** of a pixel to be classified.

The figure suggests that classifying pixel **p** as a mixture of **A**, **B**, **C**, and **D** is an ill-posed problem: **p** could either be a mixture of just **A** and **C**, or of just **B** and **D**, or it could be a mixture of all four. A unique solution can only be found if additional assumptions or rules are made to resolve the inherent ambiguity[77].

The following example of a simple heuristic rule shall illustrate this: The heuristic procedure first classifies pixels that fall completely into one of the training polygons **A**, **B**, **C**, and **D**. For all other pixels it assumes that they are a mixture of only those mixture components of adjacent, already classified pixels. For example, a mixed pixel located between a "forest" and "pasture" would be assumed to be a mixture of these landuses.

8.3.2 Classified Mixture Fields to Feature Partitions

The transformation from classified mixture fields to feature partitions of the same resolution and object partition has been discussed in detail in the sections 4.4.4 and 5.2.2. In addition, this section discusses issues of finite implementation.

Certain finite mixture field formats naturally translate into certain finite feature partition formats. This eliminates the need for finite approximation of the resulting geometry with the related increase of uncertainty. Table 8.2 gives an overview of naturally related mixture field and feature partition formats. Recall from the above section 8.1 on meta data that finite mixture field formats are described by the set of sampling locations, and their interpolation method.

Sampling Locations	Interpolation Method	Related Feature Partition Format
regular grid	nearest-neighbor	raster (same grid)
regular grid	linear	vector
irregular	nearest-neighbor or linear	vector

Table 8.2: Mixture field formats with their naturally related feature partition formats.

Nearest-neighbor interpolation in a regularly sampled mixture field keeps the mixture field value constant in each raster cell. Classification of such mixture fields thus assigns whole cells at once (rather than individual disks) to a mixture class. The resulting feature geometry therefore naturally results in a raster format that is based on the same grid as the field's sampling locations.

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77 Note that the ambiguity is caused by the use of a property space that is lower than the number of geometric objects that have to be classified. This situation is characteristic for all practical cases.

In the case of linear interpolation[78], the surfaces that express the minimal percentages of a single mixture component as a function of location are locally represented by plane segments. In the feature abstraction, these plane segments are intersected with thresholds that are determined by the required levels of homogeneity. The intersection of plane segments with such horizontal planes always results in straight line segments. The geometries of the resulting features thus become the polygons of a vector approximation.

The same is true for irregular sampling with nearest-neighbor interpolation where object classification deals with whole triangles at a time. Since triangles are bound by straight line segments, unions of such triangles are always vector polygons.

In the case of higher order interpolation methods, the resulting feature geometries are not naturally related to either raster or vector format. The easiest way of deriving a finitely representable feature partition from such mixture fields is to first transform them into a mixture field format that has a finite counterpart in the feature domain. For example, let us look at a relatively coarsely sampled mixture field that uses a higher order interpolation method in order to keep the interpolation error low as compared to a simpler interpolation method. This mixture field can be transformed into a more densely sampled mixture field that uses linear or nearest neighbor interpolation (see section 8.4.1). Most of the uncertainty caused by the simpler interpolation method is then compensated for by the denser sampling. The derived mixture field can now easily be transformed to a feature partition of the naturally related format.

Transformations from mixture fields to feature partitions propagate uncertainty by using a worst case scenario for the feature abstraction (see section 5.2). If a transformation to a naturally related finite format is used, finite approximation of the resulting geometry is unnecessary and the transformation therefore avoids to introduce additional uncertainty.

8.3.3 Feature Partitions to Stand-Alone Features

The geometry of stand-alone features was defined as the set of all coarser-resolution disks that contain at least one higher-resolution disk of a feature in the underlying feature partition (see section 5.3). In this chapter, this abstraction process is implemented in two steps by two primitive transformations: the first step transforms a feature partition to a stand-alone feature of equal resolution (see this section), and the second step transforms this stand-alone feature to a coarser resolution (see section 8.2.3).

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78 independently of the sampling strategy.

This first step is thus defined by modifying the above definition such to the special case where the coarser-resolution disks are of the same size as the higher-resolution ones. The geometry of such a stand-alone feature is then equal to the geometry of the characteristic feature in the feature partition. A primitive transformation from a feature partition to a stand-alone feature thus simply extracts one of the features. Implementations in raster and vector formats are thus straight forward.

8.4 Primitive Transformations across Finite Formats

While in current GISs, transformations across finite formats (i.e., conversions between raster and vector format) attempt to preserve the vaguely defined visual impression, this section describes transformations that preserve knowledge about the world--except for a slight increase of uncertainty. It is obvious that the preservation of knowledge about the world is impossible if the resolution of the representations is not taken into account. While in current GISs, concepts such as resolution or uncertainty are defined differently for the raster and vector domain, the resolution-limited approach uses the same concept in both domains. This conceptual compatibility of formats allows for much more precise definitions of transformations. This is for example evident in the well-defined propagation of uncertainty across formats. Furthermore, a concatenation of a transformation and its inverse yields a result that is contradiction-free to the original representation and solely has an increased uncertainty. For example, this makes it possible to convert a raster representation to a vector format and back to raster, without causing any contradictions between the original and resulting representation.

8.4.1 Transformations between Finite Mixture Field Formats

The finite format of mixture fields is specified by the set of sampling locations and the interpolation method (see section 8.1). All transformations between different mixture field formats are achieved by resampling. Since resampling is well known from the conventional approach, this section focuses on the issues of resolution and propagation of uncertainty.

Transformations across finite formats preserve the spatial resolution of the mixture field knowledge. Source and target representations thus use the same resolution-limited space. This shows again that "raster resolution" is a different concept from the resolution in this thesis: "Raster resolution" is called **sampling density** in the proposed spatial theory and is a parameter of finite approximation. Since the transformations discussed in this section are primitive, they only affect the finite format while leaving resolution unaffected.

Propagation and increase of uncertainty are of major concern during resampling. Figure 8.6 illustrates the problem. The uncertainty in the sampling locations is shown by black disks. Interpolation between sampling locations introduces additional uncertainty that usually increases with increasing distance from the sampling locations. The figure shows how resampling increases uncertainty in the resampled locations and consequently for the whole mixture field .

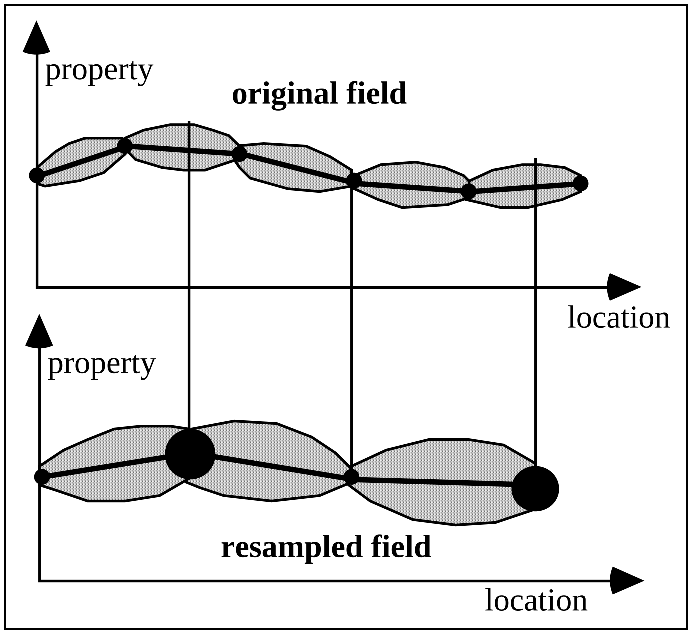

Figure 8.6: Propagation of uncertainty during the resampling of mixture fields.

The actual increase of uncertainty introduced by resampling depends on the represented real world phenomena, the sampling density, and the interpolation method. For example, different phenomena exhibit different rates of mixture change between sampling locations, and higher sampling densities with a simple interpolation method may be comparable to lower densities with a more sophisticated interpolation.

8.4.2 Transformations between Finite Feature Formats

Transformation between raster and vector formats of the geometries used for feature partitions and stand-alone features are well known from current GISs. Their major problem is their incapability of propagating meta data on resolution and uncertainty. Since the primitive transformations across finite formats leave resolution unaffected, this section focuses on the propagation and increase of uncertainty.

Chapter five has pointed out that the represented geometries in feature partitions and stand-alone features are either kernels or containers. Uncertainty can therefore be managed by the simple requirement that the geometries in the target representation are kernels or containers of the source geometries. Since the container and the kernel concepts are transitive, a kernel/container approximation of the kernels/containers of the source representation are still kernels/containers of the original infinite geometry. Figure 8.7 illustrates this for a vector kernel that is re-approximated by a raster kernel. It is obvious, that the raster kernel, designed as a kernel of the vector kernel, is necessarily also a kernel of the original feature geometry. The geometric differences of source and target kernels introduce additional uncertainty that is visualized by the gray area in the figure.

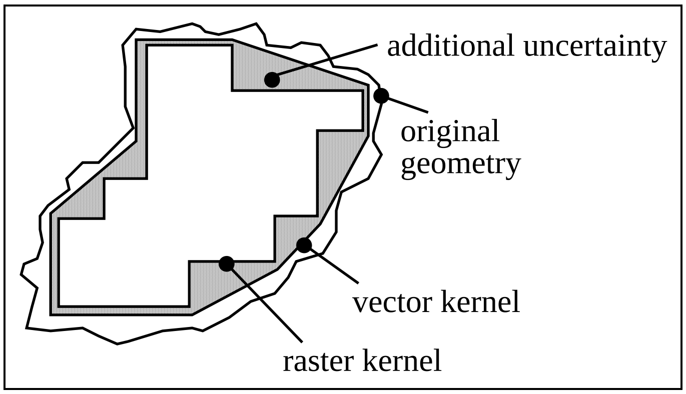

Figure 8.7: Propagation of uncertainty in a vector to raster conversion.

Note that the above requirement for preserving kernel and container properties does not completely determine transformations across formats. In the raster domain, the source and target data of finite representation completely determine the transformation since the relative position of source and target grid are known. The finer the target grid is chosen, the less uncertainty is additionally introduced. In the case of vector representations, the target meta data of minimal line segment length does not uniquely determine the transformation. This is not a problem, however, since all possible results are compatible with each other and differ solely in their amount of uncertainty that properly represented. Vector kernels/containers with a comparable amount of uncertainty are thus conceptually equivalent for all practical purposes.

8.5 General Transformations as Combinations of Primitive Ones

Most of the above primitive transformations affect only a single aspect of meta data. General transformations must therefore be composed as a composition of primitive ones. For example, a transformation from a mixture field representation to a coarser resolution feature partition can be composed of a resolution change of the mixture field followed by a change of representation type.

The decomposition of a general transformation into a series of primitive ones is not unique, however, since several sequences of primitive transformations are possible. For instance, the above example could alternatively be solved by first changing the representation type and only then changing resolution. As evident in this example, different decompositions differ in their amount of uncertainty increase and system response time.

An attractive method for managing decomposition alternatives are query optimizers as known from the discipline of data bases [Jarke, 1984] [Graefe, 1993]. They are based on rules that express when decompositions are conceptually equivalent and estimations of the uncertainty increase and processing time of primitive transformations. Optimizers then find the decomposition that minimizes either uncertainty or processing time while staying within specified constraints on the other criteria. Piwowar and LeDrew [1990] describe a very similar approach to data integration without using existing query optimizer concepts.

Query optimizers can be used for more that just the decomposition of single transformations: In a multi-format environment, a whole query involves several GIS operations and transformations and requires an optimizer for its efficient decomposition (see section 10.2). The decomposition of transformations can be implemented as part of such an overall query optimizer.

9 Data Integration in a Consistent Spatial Theory

The previous chapters have described a consistent spatial theory. This chapter demonstrates the potential of such a consistent theory for data integration with three examples. Namely, the consistency makes far-reaching system support and automation possible.

These advances in the field of data integration are possible based on more knowledge that is available in machine interpretable form: In particular, this chapter shows how the meta data can be used by a system to make intelligent decisions of how to deal with available data. For example, the system can offer substantial support in the fitness for use assessment (see section 9.1). The consistency of meta data and transformations allows translation of such decisions to actions. For example, data sets can automatically be converted to the format required by a certain analysis (see section 9.1). Systems that can access more knowledge can relieve users from certain responsibilities. For example, the resolution conscious overlay proposed in section 9.2 avoids slivers and relieves users from the task of sliver removal. It is also possible to hide all issues of finite format from users. This is discussed in section 9.3.

The chapter is divided into three parts. Each gives examples of the potential of a consistent framework for data integration. The first part discusses the integration of differently resolved data sets in a suitability study. Since spatial overlay is a major mechanism for data integration, the second part investigates how the explicit modeling of resolution allows the definition of an overlay method that avoids slivers and propagates uncertainty. The third part explores the potential of format free user interfaces.

9.1 Resolution-Sensitive Suitability Studies

This section uses the example of a suitability study to demonstrates the potential system support for the integration of data sets of different resolution and shows how meta data help to express suitability requirements. The first section gives a brief overview of the suitability study. Then, resolution and level of homogeneity are used to express minimal data quality requirements. Comparing these requirements to the meta data of available data sets allows evaluation of different degrees of fitness for use. The next section shows how to combine all data sets and how meta data propagates in the applied operations. The example points out the potential of system support for the choice of adequate data sets.

9.1.1 Suitability Example Overview

A simple example of finding suitable locations for a waste disposal site is used in this section. For reasons of simplicity, only three criteria are used: (i) maximal slope of the terrain, (ii) predominantly impermeable soils, and (iii) minimal size of the suitable area. The available data sets are an unclassified mixture field that contains slope data, and a feature partition that informs about the permeability of soils. The former data set was derived from a digital terrain model, the latter by reclassification from a soils feature partition.

9.1.2 Resolution Requirements of Queries

A suitability study must require minimal data quality in order to yield meaningful results. This section shows how this can be expressed by a **characteristic resolution** for the unclassified mixture field of slope, and a **minimal resolution** and homogeneity for the soils feature partition.

Like many other spatial phenomena and processes, slope[79] is a highly resolution-dependent measure [Buttenfield, 1989c]. A slope requirement therefore has to be expressed at a suitable resolution. This paragraph shows how this resolution is often characteristic for the processes involved. It is therefore called **characteristic resolution**. Let us assume that the slope requirement shall guarantee easy leveling of the waste disposal site with bull dozers. Slope measured at a resolution comparable to the size of the whole waste disposal site expresses how easy it is to level the whole site: if it is relatively flat, it will always be possible to fill in valleys with the material of hills. If the same is true at slightly higher resolutions, it is guaranteed that valleys can be filled in locally without the effort of displacing material over long distances. Slope at a resolution higher than the size of a bull dozer does not express much about the ease of leveling the terrain. This discussion showed how the characteristic resolution of slope data is related to the involved processes. Slope data at a too high or too low resolution would be meaningless for the study.

To support users in the specification of the characteristic resolution or range of resolutions, a resolution-conscious GIS can randomly generate and visualize possible terrain that just satisfy the slope requirement at a user defined resolution. This would allow users to intuitively choose the characteristic resolution and would avoid a theoretical derivation. In current GISs, users have neither well defined meta data to express their requirements, nor system support in choosing the characteristic resolution for their requirements. The proposed spatial theory with its machine readable meta data can thus provide a superior user support that helps to avoid meaningless studies that use data at inappropriate resolutions.

The proposed meta data also allows expression of minimal requirements for the second criteria of impermeable soils. In order to prevent potentially hazardous

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[79] A resolution-limited slope measure could for example be defined by the slope of a plain that locally approximates the terrain surface in the area of a resolution-limited disk.

materials from contaminating the ground water, suitable sites are predominantly covered by impermeable soils. Resolution and homogeneity together specify the maximal overall percentage of permeable soil (i.e., inhomogeneity) and the maximal size or grain of regions with such undesirable soil. Areas with permeable soil must be "sealed" with engineering artifacts. It is conceivable that larger permeable areas have to be treated with different engineering methods at different cost. Resolution and homogeneity thus obviously affect the overall cost of constructing a waste disposal. Their actual values can thus be chosen to include only fiscally feasible locations. Again, a resolution-conscious GIS can support users in choosing minimal requirements by visualizing the real world meaning. A similar visualization was proposed by Leung and Goodchild [1992] in the context of uncertainty (see section 7.2.6). This visualization would allow users to use intuition rather than analytically deriving the minimal requirements.

It is conceivable that in the future, the propagation of uncertainty will be well enough understood to make it possible to specify the minimal data quality of GIS products rather than that of the data sets from which the product is derived. The scale of a query could then be specified indirectly via the quality of the end result.

9.1.3 Fitness for Use Assessment

So far, users may use meta data to specify the minimal requirements of a query. In a multiple representation environment, let us assume several data sets are available to answer the query. One or several data sets may directly or after certain transformations (see chapter eight) satisfy the requirements of the query. A central problem in this scenario is the assessment of fitness for use [Goodchild, 1992b]. This section discusses this assessment for the slope and the soil criteria of the suitability study. It further points out the potential of system support in this area.

The slope criteria requires data at the characteristic resolution of the involved processes. If a data set is fit for use, a transformation must exist that maps from its original resolution to the characteristic resolution. Chapter eight showed that this is feasible for all data sets of a resolution equal or higher than the characteristic one. Suitable data sets will differ in their uncertainty of the slope measure. A query optimizer [Jarke, 1984] [Graefe, 1993] can help to select the most adequate data set (see also section 8.5).

In the case of feature partitions, fitness for use assessment is somewhat more elaborate since **four degrees of fitness are distinguishable** depending on how the available resolution and homogeneity compares to the requirements: In the best case, suitable areas can be identified for certain; in the two intermediate cases this is not possible but areas that are certainly unsuitable can be found; in the worst case, the available resolution and homogeneity are too low to derive any relevant information about suitability. The following four examples illustrate the possible degrees of fitness for use:

(1) The first example represents the best case. Assume that permeable soil was required to be at a resolution of 30 meters with a homogeneity of 80% and that the available data set has a resolution of 20 meters with a homogeneity of 90%. Chapter eight has shown that it is possible to find a transformation that converts the data set to the desired resolution and homogeneity. In the worst-case approach of feature partitions, features represent areas that are for certain either suitable or unsuitable. The transition zone could theoretically have either property. If the requirements of a minimal size of the waste disposal site are considered, however, transition zones between unsuitable features can be identified as certainly unsuitable: the minimal size guarantees that suitable real world areas always show up in the form of suitable features. The absence of suitable features thus implies the absence of suitable areas. Adjacent to suitable features, however, additional suitable areas may hide in the transition zone. The search for suitable areas can thus be answered in the form of kernels representing suitable features that mark areas that are certainly suitable, and containers given by the union of suitable features and surrounding transition zone, that contain all potentially suitable areas.

(2) Sometimes, the available data set can have a lower resolution and homogeneity than required. If a minimal size for the waste disposal site is required, it is still possible to derive valuable information, however. Assume that the required resolution and homogeneity is again 30 meters and 80%, and that the minimal size is 300 by 300 meters. Let the available data set have a resolution of 60 meters and a homogeneity of 75%. Clearly, a transformation to the required resolution and homogeneity is impossible since it would have to add information. Let us therefore study what information can be derived at the available resolution and homogeneity: Suitable features are likely to be suitable also at the required level of detail, but a certain evaluation of suitability is no longer possible. Unsuitable features, however, can be identified with certainty to be unsuitable: The homogeneity of 75% allows at most 25% of impermeable soil in every 60 meter disk of unsuitable features. Clearly, a 300 by 300 meter suitable site could not hide in the inhomogeneity of an unsuitable feature. The transition zone can be treated as in case (1) above: The required minimal size of the site would make at least some 60 meter disks part of a suitable feature. Suitable areas can thus again only hide in the transition zone around suitable features. The search for suitable areas can thus be answered in the form of containers given by the union of suitable features and surrounding transition zone, that contain all potentially suitable areas. While kernels for certainly suitable areas are not derivable, the containers allow a significant limitation of a more detailed search.

(3) The next worse degree of fitness for use differs from case (2) by the suitability information contained in the transition zone. Assume that the query requirements are unchanged from case (2) but the available data set now has a resolution of 400

meters and a homogeneity of 75%. The minimal size of the disposal site of 90,000 m^2 at the required 80% homogeneity would contain at least 72,000 m^2 of impermeable soil. This corresponds to about 57% of the area of 400 meter diameter disks, which is sufficient to prevent suitable areas from hiding in unsuitable features of 75% homogeneity, but too low to be predominant in suitable features. Unsuitable features can thus still be certainly identified as unsuitable, but the above reasoning about the transition zone does not apply anymore: The whole transition zone now has to be considered a potential hiding place for suitable areas. The search query is thus answered by a container that includes all suitable features and the complete transition zone.

(4) If resolution and homogeneity are further lowered, the minimal size of suitable areas can be absorbed in unsuitable features. It is then not possible anymore to certainly exclude suitable areas and thus limit a more detailed search. The corresponding data set is thus completely unfit for use.

Since the above assessment of fitness for use is completely based on the proposed meta data, a complete automation of the process is possible. A comparison to conventional meta data such as "cartographic scale" demonstrates the advantages of machine interpretable meta data. Extensive system support in assessing fitness for use becomes increasingly important in multiple-representations environments: here, several data sets may have the same degree of fitness. A major problem is then to select the optimal data set [Bruegger, 1989]. Section 8.5 has proposed to use query optimizers to find optimal decompositions of transformations. This optimization can be extended to also select the best suited data set. Besides processing time and data quality, also fiscal cost for the available data sets (e.g., in a network environment) can be used in the optimization. The rules used in such optimizers is a topic of future research.

9.1.4 Combination of Suitability Criteria

The previous sections have shown how different suitability criteria are formulated at different levels of resolution and homogeneity. This section is concerned with combining all criteria to yield a single feature partition of suitable and unsuitable features. In the conventional approach, feature partitions for each suitability criteria are simply overlaid--in the resolution conscious approach, their resolution differences have to be taken into account.

An overlay of layers that differ in resolution is obviously undesirable. The overlay has to be performed at a single level of resolution. Since the single criteria define suitable areas in the real world, it is natural to represent these areas in Euclidean space at infinite resolution. This is compatible with the visualization of the real world meaning of resolution-limited knowledge that was suggested in section 9.1.2 in support of choosing minimal requirements.

Several possibilities for defining suitable areas at infinite resolution based on resolution-limited representations exist. The following two possibilities are the most prominent: (1) If a resolution-limited disk satisfies the criteria, the center point of the disk is considered suitable in the real world; or (2) if a disk satisfies the criteria, every Euclidean point in this disk is considered suitable.

The result of the suitability classification of the original feature partitions thus results in a set of infinitely resolved feature partitions, where suitable features represent areas that certainly satisfy a single criteria, unsuitable features mark certainly unsuitable areas, and the transition zone could be suitable or unsuitable. Since all feature partitions are at the same resolution, they can now easily be overlaid.

The following rules specify the semantics of the overlay:

- if a point is part of a suitable feature in **all** feature partitions, it belongs to a suitable feature in the resulting feature partition.
- if a point is part of an unsuitable feature in **at least one** feature partition, it belongs to an unsuitable feature in the resulting feature partition.
- if a point belongs to the transition zone in at least one feature partition and to suitable feature in the others, it is part of the transition zone in the resulting feature partition.

The resulting feature partition again marks certainly suitable/unsuitable areas with features and potentially suitable areas with the transition zone.

The minimal size criteria can be used after the overlay to eliminate undesirable small suitable areas. If the geometry of the resulting layer is still too detailed, a transformation can transform the results back to limited resolution.

9.2 Sliver-Free Overlay

As a second example of the benefits of resolution consciousness in GISs, this section demonstrates how overlay in the resolution-limited approach avoids the problem of slivers. The overlay discussed here is different from that in the above suitability example (see section 9.1.4) since it operates at limited resolution. An example of such an overlay would be landuse change detection, where two feature partitions show landuse in different years at the same resolution.

To show that sliver problems are avoided, a first section analyses the origins of slivers in conventional representations. The second section shows how the proposed knowledge representation helps to avoid slivers. Since overlay processing is closely related to topologic integration, the implications of resolution-limited overlay processing in this area are discussed in the third section.

9.2.1 Origin of Overlay Slivers

Sliver problems occur when two different data layers describe real world phenomena with locally (i) equal or (ii) similar spatial distribution. In the former case, geographic objects of the two layers share parts of their boundaries in geographic reality; in the latter case, such boundary parts are mutually close (relative to resolution). This section discusses how such real world situations are represented in the conventional approach and how slivers have their cause in representation decisions.

One possible cause of slivers is the overlay of layers that differ in their level of resolution or level of homogeneity. The previous chapters have shown how the (identity and) geometry of features change with level of resolution and homogeneity. A single boundary section of the physical world can therefore be mapped to two significantly different representations at different scale. In an overlay, these differences show up as slivers. While common practice tries to avoid this situation, a lack of precise meta data and automated resolution change often makes living with the problem the only practical solution.

A second reason for slivers is the conventional treatment of uncertainty in representations--which basically consists of ignoring it. Conventional representations use the point-sharp boundaries of Euclidean point sets. The boundary location then depends on the approximation error in the finite and scaled representation: (i) In Sinton's feature abstraction, small errors in the mixture field property propagate to small errors in the boundary location; (ii) in addition, the error introduced by finite approximation of the general shape of the boundary introduces error in the location.

Even if uncertainty, i.e., the magnitude of error, is the same for two data layers, the actual errors are likely to be different. Figure 9.1 shows an example where the same boundary section is approximated differently in two finite representations that originate from independent digitization. Different realizations of error in two layers thus lead to different boundary representations. In an overlay, they show up as slivers. This problem is unavoidable and inherent in the conventional treatment of uncertainty.

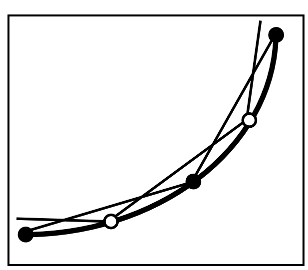

Figure 9.1: Two different finite approximations (one with black, the other with white vertices) of a boundary section (thick line) introduce different locational errors in the boundary representation.

9.2.2 Resolution-Limited Overlay

While the previous section showed that overlay slivers are unavoidable in the conventional approach, this section shows that they do not exist in resolution-limited knowledge representation due to the explicit modeling of uncertainty. The first cause of slivers, i.e., overlay of differently resolved layers, can be avoided in the resolution-limited approach due to the availability of meta data and transformations (see chapter eight). The discussion therefore focuses on the second cause, i.e., the treatment of uncertainty. Note that while the following argument is made in the context of knowledge representation, slivers in a cartographic or visualization sense are also avoided. This is evident when considering that overlay results are feature partitions that guarantee minimal dimensions in their visualizations (see chapter seven).

Conventional overlay is conceptually an intersection of point sets in Euclidean space. While points in the intersection of the point sets A and B are supposed to carry both these attributes, this is not true for an average-case based uncertainty model: The approximation error can cause points close to the boundary to carry wrong attributes. It is therefore difficult to reason about the attributes of points in an overlap polygon. In contrast to this conventional approach, resolution-limited knowledge representation allows a more precise knowledge of attributes: The worst-case approach based uncertainty model leads to transition zones with an unknown attribute rather than to an arbitrary change of attributes that yields a convenient sharp boundary.

The precise attribute knowledge allows the definition of spatial overlay in terms of both, attributes and location, rather than using a purely geometric definition: All disks of an intersection polygon must then certainly fall into the mixture classes of **both** related features of the original layers[80]. This means that disks that fall in the transition zone in one of the two layers cannot be part of a feature: Since at least part of their attribute is unknown, they have to be part of the transition zone of the resulting layer.

Since the resulting transition zone is then the union of the transition zones of the input layers, overlay operations obviously increase uncertainty. This may seem impractical, but I believe it is unavoidable and reflects what we really know about the world. While the increase of uncertainty may not "look" good in a cartographic mind set, it reflects an honest approach that expresses what is really known. Answers from an "honest" GIS may support decision making much

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80 Note that the semantics of this overlay is different from those in the suitability study example (see section 9.1.4). This is evident when considering unsuitable spatial objects: In suitability studies, a disk (point) that is part of an unsuitable spatial object in the resulting layer does **not** have to be unsuitable in all the input layers.

better, than the good looking answers of an overly optimistic and appearance based GIS.

The resolution-limited approach guarantees that the smallest possible intersection polygons are at least one resolution-limited disk wide. Since both attributes are required to be of a certain level of homogeneity, it requires a minimal size of the related geographic objects in geographic reality; too small intersection areas of geographic reality get absorbed in the transition zone or other features. This approach models the human cognition that distinguishes between significant overlaps and spurious polygons whose size depends on resolution. The similarity of resolution-limited overlay to human reasoning demonstrates how resolution-conscious GISs behave more intuitively and are therefore easier to use.

9.2.3 Uncertainty and Topologic Integration

Topologic integration is often seen as providing pre-computed overlays. We have seen that one of the major differences of the conventional and resolution-limited definition of overlay is the treatment of uncertainty. The conventional topologic integration of a large number of thematic layers is defined for the average-case model of uncertainty. This section discusses the implications of worst-case uncertainty modeling to topologic integration.

Topologic integration is known from conventional GISs such as TIGER [Broome, 1990] or TIGRIS [Herring, 1987]. It is theoretically founded in the context of certain, infinitely resolved spatial knowledge that is expressed in Euclidean geometry. In the context of a worst-case based uncertainty model, topologic integration is not practical since the single integrated layer would consist predominantly of transition zone. Resolution-limited knowledge representation and other models that take uncertainty into account must therefore favor a layered GIS architecture.

Work on automatic sliver avoidance or removal in the conventional modeling approach [Pullar, 1991] [Pullar, 1993] [Zhang, 1990] give evidence that topologic integration is problematic with all kinds of explicit uncertainty modeling. It demonstrates how topologic decisions about coincidence depend on the number and sequence of overlaid layers. Further, they show how uncertainty increases with the number of overlaid layers.

While total topologic integration is not feasible in the resolution-limited approach, it is still possible to explicitly share common boundary parts among layers. For example, school districts and census tracts often follow county and state boundaries. The equivalence of shared boundary sections can be expressed by topologic relations between layers. Note that while in the general case, topologic integration increases the transition zone area, this kind of integration can be used to increase certainty by forming the union of kernels and thus narrowing the transition zone.

9.3 Format-Independent User Interfaces

The ultimate degree of raster/vector integration is a GIS that hides differences of finite parametrization from its users [Maguire, 1991]. This section shows how a consistent spatial theory can support this goal: The ability of a system to intelligently deal with format issues relies heavily on the consistency of representations, meta data, and transformations.

Such fully integrated GISs offer users a **format-independent view of data and operations**. Different **types of data** are then characterized by their conceptual characteristic rather than their finite parametrization. The chapters four and five have discussed such conceptual characteristics at a format-independent level. The format-independent view of operations requires the support of **generic operations** that are defined at a format-independent level but can be implemented in the raster and/or vector domain. This thesis provides format-independent models for such definitions.

While users see only **generic data sets and operations**, the system manages actual, formatted data sets and executable, formatted representations. In order to hide format issues from users, such a system must be responsible for the following two tasks:

- translation of generic operations to executable operations of an adequate format
- if necessary, conversion of data sets to different formats (with transformations)

The latter point shows the importance of transformations in this context. Since a system determines the necessary transformations based on the available meta data, the latter task can only be automated if transformations are determined by source and target meta data. The absence of such consistency in current approaches is the major impediment to the implementation of fully integrated GISs.

Both of the above tasks are well known in the field of programming languages in the form of the polymorphisms "**overloading**" and "**coercion**" [Cardelli, 1985]. Overloading expresses that the same operation identifier (such as "+") is used for several executable operations (such as integer and real addition) and is instanciated according to the types of the operation arguments. Coercion allows the use of operation arguments (such as an integer variable) that have to be converted to another format (e.g., from integer to real) in order to be usable by an executable operation (such as real addition). The necessity of a type conversion is then automatically detected and the adequate conversion routine automatically executed. Prime examples of overloading and coercion is the addition operand ("+"). It is a generic operation that can be instanciated as a integer or a real addition (overloading). If one of the arguments is of type real and the other integer, the integer is automatically converted to a real before executing a real addition (coercion).

While these polymorphisms are used in programming languages to provide an easier interface to programmers, they can equally be used in GISs to provide format-independent, and thus easier to use, interfaces. In contrast to programming languages, the implementation of coercion and overloading are more complex in the domain of finite spatial formats. This is evident in the fact that a single query can be decomposed in multiple equivalent series of executable operations. The following simple example illustrates this: Assume that a raster and a vector format feature partition shall be overlaid. Obviously, this can be executed in two ways: convert the raster representation to vector and then use a vector overlay, or convert the vector representation to raster and then use a raster overlay. While conceptually, both ways are equivalent, they differ in the finite format of their result, their execution time, and the uncertainty of the result. It is easy to imagine how the complexity increases when more than two representations are overlaid or when additional operations are involved. To make things worse, certain operations may only be implemented in a single finite format.

Query optimizer [Jarke, 1984] [Graefe, 1993] are designed to manage such problems. As described in section 8.5, such optimizers are based on rules of equivalency and estimations of processing time and uncertainty increase of different processing steps. The following gives some examples of possible equivalency rules used for the implementation of a format-free user interface.

The examples use the following variables and operations: V1 and V2 are vector format feature partitions; R1 and R2 are raster format feature partitions; VtoR and RtoV are vector to raster and raster to vector conversions, respectively; and VOL and ROL are the executable overlay operations in vector and raster format, respectively.
(1) ROL(R1, R2) = ROL(R2, R1)
(2) VOL(V1, V2) = VOL(V2, V1)
(3) ROL(R1, R2) = VOL(RtoV(R1), RtoV(R2))
(4) VOL(V1, V2) = ROL(VtoR(V1), VtoR(V2))
(5) ROL(VtoR(V1), R2) = VOL(V1, RtoV(R2))

A fully integrated GIS uses such a query optimizer by first translating the format-free user query into one possible executable format, and then let the query optimizer use equivalency rules to convert the initial query decomposition into the optimal one.

10 Conclusions

The conclusions are organized in two parts, a summary of major results and an outline of future work.

10.1 Major Results

This thesis has identified inconsistency of the current spatial theory as one major cause of **problems observed during data integration** in current GISs. It has proposed a framework for the design of a **consistent spatial theory** (see chapter three). In particular, representations are derived by an abstraction process from the model of geographic reality, meta data are the parameters of these abstractions, and transformations are defined in terms of the abstraction processes of the source and target representation. The use of abstraction processes allows precise control of the knowledge content of different representations.

Modeling in the domain of finite (and therefore formatted) data models has been identified as the second problem of data integration. To overcome this problem, this thesis has defined representations, meta data, transformations, and uncertainty introduced by finite approximation in an **infinite, format-independent domain**. This approach allows application of concepts such as resolution and transition zone equally in the vector and the raster domain. This compatibility of concepts between formats is a prerequisite for the integration of data sets.

Failure to strictly **preserve the relationship between geometry and attributes** has been identified as a cause of problems in several aspects of spatial modeling. Sinton [1979] has formalized this relationship by concepts that define the geometry in terms of attributes or vice versa. The proposed spatial theory strictly applies Sinton's concepts in the abstraction processes that result in representations. The relationship between geometry and attributes is also preserved during the approximation process necessary for finite representation. A change of geometry in the approximation process is then inseparable from a change in attributes (see chapter five).

The proposed spatial theory is distinguished from previous work by **explicitly modeling the effects of limited sensor resolution on the represented spatial knowledge**. The proposed representations therefore represent knowledge at the limited level of detail that is available from sensors. The proposed resolution concept also allows the specification of data quality requirements (see section 9.1), the definition of resolution-conscious operations such as "resolution-conscious overlay" (see section 9.2), and guarantees displayability within the limitations of graphic media (see chapter seven).

The thesis has proposed **four types of representations** to express different types of spatial knowledge at different levels of detail. Namely, these representations are "unclassified mixture fields", "classified mixture fields", "feature partitions", and "stand-alone features". The related meta data and an exhaustive set of transformations between such representation has been defined in chapter eight.

Chapters six, seven, and eight have demonstrated the **validity of the proposed concepts**. In particular, chapter six has demonstrated that the proposed representations can be finitely represented in data models. Chapter eight has described finite implementations of the proposed transformations. Chapter seven has shown that the represented spatial knowledge can be visualized.

Chapter nine has given examples of the **potential impact of the proposed spatial theory on data integration**. The discussion has included specification of minimal data quality requirements, fitness for use assessment, automatic preparation of data sets for analysis, resolution-conscious overlay that avoids slivers, and the design of a format-free GIS user interface based on query optimization.

This thesis directly contributes to two of Goodchild's four "hard challenges and opportunities in GIS-related research" [Goodchild, 1992b]:

Challenge 2: "To devise a system of theory, terminology, and meta data that will support improved sharing of spatial data". Many problems of data sharing boil down to the question of what information about the world is actually contained in a given data set. The proposed system of representations and meta data directly answers this question in respect of resolution issues. Consequences of this approach that are relevant in the context of data sharing are machine-interpretable meta data (see chapter eight), necessary transformations across level of resolution and format (see chapter eight), system supported fitness for use assessment (see section 9.1.3), and the integration of fields and objects in both, raster and vector format, in a single consistent theory.

Challenge 3: "To devise a GIS and spatial language that will present the user with a view of continuous fields, hiding internal discretization[81] except where necessary". Since discretization manifests in different finite formats, Goodchild asks for a format-independent user interface. As reasoned above, this thesis lies the foundation for the design of such interfaces.

10.2 Future Work

This section discusses topics which are closely related to the presented research but were excluded from this thesis.

Since this thesis focused predominantly on representation issues, GIS operations have clearly been neglected. While some operation examples were given in chapter nine, a

81 This thesis used the term "finitization" rather than "discretization"

comprehensive treatment of GIS operations in a resolution-limited environment is necessary. This includes format-independent definitions of operations, the specification of their semantics in terms of resolution, and their propagation of meta data.

Fully integrated GISs with **format-independent user interfaces** have been outlined in chapter nine. While the proposed consistent spatial theory provides the basis for such GISs, the necessary definitions of generic operations and equivalency rules are left for future work.

An extension of a query optimizer of a format-independent user interface adapts the concept to support **access of information in a multiple representation environment**. Here, several alternative data sets are available for each query. An optimizer extension would allow use of the optimal input data sets for each query.

GIS operations usually impose certain requirements on their argument representations. For example, the overlay operation can require both input layers to be of equal resolution. Such requirements can be formalized by a **GIS type system**. It specifies argument types for operation arguments, and performs type checks. The support of polymorphisms can increase user friendliness of the interface. For example, coercion could be used to automatically transform the argument layers of an overlay operation to equal levels of resolution. A type system could be implemented as a further extension of a query optimizer. A simple GIS type system for representation types is currently developed as a masters research under my supervision [Roth, 1994].

Section 9.1.3 discussed how coarsely resolved representations can be used to significantly limit more detailed searches for areas with certain properties. In an environment with both coarse and detailed representations, this potentially makes **hierarchical reasoning** possible that incorporates several representations at different level of resolution. Research has to show how efficient such hierarchical reasoning is.

This thesis explicitly modeled uncertainty introduced by limited resolution and finite representation. It excluded **other kinds of uncertainty**, such as that introduced by the confusion of different mixture classes in remote sensing classification. An extension of the knowledge representation scheme to include other kinds of uncertainty would improve its practical value.

The proposed representation schema is based on two-dimensional space. All the concepts used are easily extendible to **three- or higher-dimensional** spaces. Future research could investigate such extensions.

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Appendix "A"--Intersection Areas of Disks with Half-Planes and Disks

A.1 Intersection of Disk with Half-Plane

This section is concerned with calculating the intersection area and overlap area of a disk intersecting with a half-plane (see figure A.1).

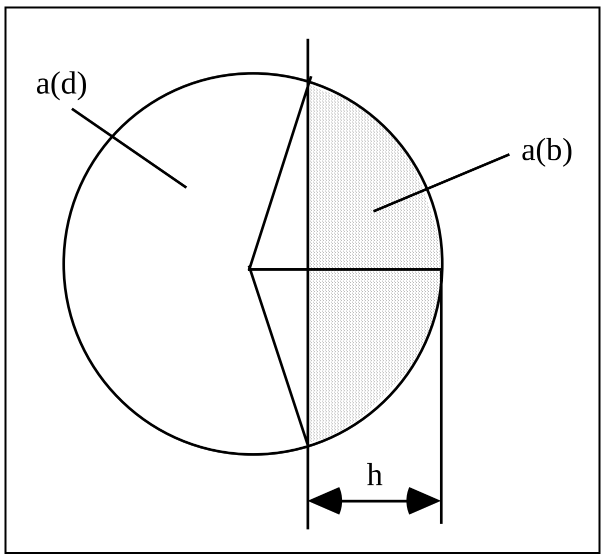

Figure A.1: Disk segment created by intersection with half-plane.

Figure A.2 introduces the necessary symbols. O, A, B, C, and D. are distinct points. The symbols r, d, and h represent distances between such points. In particular, r is the radius of the disk, measured for example between O and A; d is the distance between O and D, and h the distance between D and C. φ is half the opening angle of the sector, i.e. the angle between OA and OC.

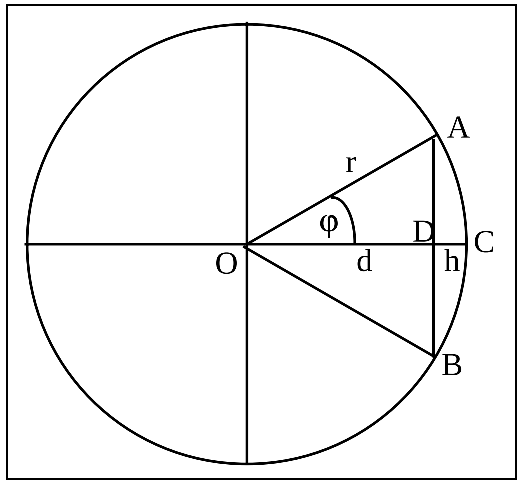

Figure A.2: Symbols related to a disk sector.

We will be interested in the following areas:
a(d): the area of the whole disk
a(t): the area of the triangle between O, A, and B
a(s): the area of the whole sector involving the points O, A, C, and B.
a(b): the area of the disk segment deliminated by the arc (A,C,B) and the line (A,B)

With the radius r = 1, the following equations hold:

$a(d) = \pi r^2 = \pi$

$a(t) = \sin\varphi \cos\varphi$

$a(s) = \pi r^2 \varphi/\pi = \varphi$

$a(b) = a(s) - a(t) = \varphi - \sin\varphi \cos\varphi$

$\mathbf{a(b) / a(d) = \varphi / \pi - \sin\varphi \cos\varphi / \pi}$

$\mathbf{h = h/r = r - d = 1 - \cos\varphi}$

We use the last two formulas to calculate tables A1 and A2, where the intersection area of a half-plane and a disk are related with its overlap depth. A1 is organized by even values of intersection area, A2 by even values of overlap depth.

h / r [%]	**a(b) / a(d)** [%]	φ [degrees]
11	2	27
20	5	36
32	10	47
42	15	54
51	20	61
59	25	66
67	30	71
76	35	76
84	40	81
88	42.5	83.22
93	45	86
96	47.5	87.8
100	50	90

Table A.1: Arc hight as a function of disk-segment area.

h / r [%]	**a(b) / a(d)** [%]	φ [degrees]
10	2	26
20	5	37
30	10	46
40	14	53
50	20	60
60	25	66
70	32	73
80	37	78
90	43	84
100	50	90

Table A.2: Disk-segment area as a function of arc hight.

A.2 Intersection of Two Disks of Equal Size

This section is concerned with calculating the non-intersecting area a(n) of a disk that intersects with an equally sized disk (see figure A.3). This area a(n) shall be expressed as a function of the distance d between the two disks. It is obviously closely related to the above problem, since the intersection can be constructed from two half-plane intersections:

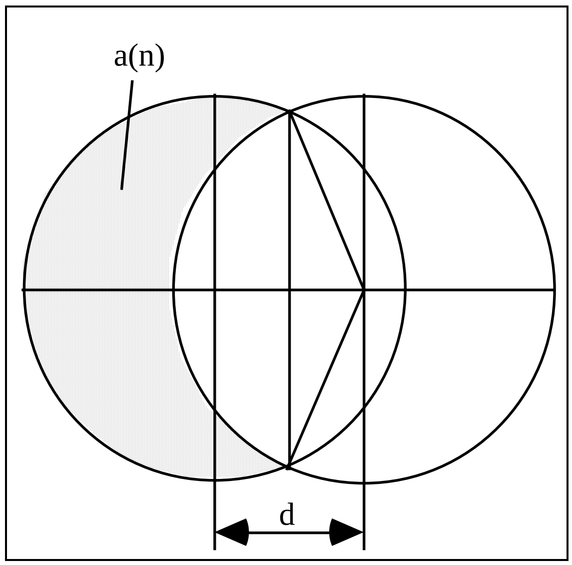

Figure A.3: Intersection of two disks of equal radius.

a(n) = a(d) - 2 a(b)
a(n) / a(d) = 1 - 2 a(b) / a(d)

The distance d between the disk centers is related to the above used arc hight h by the following formula:
d/2 = r - h and thus
h = 1 - d/2 or
h/r = 1 - 1/2 d/r

Table A.3 shows the non-intersecting area as a function of d and is derived from table A2 using the above formulae.

d / r [%]	a(n) / a(d) [%]
0	0
20	14
40	26
60	36
80	50
100	60
120	72
140	80
160	90
180	96
200	100

Table A.3: Non-intersecting area of two disks as a function of distance of centers.

A.3 Intersection of Two Disks of Different Radius--Special Case

In this section we are interested in the intersection of two disks of different radius. For reasons of simplicity, we only consider the case where a smaller disk is totally contained in the larger one (see figure A.4). The following shows how to compute the ratio of areas as a function of the ratio of radii:

$a_1 = \pi r_1^2$

$a_2 = \pi r_2^2$

$a_1/a_2 = r_1^2 / r_2^2$

and thus

$a_1/a_2 = (r_1/r_2)^2$ and

$r_1/r_2 = \sqrt{a_1/a_2}$’

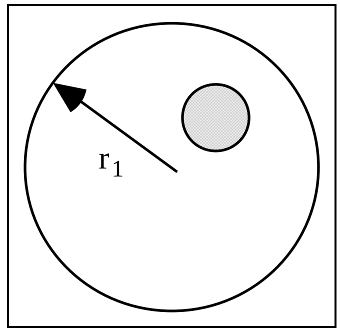

Figure A.4: Small disk contained in larger disk.

The following table A4 lists some points of this function:

ratio of area [%]	**ratio of radii** [%]
5	22
10	32
15	39
20	44

Table A.4: Ratio of radii in fuction of ratio of areas.

Appendix "B"--Derivation of Low Resolution Mixtures from Higher Resolution Ones

Mixture components express how many percent of a resolution-limited disk's area are affected by a given geographic object. This areal percentage is thus defined at the infinite resolution of geographic reality. When transforming knowledge from one resolution to another, areal percentages have to be derived from already resolution-limited mixtures, rather than from geographic reality directly. This appendix discusses how to estimate areal percentages of mixture components from limited-resolution representations and how much uncertainty is introduced in the process as compared to deriving them directly at infinite resolution.

The mathematical tools used for this discussion come from linear system theory [Castleman, 1979, pp. 139]. In particular, the reasoning uses convolution and its properties of commutativity and associativity (see [Castleman, 1979, p 145-148]). To apply these tools, a first section shows how the areal percentage of a single mixture component can be modeled by a convolution at infinite resolution. This process yields the true percentage of the mixture component. The second section describes a simple method to estimate the same mixture component indirectly from a resolution-limited representation also using a convolution. It further discusses the difference between the true and estimated component. Section 3 looks at the worst-case scenario to give an estimation of the maximal possible estimation error. Section 4 outlines a more sophisticated estimation method that should drastically decrease the maximal error.

B.1 Mixture Percentage as a Convolution at Infinite Resolution

To derive mixture percentages in disks as a convolution, we first have to describe the spatial distribution of geographic objects in the form of functions. We here assume a geometric interpretation of geographic reality where every point in Euclidean space is associated with exactly one geographic object. (The relations between this geometric interpretation and the physical one were discussed in section 4.3.2). The distribution of a single geographic object can then be described by an object function $o(x,y)$ that has the value 1 in points associated with the geographic object and 0 otherwise. Figure B.1 gives an example of such a function in a map and a cross-section view.

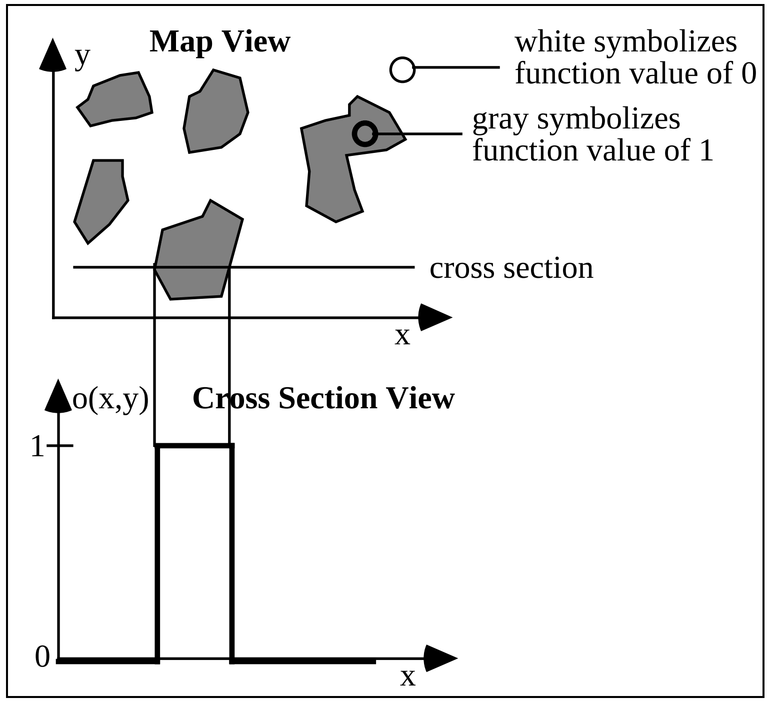

Figure B.1: Example of an object function.

The mixture percentage **m** of this geographic object within a resolution-limited disk **d** can now be computed as the integral over the disk area of the object function **o**, normalized by the total area **a** of the disk (see figure B.2):

$$p = 1/a \iint_{\text{in } d} o(x,y)\, dx\, dy \quad \textbf{(1)}$$

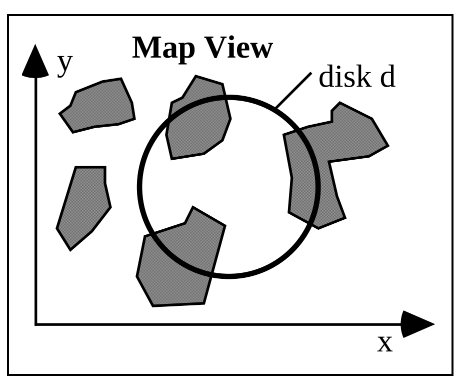

Figure B.2: Mixture percentage in a disk.

To get closer to a convolution view, we modify equation (1) such that we integrate over the whole Euclidean space. This can be achieved by multiplying the object function with a disk function D(x,y) that has the value 1/a inside the disk and 0 outside:

$$m = \iint_{\text{over whole space}} o(x,y) * D(x,y) \; dx\, dy \quad \textbf{(2)}$$

The disk function d(x,y) for a disk of radius **r**, centered on the origin is shown in figure B3 in a cross section.

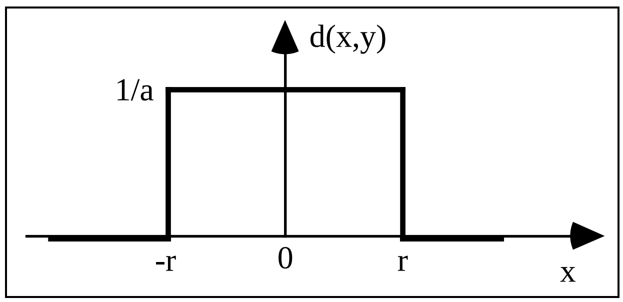

Figure B.3: Disk function d(x,y) for a disk of radius r, centered on origin.

If the original disk of figure B.2 was centered on the point (u,v), equation (2) can be rewritten using the disk function d of the origin, rather than D centered on (u,v):

$$m(u,v) = \iint_{\text{over whole space}} o(x,y) * d(u - x, v - y) \; dx\, dy \quad \textbf{(3)}$$

By writing m(u,v), we expressed that the equation calculates the mixture for the disk in the location (u,v). The general equation for a two-dimensional convolution is given by the following equation[1]:

$$h(u,v) = \iint_{\text{over whole space}} f(x,y) * g(x - u, y - v) \; dx\, dy \quad \textbf{(4)}$$

---

1 Adapted from Castleman's equation (41) from page 148 by renaming of variables.

The calculation of a mixture component given in (3) is thus a convolution and can be written in the abbreviated form
m = o * d **(5)**
where * is the convolution operator (see Castleman, equation 33, page 146). To get the mixture percentage of a single mixture component of a single disk, we can use equation (3) with fixed values for u and v.

B.2 Simple Estimation of Mixtures from Resolution-Limited Fields

While the previous section gave a equation for calculating mixture components, the object function at infinite resolution is usually not available. We therefore assume that the only available information is given by a resolution-limited field representation. For the purpose at hand, we look at a single mixture component of this field and attempt to estimate the mixture percentage of the same component in a field of coarser resolution.

Assume that the disk function d with radius r characterize the resolution of the available field information, while the disk function D with radius R characterizes the resolution of the requested field. The available information about a single mixture component was expressed above in equation (5). The requested field information can then be expressed by an equivalent equation (6) that uses a coarser resolution disk function:
m = o * d **(5)** : available information
M = o * D **(6)** : requested information
Since o is not available, M can be approximated by the following estimate E:
E = m * D **(7)** : possible estimate

The next section will reason that E is a good estimate if the R is large compared to r. Here we will compare the geometric meaning of the equations (6) and (7) in order to understand the estimation better.

Using (5), equation (7) can be rewritten as:
E = (o * d) * D **(8)**

Since convolution is both commutative and associative (see Castleman, equations 32 and 39 on the pages 146 and 148), equation (8) can be rewritten as follows:
E = o * (D * d) **(9)**

The most common interpretation of convolution is filtering with weighted averaging in a moving window. Equation (6) can thus be interpreted as a low-pass filtering of the original object function. The filter kernel D averages with constant sensitivity over the disk area. In this context, the difference of the requested information M and the estimate E, are expressed in using different kinds of low-pass filters: D for M, and (D * d) for E. The filter kernel (D * d) can itself be seen as the original kernel D low-pass filtered with d. Figure B.4 illustrates the original and filtered version of the kernel in a cross section.

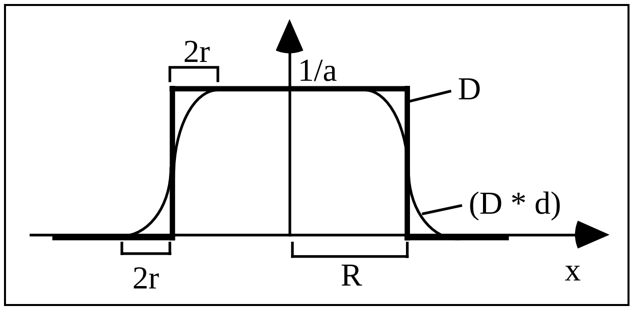

Figure B.4: Original (D) and smoothed (D ***** d) version of filter kernel.

Figure B.5 shows how the smoothed filter kernel affects the computation of the mixture percentage. A comparison with figure B.2 shows that the estimate E differs from the true mixture M by integrating over the object function o with a changes sensitivity factor (see equation 2). Compared to the D, (D ***** d) is not sensitive enough just inside the boundary of the original disk, and is too sensitive in a ring just outside that boundary.

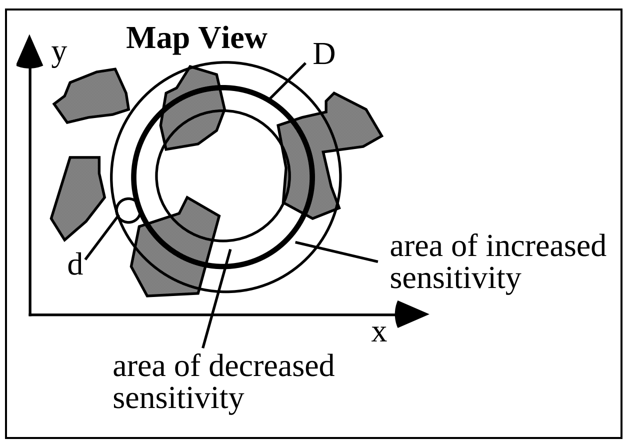

Figure B.5: Effects of smoothed filter kernel on computed mixture percentage.

B.3 Maximal Error of Simple Estimation Method

The error made in estimating mixture percentages from resolution-limited fields depends on the object function **o**. The following examples shown in figure B.6 illustrate this:

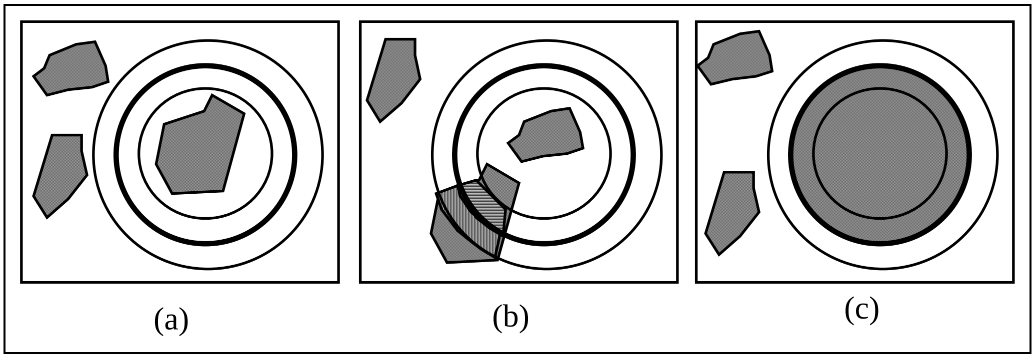

Figure B.6: The approximation error depends on the object function: If the component is not present in the rings, the approximation is equal to the true value (a); In configuration (b), what is counted with too small sensitivity in the inner ring (horizontal hatch) can be partly compensated for by sections counted with too high sensitivity in the outer ring (vertical hatch); Configuration (c) shows the worst case, since the too small sensitivity effects the maximal possible component area and the outer ring cannot compensate due to the absence of the component.

We use the worst case scenario (see B.6.c) to determine the maximal estimation error. The true mixture value M is determined by integration given in equation (2):

$$M = \iint o(x,y) * D(x,y) \, dx \, dy \quad \textbf{(2)}$$

Since D is zero outside the disk, only the area inside the disk is of interest. In the worst case scenario, **o** is 1 everywhere inside the disk. M therefore is the integral over D, i.e., the volume of a cylinder with radius R and height 1/a. This true mixture value compares to the estimate E that is the volume under the smoothed kernel shown as a cross section in figure B.7. Note that the outer ring is "cut off" since the object function **o** is zero in that area.

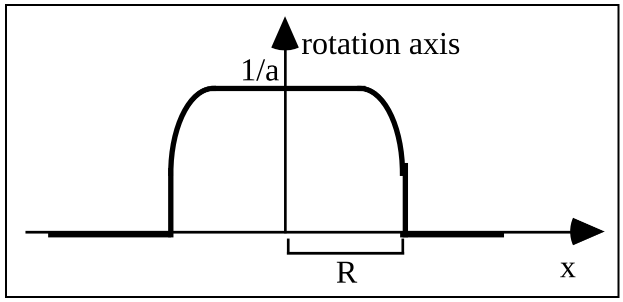

Figure B.7: The estimated mixture percentage in the worst case scenario is the volume under the body above (shown in a cross section).

The estimate E is thus too small by a percentage that is given as the ratio of the volume differences. Since the volume of the cylinder is 1 (by definition of D), the error percentage is directly the volume difference. The error volume is shown in figure B.8 in a cross section.

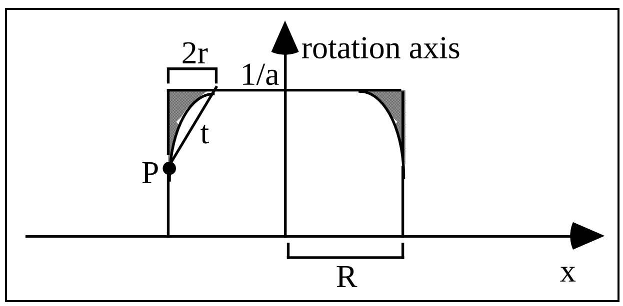

Figure B.8: Error volume in a cross section.

The error is bound by the rotated triangle **t** shown in figure B.8. The position of point P depends on the ratio of the radii r and R. Table A.3 in appendix A shows that in the case of r=R, P is located in a height of 40% of the total cylinder height[2]. The smaller disk d is usually much smaller than D, however. When d becomes increasingly small compared to

---

2 The value of the smoothed filter kernel is given by the intersection area of the original disk D and the smoothing disk d in the position where the center of d lies on the boundary of D. This means that the distance between the disks' center points is equal to one radius. Table A3 shows 60% non-intersecting area, which implies an intersection area of 40%.

D, the height of P approaches 50% of the cylinder height[3]. In the following, we use the worst case of P in a height of 40% to compute an error bound.

To simplify the computation of the error bound further, we can cut the "ring" of the error volume and bend it straight. This is illustrates in figure B.9. The initial error volume is shown in (a), while the situation after is shown in (b). If the deformation stretches the inner edge while preserving original length of the two outer edges, the total volume slightly increases. This is compatible with the concept of an error bound.

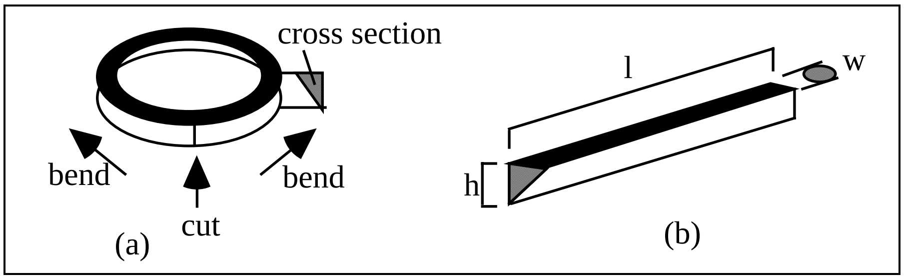

Figure B.9: A deformation of the error volume simplifies the computation of its volume.

The dimensions of the deformed error volume are shown in figure B.9: the length **l** is equal to 2•R, the width **w** is given by the smaller disk **d** with a diameter of 2r, and the height is 60% of the total height 1/a of the cylinder. The area of the cylinder foot print denoted by **a** is •R^2. The height h is thus 0.6 / •R^2. The total volume **v** is then given by the following formula:

v = 1/2 * h * w * l = 1/2 * 0.6 / •R^2 * 2r * 2•R = 1.2 * r / R

Table B.1 shows error bounds computed with this formula for different ratios of r and R.

---

3 If D is very much larger than d, D's curvature can be neglected and it can be approximated by a half-plane. Table A1 in appendix A shows that the intersection area in this case is 50% of d.

ratio of radii r and R	estimation error smaller than:
R = 4 r	30%
R = 6 r	20%
R = 8 r	15%
R = 12 r	10%
R = 24 r	5%
R = 60 r	2%
R = 120 r	1%

Table B.1: Error bounds for the estimation of mixture components depending on the resolution difference of the source and target representation.

The table shows that a resolution change from 10 meters to 120 meters introduces a maximal mixture uncertainty of 10%.

The above table gives a rough bound of the estimation error. Figure B.8, the conservative positioning of point P, and the increase of volume in the bending process suggest that the actual maximal error is smaller by about a factor of 1.5 to 2. Further, this error is only reached in the worst case scenario that is rather unlikely. In most cases, the estimation error will be rather small since too small and too high sensitivity even out as in figure B.6.b.

A more precise calculation of the maximal estimation error could be produced with an image processing package: D and d can be created analytically as images. D can then be filtered with d. The result can be masked with D to cut off the outer ring. The volume of the smoothed cylinder can be computed by summing up the cell values. The precision of this numeric integration depends on the resolution of the raster approximation.

B.4 More Sophisticated Estimation Method

This section outlines a more sophisticated estimation method that reduces the maximal error drastically. Working out the details of this method is left for future research.

While the above simple estimation method used (d ***** D) as an approximation for D, the more sophisticated method attempts to yield the true values (in the case of certain fields). When using uncertain fields, a small error in the estimate can be expected.

Image restoration [Castleman, 1979, chapter 14] attempts to eliminate the effect of an undesired filter by a convolution with its inverse. A possible problem is that the inverse filter does not always exist and can then only be approximated. We use this approach of inverse filtering to compute the true value of M:

$M = o * D = (o * d) * (d^{-1} * D)$

In this equation, the available representation is expressed by $(o * d)$, while $(d^{-1} * D)$ is the sought filter necessary to compute M. While d^{-1} does not exist, we show here that $(d^{-1} * D)$ exists in the one-dimensional case when R is a multiple of r:

In the frequency domain, both d and D correspond to a sin(x)/x type function[4]. More precisely, if d transforms into sin(x)/x, D becomes sin(nx)/(nx) where n is an integer expressing the ratio of R/r. The inverse d^{-1} of d then becomes x/sin(x), since this eliminates the effects of d. While d^{-1} has discontinuities where sin(x) goes to zero, this problem is avoided in $(d^{-1} * D)$, since sin(nx) goes to zero in the same locations and sin(nx)/sin(x) is always finite[5].

With the spectrum of the filter $(d^{-1} * D)$ being finite, an inverse Fourier transform is no problem and the filter therefore exists. It is actually easy to reason in the spatial domain alone that such a filter exists (see figure B.10): In a single location x3, the filter D basically integrates the object function over a (one-dimensional) spatial interval I. The filter d does the same thing but for a smaller interval. For example, i1 is the integration interval of d in position x1. It is now possible to construct the integration given by D in location x3, as the sum of several integrations given by d in the locations x1 through x5. These locations are chosen such that the related intervals just meet at their edges.

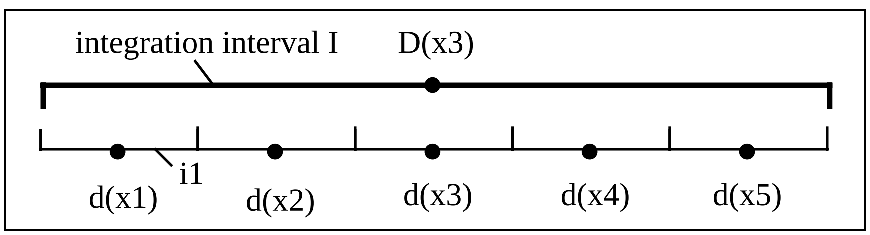

Figure B.10: Relations between filters at different resolution.

Adding up $(o * d)$ in the locations x1 through x5 is equivalent to a convolution[6] of $(o * d)$ with a discrete filter that has non-zero sensitivity only in the locations corresponding to x1 through x5. This filter is thus obviously equal to $(d^{-1} * D)$. Note that the discrete

4 For simplicity, constants are not considered in the discussion.

5 Sin(nx)/sin(x) is n for x = 0.

6 evaluated in the fixed position x3

character of this filter is not surprising considering that the spectrum of $(d^{-1} * D)$ is periodic.

We have thus shown that $(d^{-1} * D)$ exists in the one-dimensional case and is of a very limited width. I believe that the presented reasoning can be generalized for the two-dimensional case. Instead of using the one-dimensional Fourier transform, the rotation symmetric filter kernels can be treated with the Hankel transform [Castleman, 1979, page 184]. The close similarity of the two-dimensional Hankel transform to the one-dimensional Fourier transform suggests that a two-dimensional version of $(d^{-1} * D)$ has similar properties to its one-dimensional equivalent.

Appendix "C"--Algebraic Specification of Certain Resolution-Limited Representations

This appendix algebraically specifies some of the key terms of this thesis that are defined informally in chapter four. Algebraic specification is accepted in computer science as a high level, formally precise description of models [Woodcock, 1988] [Thomas, 1988] [Horebeek, 1989] [Liskov, 1986]. The format of the specification is very similar to that used in [Dorenbeck, 1991].

An algebraic "sort" is similar to the object-oriented concept of "class" [Meyer, 1988]. However, algebra does not distinguish between "objects" and "values" but rather treats everything as a "value". For example, it is not possible to modify an "object" while preserving its identity but rather every possible state of an object is treated as an entity of its own. A sort is a set that contains all such entities, i.e., all allowable states.

A sort is specified in three steps: (i) the sort is named, (ii) the signatures of operations on the sort are specified, and (iii) equations capture the semantics of such operations. The third step is simplified by providing variables that can be used in the equations.

Rather than enumerating all elements of a sort, these elements are defined axiomatically (implicitly) by the operations and equations (also called axioms).

Genericity and inheritance are used to keep the specification more compact. The use and difference of genericity and inheritance are discussed in detail in [Meyer, 1988].

In case of genericity, a sort can be defined generically such that the elements of the sort depend on the parameter(s) of the generic definition. The process of fixing the parameters of a generic sort is called "instantiation". All instances of a generic class feature all operations and equations of the generic class, in a form that is adapted to fit the instantiation parameters. It is important for this thesis that different instantiations of parameters create different sets (i.e., sorts). An example of a generic sort is "Set" with the parameter "Element". Element can then take on values such as "point" or "object". Evidently, sets of points are different from sets of objects. The instantiated sort can be specified as "instantiation of <generic sort> (<parameter1>, <parameter2>, ...)". For example, a set of points is specified as "sort PtSet: instantiation of Set(Point)". If generic sorts are used in the operations and equations, an "abbr" statement can define abbriviations of the full sort identifiers. For example, "abbr Disk = Disk(resolution)" allows use of the simple terms "Disk" instead of "Disk(resolution)".

Inheritance creates a new sort that inherits all operations and equations from its "super-sort" but specializes the semantics of the super-sort by defining additional operations and/or equations. For example, the sort "value" represents real numbers and the sort "percentage" is specified as "Sort Percentage: specialization of Value" with an additional equation that limits Percentage values to the interval [0, 1].

In their most detailed format, operations are specified by their sort and their operation name: for example "Value.add" is the operation with the name "add" as specified in the sort "Value". Within the specification of a sort, the sort as prefix to the operation name is omitted since it is clear from the context. Further, when the sort prefix is clear from the argument sort or the uniqueness of the operation name, the following specification omits the sort prefix. The shorter specification text then improves readability.

Operations that are defined for a single argument are sometimes applied to whole sets of such arguments. In this case, the operation has to be applied to every element in the set which results again in a set that contains all operation results.

The following specification defines key terms of chapter four. Sorts such as "Boolean" that are well known from mathematics are not completely specified. If necessary for clarity, the signature of operations is given; the equations are always omitted, however. The specifications are followed by a brief explanation.

{------------------------------ Boolean ------------------------------}

Sort **Boolean** {as known from mathematics}

{------------------------------ Id -- Identifier ------------------------------}

Sort **Id** {unique identifier}

{------------------------------ Set ------------------------------}

generic Sort **Set** with parameter **Element** {as known from mathematics}

Ops	**makeEmpty:**		-> Set
	addElem:	Set x Element	-> Set
	isElement:	Element x Set	-> Boolean
	isSubSet:	Set x Set	-> Boolean
	isPartition:	Set x Set of Sets	-> Boolean
	isSingleton:	Set	-> Boolean
	areDisjoint	Set x Set	-> Boolean
	union:	Set of Sets	-> Set
	intersection:	Set x Set	-> Set
	noOfElems:	Set	-> Integer

{------------------------------ Value -- real numbers ------------------------------}

Sort **Value**

Ops	**sum:**	Set of Value	-> Value
	ratio:	Value x Value	-> Value
	isSmallerEqual:	Value x Value	-> Boolean
	abs:	Value	-> Value
	diff:	Value x Value	-> Value
	min:	Set of Value	-> Value
	isZero:	Value	-> Boolean
	nonZero:	Value	-> Boolean

{------------------------------ Percentage ------------------------------}

Sort **Percentage**: Specialization of **Value**

Var p: Percentage

Eq 0 • p • 1

{------------------------------ Point ------------------------------}

Sort **Point**

Ops	**make:**	Value x Value	-> Point
	x:	Point	-> Value
	y:	Point	-> Value

Var xx, yy : Value

Eq x(make(xx, yy)) = xx

y(make(xx, yy)) = yy

The sort Point can be interpreted as a two-dimensional vector space that can underlie several different metric space that define different distance functions.

{------------------------------ PtSet -- Point Set ------------------------------}

Sort **PtSet:** instantiation of **Set(Point)** {point sets}

{------------------------------ EuclSpace -- Euclidean Space ------------------------------}

Sort **EuclSpace:** Specialization of **Point**

Ops	**distance:**	Point x Point	-> Value	{Euclidean distance}
	area:	PtSet	-> Value	
	isOpen:	PtSet	-> Boolean	{point set topology}
	isClosed:	PtSet	-> Boolean	
	boundary:	PtSet	-> PtSet	
	isPartition:	Set of PtSet	-> Boolean	
	medAxis:	PtSet	-> PtSet	{medial axis}
	cg:	PtSet	-> point	{center of gravity}

The operations listed in the specificaton are only examples of all possible operations of Euclidean Space.

{------------------------------ Disk ------------------------------}

generic Sort **Disk** with parameter **resolution: Value**

Specialization of **PtSet**

Ops **make:** Point -> Disk

centerPt: Disk -> Point

Var center, p: Point

Eq (1) isSmallerEqual(distance(center, p), (resolution/2)) =>

isElement(p, make(center))

(2) centerPt(make(p)) = p

Disks are Euclidean point sets that are disk shaped. An individual disk is specified by its center point and the radius which is aways half of the sort parameter "resolution". An instantiated sort Disk(resolution=constant) contains infinitely many disks, all of the same radius.

{------------------------------ Region ------------------------------}

generic Sort **Region** with parameter **resolution: Value**

Instantiation of **Set(Disk**(resolution)**)**

{-------------------------- ResLimSpace -- Resolution-limited Space ---------------------------}

generic Sort **ResLimSpace** with parameter **resolution: Value**

Specialization of (Instantiation of **Disk**(resolution))

Abbr Disk = Disk(resolution), Region = Region(resolution)

Ops **distance:** Disk x Disk -> Value

area: Region -> Value

boundary: Region -> Region

isPartition: Set of Region -> Boolean

Var d_1, d_2: Disk; r: Region; sr: Set of Region

Eq (1) distance(d_1, d_2) = EuclSpace.distance(center(d_1), center(d_2))

(2) boudary(r) = Disk.make(EuclSpace.boundary(Disk.center(r)))

(3) isPartition(sr) = EuclSpace.isPartition(Disk.center(sr))

ResLimSpace(resolution) is a subset of Disk(resolution) with additional operations and equations. The specialization of Disk to ResLimSpace is comparable to that from Point (i.e., a vector space) to EuclidSpace (i.e., a metric space). The operations of ResLimSpace are structurally equivalent to those of EuclidSpace, i.e., there exists a homomorphism [Gill, 1976] between the two sorts. The 1:1 relationship between elements of the sorts is given by the opeartion Disk.centerPoint and Disk.make (in both directions). The 1:1 relationship between operations is evident in the same operation name. The homomorphism is used in the design of the equations (1), (2), (3). The operations listed in the specification are only examples of the full set of possible operations.

{------------------------------ Atom ------------------------------}

Sort **Atom**

Ops **location:** Atom -> PtSet

id: Atom -> Id

property$_1$:

...

property$_n$:

method$_1$:

...

method$_n$:

Atoms are the smallest discrete entities in the world. They are characterized by their location.

{------------------------------ GeoReal -- Geographic Reality ------------------------------}

Sort	**GeoReal**: Specialization of (Instantiation of **Set(Atom)**)		
Ops	**make:**		-> GeoReal
Eq	isPartition(atom.location(make)		

Geographic reality is basically a set of Atoms with the additional requirement that the locations of atoms must form a partition of EuclideanSpace.

The proposed model of geographic reality is very similar to Goodchild's model in case of using only nominal values. The difference is that the model of this thesis considers contiguous areas of a constant nominal value entities.

{------------------------------ Obj -- Object ------------------------------}

Sort	**Obj:**		
Ops	**makeAtm:**	Atom	-> Obj {single Atom}
	makeAM:	Set of Obj	-> Obj {abstr. mechs.}
	partOf:	Obj x Obj	-> Boolean
	makeGeom:	PtSet	-> Obj {admin.objects}
	location:	Obj	-> PtSet
	contained:	Atom x Obj	-> Boolean
	isObjPartit:	Set of Obj	-> Boolean
	id:	Obj	-> Id
	property$_1$:		
	...		
	property$_n$:		
	method$_1$:		
	...		
	method$_n$:		

Var a: Atom; o: Obj; OS: Set of Obj; geom: ptSet

Eq (1) location(makeAtm(a)) = Atom.location(a)

(2) location(makeAM(OS)) = union(location(OS))

(3) isElement(o, OS) => partOf(o, makeAM(OS))

(4) contained(a, o) = isSubset(atom.location(a), location(o))

(5) contained(a, makeGeom(geom)) <=> isElement(cg(atom.location(a)), geom)

(6) isObjPartit = isPartition(location(p))

Objects are directly or indirectly composed from atoms. MakeAtm creates an object from a single atom. MakeAM is designed to create higher level objects using the abstraction mechanisms "classification", "generalization", "aggregation", and "association". PartOf tests whether a lower level object is part of a higher level one.

In the cases of classification and generalization, a new (super) class is defined. The related higher level object is the association of all objects that are members of this class (i.e., an instance rather than a class). The higher level object is thus based on common behavior of its component objects. The behavior of objects is described with the opeartions $property_i$ and $method_i$. Note that most programming languages limit the common behavior used in classification and generalization to methods and excludes properties that are expressed in terms of values. For example, the class "teenagers" whose members have the property "age = [10, 20)" in common is usually not supported by programming languages. Since they are common practice in GIS, this specification allows also property based classifications and generalizatons.

In case of using the abstraction mechanisms aggregation and generalization for makeAM, a crisp object is defined based on strong relations/interactions between its lower level components.

MakeGeom is designed to deal with administrative units such as parcels, districts, states, or countries that are usually defined as point sets in Euclidean space. MakeGeom transformes this geometric definition into an object that contains all atoms that are (fully or predominantly) inside the specified geometry. Note that the location of such an object is slightly different from the pointset used in its definition, since it is the union of atom locations (see equation (5)).

The provided operations allow the creation of an unlimited number of objects at different levels of abstraction. These objects are potentially overlapping. IsObjPartit is therefore provided to check whether a set of objects forms a partition of geographic reality, i.e., that the objects' locations form a partition of Euclidean space.

{------------------------------ ObjPart -- Object Partition ------------------------------}

Sort **ObjPart**: Specialization of (Instantiation of **Set(Obj)**)

Var p: ObjPart

Eq isObjPartit(p)

An object partition is a set of objects that partitions geographic reality.

{------------------------------ Mixture ------------------------------}

generic Sort **Mixture** with parameter **objP: ObjPart**

Ops **make:** Set of (Obj x Percentage) -> Mixture

compnts: Mixture -> ObjPart

compPerc: Mixture x Obj -> Percentage

distance: Mixture x Mixture -> Percentage

Var m, m_1, m_2: Mixture; o: Obj; p: Percentage;

s: Set of (Obj x Percentage)

Eq (1) compnts(m) = objP

(2) isElement([o, p], s) <=> isElement(o, compnts(make(s))) and

compPerc(make(s), o) = p

(3) sum(compPerc(m, compnts(m))) = 1

(4) isNotElement(o, compnts(m) => compPerc(m, o) = 0

(5) distance(m_1, m_2) = 0.5 * sum(abs(diff(compnts(m_1), compnts(m_2))))

A mixture is an assignment of a percentage to every object contained in a object partition. A distance operation is defined to measured the similarity between two mixtures. Minimal distance is 0 and means that the mixtures are the same; maximal distance is 1 (or 100%) and expresses maximal disimilarity.

{------------------------------ ArbArea -- Arbitrary Aarea ------------------------------}

Sort **ArbArea**

Ops **make:** ObjPart x PtSet -> ArbArea

location: ArbArea -> PtSet

mixture: ArbArea -> Mixture

Var o: Obj; op: ObjPartit; geom: ptSet

Eq (1) compnts(mixture(make(op, geom))) = op

(2) isElement(o, op) => isElement(o, compnts(mixture(make(op, geom))))

(3) isElement(o, op) =>

compPerc(mixture(make(op, geom)), o) =

ratio(area(intersection(location(o), geom)), area(geom))

An arbitrary area is defined by an arbitrary point set in geographic reality. The determination of the properties and behavior of such an area requires the relation to behavior carrying objects. The behavior of an arbitrary area is thus approximatively described by a mixture of objects. The mixture percentages of objects are given by the area that they cover within the arbitrary area (see equation 3). This description is at a higher level of abstraction if higher level objects are used instead of atoms. The behavior description is an approximation since some objects are only partly contained while the bahavior of objects is usually not dividable.

{------------------------------ MixField -- Mixture Field ------------------------------}

generic Sort **MixField** with parameter **resolution: Value**

Abbr Disk = Disk(resolution)

ops **make:** objPart -> MixField {sensoric perception}

mixture: MixField x Disk -> Mixture

isClassifd: MixField -> Boolean

obsProp$_1$: MixField x Disk -> $propType_1$

...

obsProp$_n$: MixField x Disk -> $propType_n$

Var d: Disk; op: ObjPart

Eq (1) compnts(mixture(make(op), d)) = op

(2) mixture(make(op), d) = ArbArea.mixture(op, d)

(3) isClassifd(make(op)) = greaterThan(noOfElems(op), 1)

(4) obsProp = assumed to propagate from objects contained in area

Sensoric perception (make) at the resolution specified by the parameter extracts mixtures and observable properties for every disk of resolution-limited space. The mixtures are derived from knowledge about the world that is provided in the form of an object partition. A classified mixture field uses two or more objects in the object partition and thus in the mixture. Mixtures are thus the major property of classified mixture fields. ObsProp stands for observable properties that have different values for every disk. They are important for unclassified mixture fields that do not distinguish different objects and whose mixtures degenerate to the trivial form "100% atoms". Unclassified mixture fields and observable properties are included in this specification to allow classifications that map unclassified mixture fields to classified mixture fields. This classification is based in the relations between observable properties and objects in the object partition objP that cause such properties (equation (4))

{------------------------------ MixClass -- Mixture Class ------------------------------}

generic Sort **MixClass** with parameter **objP: ObjPart**

Specialization of (Instantiation of **Set(Mixture(objP))**)

Ops **distance:** MixClass x MixClass -> Percentage

Var c_1, c_2: MixClass

Eq distance(c_1, c_2) = min(Mixture.distance(c_1, c_2))

{------------------------------ MixPart -- Mixture Partition ------------------------------}

generic Sort **MixPart** with parameter **objP: ObjPart:**

Specialization of (Instantiation of **Set(MixClass(objP))**)

Abbr MixClass = MixClass(objP), Mixture = Mixture(objP)

Ops **makeStandAlone:** Obj -> MixPart

Const transitionZone: MixClass

Var mp: MixPart; mc, mc_1, mc_2: MixClass; o: Obj; m: Mixture

Eq (1) isPartition(Mixture, mp)

(2) isElement(transitionZone, mp)

(3) isElement(mc, mp) => MixClass.distance(mc, transitionzone) = 0

(4) isElement(mc_1, mp) and isElement(mc_2, mp) and
notEqual(mc_1, transitionZone) and notEqual(mc_2, transitionZone)
=> MixClass.distance(mc_1, mc_2) > 0

(5) noOfElems(makeStandAlone(o)) = 2

(6) makeStandAlone(o) => isElement(o, objP)

(7) isElement(mc(makeStandAlone(o)) and notEqual(o, transitionZone)
and isElement(m, mc) <=>
nonZero(compPerc(m,o)

(8) isElement(transitionZone, makeStandAlone(o)) and isElement(m, transitionZone)
<=> isZero(comPerc(m, o))

A mixture partition is a partition of the set of all possible mixtures into mixture classes (equation (1)). The transition zone is always part of a mixture partition (equation (2)). The transition zone is neighbor of all other mixture classes (equation(2)) and thus separates all these other mixture classes (equation (4)).

MakeStandAlone makes the mixture partition for a stand-alone feature. Its semantics are captured in equation (5) through (7): The mixture partition of a stand-alone feature contains two mixture classes (5). MakeStandAlone requires that the argument object is part of the object partition given as parameter of the generic class (6). One of the mixture classes in makeStandAlone(o) contains all mixtures that have a non-zero percentage of the argument object o (7). The transition zone on the other hand consists of all mixtures that contain zero percent of the argument object (8).

{------------------------------ Feature ------------------------------}

generic Sort **Feature** with parameters **resolution: Value**, **objP: ObjPart**

Specialization of (Instantiation of **Region(resolution)**)

Abbr MixField = MixField(resolution), MixClass = MixClass(objP),

Disk = Disk(resolution)

Ops **make:** MixField x MixClass -> Feature {Sinton}

mixClass: Feature -> MixClass

Var d: Disk; f: MixField; mc: MixClass

Eq (1) isElement(d, make(f, mc)) <=> isElement(mixture(f, d), mc)

(2) mixClass(make(f, mc)) = mc

A feature is the aggregation of disks that fall into the same mixture class to a region. The mixture class describes Sinton's "control of the theme" and the resulting region correspond's to Sinton's "measured location".

{------------------------------ FeatPart -- Feature Partition ------------------------------}

generic Sort **FeatPart** with parameters **resolution: Value, objP: ObjPart**

Specialization of (Instantiation of **Set(Feature(resolution, objP))**)

Abbr MixField = MixField(resolution)

Ops **make:** MixField x MixPart -> FeatPart

makeStandAlone: MixField x Object -> FeatPart

Var mc: MixClass; mp: MixPart; f: MixField; o: Object

Eq (1) isElement(mc, mp) <=> isElement(Feature.make(f, mc), make(f, mp))

(2) makeStandAlone(f, o) = make(f, MixPart.makeStandAlone(o))

A feature partition is a set of features that are derived from a mixture field by using all mixture classes of a mixture partition (1). The partition property of the mixture domain propagates to the spatial domain of resolution-limited space. A special case of feature partitions are stand-alone features that are created by makeStandAlone (2).

Appendix "D"--Definition of Medial Axis

This appendix defines the concept of medial axis of a point set. Again, algebraic specification is used. The specification uses point sets (sort "PtSet") and closed disks (sort "Disk") that are defined in Euclidean Space.
The medial axis transform ("ma") is presented in the form of an operation that maps a point set to another point set:
ma: PtSet -> PtSet

For reasons of convenience, ma is composed of the two steps ma1 and ma2, such that
ma1: PtSet -> Set of Disk
ma2: Set of Disk -> PtSet

Let ps be a point set, then the relation between ma and ma1, ma2 is as follows:
ma(ps) = ma2(ma1(ps))

The semantics of ma2 are given by the following equations that use the variables:
ps: PtSet; sd: Set of Disk
ma2(sd) = centerPt(sd) {union of all center points of disks in sd}

The semantics of ma1 is given by the following two equations that use the variables:

d, d1: Disk; ps: ptSet

(1) union(ma1(ps)) = closure(ps)
(2) isElement(d, ma1(ps)) =>
not ∃ d1 such that isSubset(d, d1) and isSubset(d1, closure(ps))

The first equation specifies that the union of disks created by ma1 must completely cover the closure of the argument point set. The second equation requires that ma1 returnes the largest possible disks to cover the closure of ps.

Biography

Beat (Bud) P. Bruegger was born in Basel, Switzerland on June 19, 1961. He received his high school education in Muttenz, Switzerland.

He entered the Swiss Federal Institute of Technology (ETH), Zürich, Switzerland, in 1980 and obtained a diploma (comparable to a Master of Science) in Surveying Engineering in 1986 and a prize for best graduate of the year in his field. His thesis used digital image processing to derive the optical transfer function of photogrammetric imaging systems. After graduation, he spent just short of two years as a research and teaching assistant at ETH where he predominantly developed interactive surveying software.

In January 1988 he was enrolled for graduate study in surveying engineering at the University of Maine. He served as a research assistant in the Surveying Department and later in the National Center for Geographic Information and Analysis (NCGIA), Maine. As a consultant in Neuchâtel, Switzerland, in summer 1990, he developed a software package for the integration of digitized cadaster plans. During 18 month in 1991 and 1992 he pursued his doctoral research at the International Institute for Aerospace Survey and Earth Sciences (ITC), The Netherlands, on an ITC fellowship. After a short stay in Maine, he became a visiting instructor for the academic year 93/94 at the Geography Department of the University of Oklahoma, Norman, where he taught courses in geographic information systems (GIS) and cartography. He is a candidate for the Doctor of Philosophy degree in surveying engineering from the University of Maine, Orono, in August 1994.

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