Title

Meanderings of fluid streams on acrylic surfaces, driven by external noise

Authors
Birnir, Bjorn
Mertens, Keith
Putkaradze, Vakhtang
et al.

Publication Date
2008-06-16

Meandering of fluid streams on acrylic surfaces, driven by external noise

Björn Birnir

Center for Complex and Nonlinear Science
and
Department of Mathematics
University of California, Santa Barbara CA

Keith Mertens and Vakhtang Putkaradze

Department of Mathematics
Colorado State University, Fort Collins, CO

and

Peter Vorobieff

Department of Mechanical Engineering
University of New Mexico, Albuquerque, NM

Dec. 3, 2007

1 Introduction

A stream of fluid flowing down a partially wetting inclined plane usually meanders, unless the volume flow rate is maintained at a uniformly constant value, producing a recently discovered [16, 19] braiding pattern. However, fluctuations in the flow rate are inevitable in naturally occurring flows. Previous studies have conjectured that for some surfaces the meandering of a stream is an inherent instability [5, 22, 3, 4, 21, 24, 20, 13, 6]. In this paper we show that by using acrylic plate we can completely eliminate the meandering by reducing the perturbations entering the flow. By re-introducing fluctuations into our experiment, we then demonstrate that these perturbations are indeed responsible for the onset of the meandering. Another factor contributing to the meandering behaviour is the presence of isolated fluid droplets, left by the fluid stream, which maintains the meandering. If the flow rate fluctuations are eliminated, the flow universally re-establishes the non-meandering pattern, although the relaxation time is on the order of hours. We derive a mathematical model for the stream shape from first principles, which includes stream dynamics and forcing by external noise. While the deviation $h(x)$, from a straight linear stream, $h(x) = 0$, shows considerable variability as a function of downstream distance x, when an ensamble average is computed, averaging power spectrum $S(k)$ as a function of wavenumber k for several different times t, we obtain the power-law scaling $S(k) \sim k^{-5/2}$. In addition, the growth of the area $A(x)$ swept by the stream at the distance x grows as $A(x) \sim x^{1.75}$. These experimental results are in excellent agreement with our theory and with recent results on turbulent flow in rivers [1] and landsurface evolutions [2].

Meanderings of a stream of fluid on a partially wetting surface (e.g., acrylic) and meanderings of rivers are visually strikingly similar, in spite of the differences in the physical nature of these two phenomena. River meanderings are affected by many effects which are both highly complex and beyond our control: turbulence in the water, erosion of the soil on the bottom, unevenness of the soil consistency and its rheological properties, variations in flow rates due to the seasons, etc. It is not clear at present what phenomenon contributes most to the river meandering since all the effects we mentioned are highly intertwined. Two approaches to modelling of river meandering are dynamical (derivations of dynamical equations based on first principles [17, 15, 18]) and stochastic (with noise simulating the effects of turbulence and landscape variations [1, 2]). While a detailed model of the meanderings of all rivers may be a long way off, we believe that combining both stochastic and dynamic approaches to river meandering is valid.

Stream meanderings on a partially wetting surface (glass or specially fabricated plastics-(Mylar, or polyethylene terephthalate)) have attracted much attention in recent literature [5, 22, 3, 4, 21, 24, 20, 13, 6]. All studies have concentrated on the dynamical approach to this problem, treating this phenomenon as an inherent instability of the fluid rivulet. The development of a consistent theory was hampered by the fact that a dynamic model would necessarily incorporate detailed assumptions about the evolution and hysteresis of contact angle. This is the angle that the cross-sectional surface of the liquid, perpendicular to the downstream direction, makes with the plate. However, the theory of contact angle is currently in the initial stages of development [6]. The stochastic approach to this problem has not been considered before. In this paper, we show that stream meandering on an acrylic plate (that is different from surfaces considered by previous authors [5, 22, 3, 4, 21, 24, 20, 13, 6]) is purely noise driven and the shapes of the stream can be explained completely by a model that combines a dynamic approach with effect of noise. No assumptions on the nature of contact angle have to be made.

2 The Experiment

A stream of fluid flowing down an inclined plane in the laboratory is highly controllable. Our experimental apparatus consists of 2.4 meter long acrylic plate (supported by metal beams to eliminate bending) that can be inclined with respect to the horizontal at angles up to 50 degrees. We use either pure water or a water-glycerin mixture to investigate different viscosity and flux regimes. A high precision peristhaltic pump recirculates fluid running down the plate in a 2 meter tall and 10 cm narrow container positioned above the inclined plane. That container is an essential factor in efficient damping of disturbances of the flow. The water then runs from the container onto the inclined plate through a 2mm diameter tube. After the disturbances have been eliminated, after initial set-up time the rivulet assumes straight shape for all flow regimes investigated (Fig. 1b) over all downstream distance available in the experiment (2.4 meters).

To investigate the meandering flow regime, we added an electronically operated valve to our original flow system, inducing an artificial fluctuation to the flow rate. With this arrangement, we can eliminate the meandering and re-introduce it in the flow at will at any flow rate (Fig. 1c). When the valve is switched off, the stream always assumes a straight shape, although the relaxation time can be large (up to 3 hours) for small flow rates. In the few cases we observed a meandering pattern become stationary without straightening out. This is called stationary meandering in previous literature [13]. However, in our case this effect could always be attributed to the sedimentation of dirt and dust particles on the surface. If the surface is cleaned and the flow restarted with exactly the same flow parameters as before, in the absence of disturbances in the flow rate, we inevitably get a straight

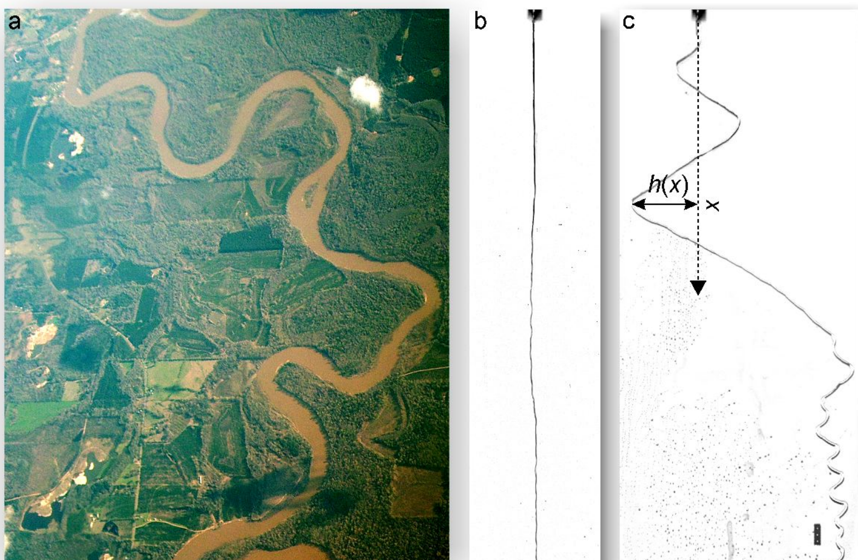

Figure 1: Natural (a) and laboratory (b,c) flows, viewed from above, flow direction in (b,c) is from top to bottom. a, Flow of Mississippi river south of Memphis. b, Non-meandering flow (constant flow rate) of a 50%-50% water-glycerine mix. c, Meandering flow of the same fluid with the same mean flow rate, where fluctuations in the flow rate are induced by a pulsed electromagnetically operated valve.

stream, whereas when the disturbances in the flow are present the stream seems to meander indefinitely.

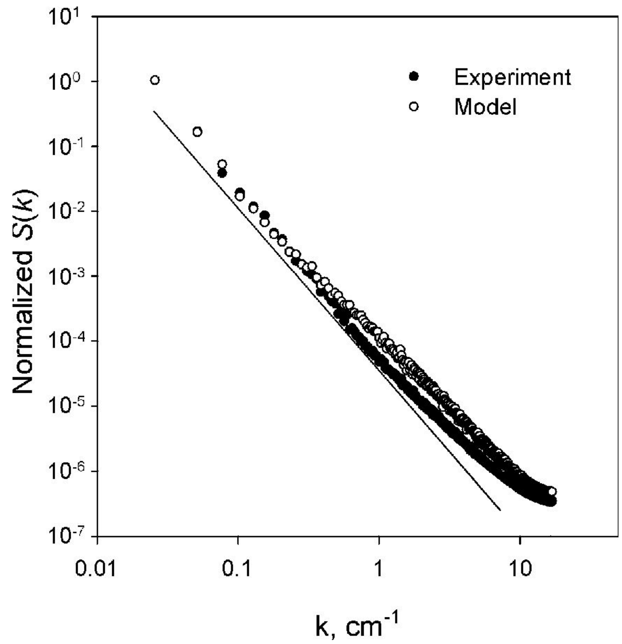

Figure 2: Power spectra of the deviations of the stream from the centerline. • - experimental results, ∘ - results predicted by the model. Solid line represents $a = -5/2$ power law. The amplitude of the noise in the model (a universal constant for all experiments) is fixed to achieve agreement at small wavenumbers k. The agreement persists over all the values k having physical relevance.

We believe the discrepancy between our findings and previous literature is due to the difference in the surface wetting properties. It was noted in [13] that surface properties play a crucial role in this phenomenon. We believe that use of readily available acrylic is advantageous since all experimental results can be explained using a rather simple and consistent theory because, as is explained below, the sources responsible for meandering can be readily understood.

First, the onset of the stream meandering is caused entirely by disturbances in the flow rate. To check this assumption, we tried alternative ways of introducing fluctuations in the stream. We have applied periodic oscillation to the position of the nozzle or single-tone acoustic forcing to the stream. Even with the nozzle position oscillating, the flow stabilizes downstream. Alternatively, an acoustic forcing

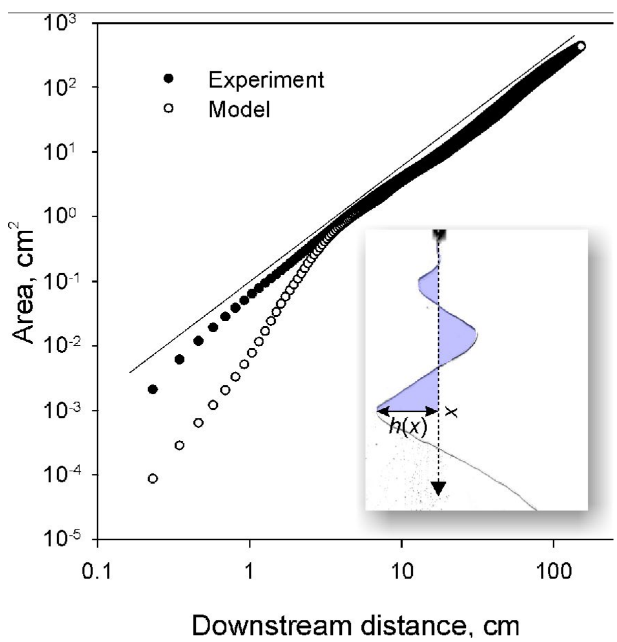

Figure 3: Area enclosed between the stream and the centerline as function of downstream distance. See inset for explanation. • - experimental results, ◦ - results predicted by the model. Solid line represents power law with exponent $7/4 = 1.75$. The same amplitude for the noise as in Fig.2 is fixed in the model. The deviation of the model from experiment is observed for distances comparable to the width of noise forcing, equal to the diameter of the stream.

of the jet with a constant frequency also has a limited effect [1, 2]. On the other hand, a disturbance consisting of 'spikes' (short interruptions of fluid flow rate) or acoustic forcing with repeatedly changing frequency causes the meandering. As long as this forcing is applied, the meandering will persist. When the forcing is removed, the meandering will die out after some transition time. Second, after the onset of meandering, the amplitude of transverse oscillations downstream is amplified by droplets which are left by the moving stream. We believe that is the fundamental property of acrylic sheet that allows for construction of a simple and successful theory. When the stream is moving, it re-encounters the droplets (seen in Fig. 1b,c), which act as transverse random forcing. This mechanism appears to be strongly affected by surface properties, which are different in our experiment from other recent meandering studies [13].

Our quantitative analysis of the stream behaviour is based on 105 pictures of the stream acquired with a high-resolution computer-controlled digital camera. The random time intervals between the pictures were sufficiently large for the flow patterns to be statistically independent. From each of these images, we can extract the deviation h of the stream from the centerline as the function of downstream distance x, see Fig. 1c. From the $h(x)$ data produced by the analysis of each picture, we computed power spectra $S(k)$, where $k = 2/l$ is the wavenumber corresponding to a spatial wavelength l. While the power spectra based on single images are rather noisy, the spectrum produced by averaging over all the $h(x)$ realizations manifests a smooth graph (Fig. 2) with apparent power-law scaling $S(k) \sim k^{-5/2}$ over the span of about two decades. Deviation from this scaling is noticeable only for $k \geq k_{\max} = 5\text{cm}^{-1}$, corresponding to physical scales smaller than the characteristic stream width (1cm). The largest physical scale we can acquire (and thus the smallest wavenumber) is constrained by the size of our experimental arrangement, (2.4m) with the inclined plane being 2.4 m long. It is also noteworthy that, while the graph presented in Fig. 2 is based on all realizations of $h(x)$, averaging as few as 30 realizations produces a similar smooth spectrum with the same power-law scaling with the same exponent $a = -5/2$. The latter is a persistent feature of all our experiments, representing a universal characteristic of the problem of the flow down a partially wetting incline. A model of experiment incorporating surface tension, fluid friction with the surface as well as friction inside the fluid and surface tension can be derived (see next section). The model incorporates the effect of noise forcing on the stream from the residual droplets on the surface and includes the noise strength as the sole fitting parameter.

As another test of our theory, in Fig. 3, we plot the area enclosed between the meandering stream and its centerline as the function of the downstream distance.

The deviation of the model from the power law is likely due to the length scale associated with the forcing (characteristic droplet size about 1-5 mm). The area grows as $x^{7/4} = x^{1.75}$ with the distance, consistent with the power law $k^{-5/2}$ of the spectrum. Thus we can conclude that the behaviour of a stream meandering down an inclined plane is dominated by the effects of the stream interacting with droplets on the plane, which can be modelled with including appropriate random forcing into the governing equations.

A compelling argument base on [2] indicates that this last result is connected to Hack's law [14] which says that the length ℓ of a main river in a river basin scales as a power of the area of the basin $\ell \sim A^{\frac{1}{1.75}} = A^{0.58}$, see conclusions below. It indicates that our experimental river shares at least some properties with real rivers. Assuming uniform drainage density, Hack's exponent (0.58) and the meandering exponent, or the fractal dimension of the river, determine all the other exponents of all the known scaling laws for landsurfaces, see [11, 7, 8, 9, 10].

3 The Theoretical Model

The derivation of the model is as follows. We assume that the stream is sufficiently thin so that its properties at the downstream distance x and time t can be described by three variables: downstream velocity u, transverse velocity v and deviation from the centreline $h \equiv 0$. The equations for the velocities (u, v) are obtained by balancing the forces (inertia, viscosity, gravity and surface tension) acting in the downstream and perpendicular to the downstream direction: Consider the equations for the flow down the plate

$$\partial_t u + u\partial_x u = -\lambda u + nuu_{xx} + \gamma/d \frac{h_{xx}}{\sqrt{1+h_x^2}} \sin\phi + \eta(x,t)\sin\phi + g\cos\alpha$$

$$\partial_t v + u\partial_x v = -\lambda v + nuv_{xx} + \gamma/d \frac{h_{xx}}{\sqrt{1+h_x^2}} \cos\phi + \eta(x,t)\cos\phi$$

where Q is the volume flux, ν is the kinematic viscosity, γ is the coefficient of surface tension, d is the stream thickness, α is the inclination of the plane with respect to vertical, ϕ is the angle between the stream and downward direction x and λ is the lubrication friction coefficient $\lambda = 3\nu u/Q$. The equation for the deviation from the linear river h is given by the fact that that for given values of velocities (u, v) the instantaneous rate of change of h is given by

$$\partial_t h + u\partial_x h = v$$

The important part of the model is the nature of the noise η, that is modeling the effect of the stationary random distributed droplets with the stream. An important thing to keep in mind here is that at a given time, the stream encounters only very few droplets. Thus, η cannot be modeled by white noise distributed over all the length of the stream. Instead, we use point forcing at random positions of the stream at random times; the results do not change for all noise models we tried as long as the noisy forcing is applied locally at any given time. The amplitude of the random noise is the only fitting parameter we have in the model. Interestingly enough, that fitting parameter is universal for all experimental realizations.

Another way of expressing this is to say that the noise is quenched, or colored, just as in turbulence. This implies that both u and v scale as the solutions of the noise-driven Navier-Stokes equation in one-dimensional turbulence, see [1]. A typical Reynolds number of the experiment is around $R = 4500$ but it it can possibly be as low as 500 (1mm layer of water and glyserin) or as high as $18,000$ (2mm layer of water only).

The equation

$$\partial_t h + u \partial_x h = v$$

can be solved in the following manner. Let

$$h = h(x - U_o t, \varepsilon t)$$

where U_o is the mean velocity in the downstream direction. This solution is a slowly modulated traveling wave. Then

$$h_t = -U_o h', \quad h_x = h'$$

and

$$h_t + u h_x = (-U_o + u) h' = \frac{(U_o - u)}{U_o} h_t = v$$

ignoring terms of order ε.

The equation

$$h_t = \frac{U_o}{U_o - u} v$$

can be solved by integration in t

$$h = U_o \int_0^t \frac{v(x,s)}{(U_o - u(x,s))} ds$$

since $h(0)=0$. Now if ℓ is a lag variable, then

$$
\begin{gathered}
h(x+\ell,t)-h(x,t) = \\
U_o \int_0^t \left(\frac{v(x+\ell,s)}{U_o-u(x+\ell,s)} - \frac{v(x,s)}{U_o-u(x,s)} \right) ds = \\
U_o \int_0^t \left(\frac{v(x+\ell,s)-v(x,s)}{U_o-u(x+\ell,s)} - \frac{u(x+\ell,s)-u(x,s)}{(U_o-u(x+\ell,s))(U_o-u(x,s))} \right) ds
\end{gathered}
$$

Thus

$$
\begin{gathered}
\int_0^1 |h(x+\ell,t)-h(x,t)|^2 dx \leq \\
C \int_0^1 |v(x+\ell,t)-v(x,t)|^2 dx + \int_0^1 |u(x+\ell,t)-u(x,t)|^2 dx
\end{gathered}
$$

or

$$
s_h(\ell,t) \leq s_v(\ell,t) + s_u(\ell,t) \tag{1}
$$

where s_f are the second structure functions, see [12], (or width functions, see [2] squared).

The equation (1) implies that if s_v and s_u scale with the exponent $3/2$, see [1], then s_h must also scale with the same exponent. If the scaling exponent $4/3$ takes over this indicates the scaling of u is dominated by turbulent Burgers shocks, see [23]. Presumably, that is what happens on concave (down) surfaces.

4 Conclusion

In this paper we have shown both experimentally and theoretically that experimental river meanderings are cause by fluctuation in the flow rate. Left over droplets on the surface then cause further meanderings whereas an experimental river does not meander on a drop-free (clean) surface when the flow rate is uniform.

The structure function s_2 of the deviation of the meandering from a center line $h(x)$ are shown to scale to the power $3/2$ in a lag variable. This corresponds to the scaling of the power spectrum $S(k) \sim k^{-5/2}$, see for example [12]. We identify the scaling exponent as the scaling exponent $3/4$ of the width function, $V^2 = s_2$ of the velocity of turbulent flow in rivers, see [1].

The scaling of the area under $h(x)$ as $x^{7/4}$ seems to resolve a puzzle left over by the scaling of the fluvial surfaces in [2]. How is the spatial roughness $\chi = 3/4$ of a riverbed, caused by turbulent flow eroding the bed, transported to the whole surface? The scaling of the area which is in accordance with Hack's [14] law indicates that the river meanderings cover the whole river basin over time thus endowing the surface of the whole basin with the same spatial roughness. It also seems that in some cases Hack's exponent determines the meandering exponent.

The overall meandering exponent for river is reported to be 1.1 not $7/4$, see [7, 8, 9, 10]. Thus our result is clearly not the whole story and there is more to the meandering of real rivers than our experimental ones.

References

[1] B. Birnir. Turbulent Rivers. *To appear in Quarterly of Applied Math*, 2008.

[2] B. Birnir, J. Hernández, and T. R. Smith. The stochastic theory of fluvial landsurfaces. *J. Nonlinear Science*, 17:13–57, 2007. DOI:10.1007/s00332-005-0688-3.

[3] J. B. Culkin. Ph.d. thesis, 1984.

[4] J. B. Culkin and S. H. Davis. *AIChE J.*, 30:263, 1984.

[5] S. H. Davis. *J. Fluid Mech.*, 98:225?242, 1980.

[6] P. G. DeGennes. *Rev. Mod. Phys.*, 57:827?863, 1985.

[7] P. S. Dodds and D. Rothman. Geometry of river networks. I. scaling, fluctuations and deviations. *Phys. Rev. E*, 63:016115, 2000.

[8] P. S. Dodds and D. Rothman. Geometry of river networks. II. distributions of component size and number. *Phys. Rev. E*, 63:016116, 2000.

[9] P. S. Dodds and D. Rothman. Geometry of river networks. III. characterization of component connectivity. *Phys. Rev. E*, 63:016117, 2000.

[10] P. S. Dodds and D. Rothman. Scaling, universality and geomorphology. *Annu. Rev. Earth Planet. Sci.*, 28:571–610, 2000.

[11] P. S. Dodds and D. Rothman. Unified view of scaling laws for river networks. *Phys. Rev. E*, 59(5):4865, 2000.

[12] U. Frisch. *Turbulence*. Cambridge Univ. Press, Cambridge, 1995.

[13] N. Le Grand-Piteira, A. Daerr, and L. Lima. *Phys. Rev. Lett.*, 96:254503, 2006.

[14] J. Hack. Studies of longitudinal stream profiles in Virginia and Maryland. *U.S. Geological Survey Professional Paper 294-B*, 1957.

[15] S. Ikeda, G. Parker, and K. Sawai. *J. Fluid Mech.*, 112:363, 1981.

[16] V. Putkaradze K. Mertens and P. Vorobieff. *Nature*, 430.

[17] L. B. Leopold and M. G. Wolman. *Bull. Geol. Soc. Am.*, 71.

[18] E. Levi. *The Science of Water*. ASCE Press, New York, 1995.

[19] K. Mertens, V. Putkaradze, and P. Vorobieff. *J. Fluid Mech*, 531.

[20] T. Nakagawa. *J. Multiphase Flow*, 18:455, 1992.

[21] T. Nakagawa and J. C. Scott. *J. Fluid Mech.*, 149:89, 1984.

[22] R. H. Weiland and S. H. Davis. *J. Fluid Mech.*, 107:261?280, 1981.

[23] E. Welsh, B. Birnir, and A. Bertozzi. Shocks in the evolution of an eroding channel. *AMRX*, 2006. To appear.

[24] G. W. Young and S. H. Davis J. *J. Fluid Mech.*, 176:1–31, 1987.