Title

SPECTROSCOPIC DETERMINATION OF STELLAR MASSES - MENE, MENE, TEKEL, ARCTURUS

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Spectroscopic Determination of Stellar Masses: Mene, Mene, Tekel, Arcturus*

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SUMMARY

The determination of stellar masses (i.e. surface gravities) from spectroscopic data requires, in principle, a complete understanding of stellar atmospheric structure, including the effects of turbulence and convection, departures from LTE and plane parallel structure, and variations in abundances of individual elements. Where very accurate data are available, as for Arcturus, it becomes clear that the uncertainties in derived effective temperatures, surface gravities, and abundances are both rather larger and somewhat more interdependent than is usually supposed. As a result, it takes surprisingly large error bars, e.g. $M = 0{\cdot}7 \pm 0{\cdot}5\, M_{\odot}$, to bracket all modern determinations of the mass of Arcturus. We discuss here why this is so and what, if anything, can be done about it.

I INTRODUCTION

Astronomy often progresses amoeba-like; the advance of one pseudopod may not move the animal very far, but it permits other parts to advance in their turn and can reveal portions of the organism that are in danger of being left behind. The publication of the *Arcturus Atlas* (Griffin 1968), presenting high resolution spectral data over a wide wavelength range, was such an advance, whose implications for our understanding of stellar spectroscopy, atmospheres, and evolution are still unfolding. Since then, various workers, applying standard techniques to the *Atlas* and other photometric data, have arrived at effective temperatures that disagree by some 300 K (4200–4500 K) and surface gravities that range over more than an order of magnitude ($\log g = 0{\cdot}9\text{–}2{\cdot}1$). The corresponding masses for Arcturus, with 1σ error bars, extend from 0·1 to 3·5 $M_{\odot}$. This is a disturbing situation for one of the most intensively studied stars in the sky.

A Workshop on the Mass of Arcturus was convened in Cambridge (1981 March 23–25) by R.E.M.Griffin (Cambridge), in response to a suggestion from B.Gustafsson (Uppsala), to investigate the problem and possible resolutions of it. We report here recent work on Arcturus and its implications for stellar astronomy in general in light of that workshop.

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2 DATA

(a) *Fluxes as a function of wavelength.* In an ideal world, the theorist would have available for Arcturus (and any other star he wanted) absolute flux as a function of wavelength with arbitrarily high precision and resolution. We don't. We have the tracings (and their digitized equivalents) of the *Atlas*; assorted other spectra, including some FTS ones of very high quality but limited wavelength coverage; and several sets of broad-band fluxes determined relative to Vega or some other standard star.

The line profile situation is relatively good. Although the *Atlas* profiles are undoubtedly contaminated by scattered light, the instrumental profile is well known and can be convolved with a calculated profile for comparison with the data. This has been checked by H.Holweger (Kiel), who compared FTS and *Atlas* profiles, and concluded that the only difference is instrumental resolution. Thus equivalent widths from the *Atlas* should be reliable. Both strong and weak lines are inevitably contaminated by blending. This, however, is a property of the star itself and cannot be removed by observational techniques, but must be taken account of in modelling. Thus, according to R.Kurucz (Center for Astrophysics) an analysis, like that of Ayres & Johnson (1977), which does not allow for blending in strong lines, should not be trusted.

The absolute flux measurements are much less satisfactory. Recent results by Frisk *et al.* (1981) differ from previous ones by several per cent, especially near the peak of the spectrum (0·6–1·2 μm), which matters most for temperature determination. It is not entirely clear that the most recent data set (best fit by $T_{eff} = 4375^{+100}_{-50}$ K) should be preferred to earlier ones (best fit by $T_{eff} = 4250^{+100}_{-50}$ K). Kurucz suspects that this may be the entire problem with Arcturus. At any rate, there is general agreement with Gustafsson's estimate that 1 per cent flux accuracy is needed to pin down effective temperature to ± 50 K (and so log g to $\pm 0{\cdot}25$ by standard techniques). This may not be possible. Even Vega, the primary flux standard, is known only to about ± 2 per cent. At short wavelengths, laboratory flux standards are a problem, because they are relatively cool; and so their emission is exceedingly sensitive to their exact temperature and its uniformity. At longer wavelengths, emission by earth, air, and apparatus can contaminate measurements. Recent work on Vega by D.Blackwell and his colleagues (Oxford) should yield much higher accuracy than the 8 per cent of their published K-band photometry (Selby *et al.* 1980), but it is not clear precisely what can be achieved either for Vega or for important secondary stars like Arcturus. Airborne measurements overcome some of the ground-based problems, but have others of their own. The situation for other, fainter, stars will in general be worse; but as they have not been measured so often, the discrepancies are less conspicuous!

(b) *The radius of Arcturus.* In order to turn a spectroscopic surface gravity into mass, we must know the radius of the star, that is, its distance and angular diameter. Arcturus has one of the better-determined parallaxes ($0{\cdot}092 \pm 0{\cdot}005$ arcsec; Woolley *et al.* 1970). Virtually all other stars will be worse off than this 5 per cent uncertainty in distance (and so in mass). The angular diameter is much less well known. There have been about half a dozen direct determinations (by assorted interferometric techniques) over the past 50 years. The measured values range from 0·018 to 0·026 arcsec for a uniformly bright disc (Augason *et al.* 1980; Blazit *et al.* 1977). Correction for maximal limb darkening can turn, e.g. 0·019 into 0·027 arcsec, according to Worden (1976); but Barnes, Blazit & Moffet (1978) report a maximum limb darkening correction factor of 1·134 for occultation and intensity interferometric techniques, modifying the measured values to, at most, 0·020–0·029 arcsec. Angular diameters can also be extracted by infrared flux techniques (Blackwell & Shallis 1977; Scargle & Strecker 1979; Augason *et al.* 1980; Manduca, Bell & Gustafsson 1981). The values are at the lower end of the directly measured range, 0·020 ± 0·001 arcsec. The compromise value of 0·024 arcsec (corresponding to a radius of 28 $R_\odot$) used by Mäckle *et al.* (1975) is thus reasonable, but must be accepted as having some 20 per cent or more uncertainty, which carries across to a 40 per cent uncertainty in mass.

(c) *Atomic and molecular constants.* Any analysis of line strengths or profiles requires accurate transition probabilities (f values, or oscillator strengths). For strong lines, one also needs damping constants, describing how atomic or molecular energy levels are broadened by the presence of nearby atoms, ions, and molecules. Both can, in principle, be obtained by calculation, by laboratory measurement, or by analysis of the sun using a solar model atmosphere. Neither are known well enough to be neglected in analysing uncertainties in spectroscopic parameters.

All damping constants in use are really solar: laboratory measurements would require longer path lengths than were possible until quite recently, and the calculated values of van der Waals broadening by hydrogen and helium atoms differ from the best-fit solar values by up to an order of magnitude. The custom, therefore, is to use calculated values multiplied by a scaling factor derived from solar data. Unfortunately, there is not universal agreement about the magnitude of the factor (compare Blackwell & Willis 1977 and Ruland *et al.* 1980). The resulting difference in log g is about 0·6 for Arcturus. In addition, such a scaling factor will inevitably have errors due to (a) the solar model, and (b) variations with parameters that differ between the Sun and the star of interest. The scaling factor is known to vary from one element to another (by a factor of four or more, e.g. Mäckle *et al.* 1975) and from one temperature to another (Blackwell & Willis 1977) in ways that we can neither predict nor explain.

Edmunds (1975a, b) and Blackwell have made the intriguing suggestion that the scaling factor for damping constants may also depend on excitation potential. If so, then any parameter (T, g, abundances, or microturbulent velocity) determined by requiring abundances derived from lines of different EP and strength to come out the same could be considerably in error.

Much better damping constants should be available soon. The Oxford group have recently started determining laboratory data for collisions with helium, from which the hydrogen collision constants can be deduced (O'Neill & Smith 1980). According to Blackwell, these measured values confirm a dependence on excitation potential of the scale factor mentioned above.

A transition probability is genuinely a property of the atom or molecule alone. Thus a good *f* value can be used anywhere, no matter how you get it. There are numerous sets of calculated and laboratory *f*s in the literature for popular ions like Fe I. But they disagree (the lab values among themselves as much as with the calculated ones) by 0·2 dex (50 per cent) or so on average (see Blackwell *et al.* 1979, 1980 for references and discussion). And there seem to be systematic trends in the disagreements with line strength, excitation potential, and (probably) other things at about the same 50 per cent level. Thus, although some relative oscillator strengths are so well-determined (0·5 per cent) that one needs 1 K temperature accuracy to make full use of them, the absolute iron abundance of any star (including the sun) remains uncertain by about 0·2 dex. And if the trends in the disagreements among sets of *f* values indicate errors that scale with line strength and ionization and excitation potential, the spectroscopic determinations of T and g will be affected. For instance, a 30 per cent systematic error in *f* values for lines differing by 3 eV in EP translates into a 150 K error in excitation temperature for Arcturus. And (pretending that all other parameters are exactly known) a similar 30 per cent error between the oscillator strengths for neutral atoms and ions distorts surface gravity by an order of magnitude ($P_e \propto g^{1/3}$; Gray 1976). Similarly, a 30 per cent systematic trend in *f* value errors with line strength results in a value for the microturbulence parameter, ξ, which is wrong by a factor of two when it is determined from the part of a curve of growth dominated by doppler broadening. Gurtovenko & Kostik (1981) believe that even the best calculated *f* values probably have errors of this sort.

Most analyses of Arcturus have used oscillator strengths tied to the Sun in one way or another, which reduces such systematic trends (otherwise even more discordant results might be expected!). But solar *f* values, like solar damping constants, are subject to uncertainties in the atmospheric model used. Resulting systematic errors may be perturbing spectroscopic parameters of other stars by an appreciable fraction of the errors just discussed. The determinations of solar *f* values and damping constants for strong lines are not independent, of course. As the lines will be of different strengths in a cooler star, the errors will not necessarily tend to cancel each other.

Laboratory measurements of transition probabilities for ionized atoms are exceedingly difficult. Kurucz believes that the calculated ones are more reliable than published measured values. The absolute errors are probably rather larger than the 0·2 dex characteristic of the best Fe I data. The Oxford group is now making furnace measurements of transition probabilities for ions which should have the same very high relative accuracy as their neutral atom data. Finally, the best laboratory data even for Fe I, does not yet extend to weak enough lines to be used for normal analysis of Arcturus. The Oxford group plans to tackle these very weak lines in both Fe I and Ti I.

3 DETERMINATIONS OF TEMPERATURE

The photospheric temperature of a star is important in its own right (for comparison with evolutionary tracks and such) and, in addition, must be known with some precision before techniques to determine log g can be applied. The temperature of Arcturus has been sought by all the standard methods, including model fitting to continuum fluxes or their gradients, spectrum synthesis of line profiles, and curve-of-growth analysis of atomic and molecular lines with different lower level excitation potentials. The average of the line profile determinations is about 130 K lower than the mean of the continuum flux determinations (Auguson *et al.* 1980), but there is considerable spread in both sets. Fig. 1, compiled by Gustaffson, shows effective temperatures as a function of publication date.

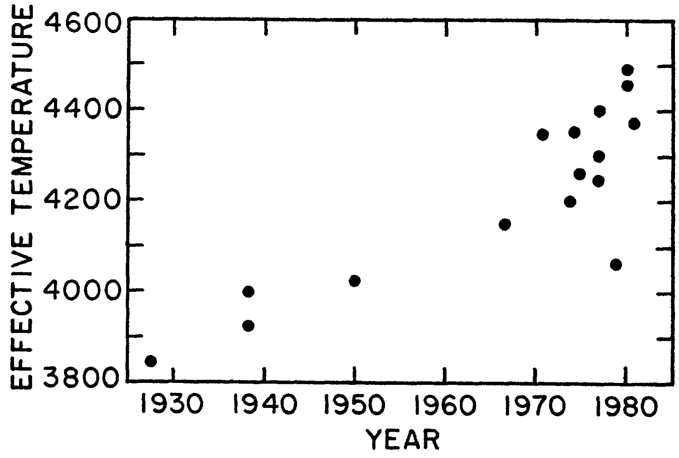

Fig. 1. Published effective temperatures for Arcturus as a function of publication date. The general upward trend probably does not have predictive power.

Continuum temperatures are derived by matching observed broad-band fluxes to the predictions of a grid of calculated model atmospheres. In the current state of the art, uncertainties due to the models and to the data are comparable. Blackwell & Willis (1977) found, for instance, that switching from Carbon & Gingerich (1969) to Bell, Kingston & McIlveen (1976) atmospheres lowered the effective temperature best fitting their relative fluxes from 4500 to 4400 K. On the other hand, the models used by Frisk *et al.* (1981) match their own absolute fluxes best at 4375 K, but provide an equally good fit to earlier published photometry (e.g. Honeycutt *et al.* 1977) at a temperature near 4250 K. $T = 4250$ K also provides the best fit to the V–I and V–K colours of Arcturus, according to D.Rabin (Cambridge), who remains puzzled that any uncertainty in model atmospheres or anything else can possibly perturb this by as much as 200 K, given that $d(V\text{–}K)/dT_e = 0^{\mathrm{m}}{\cdot}07/50$ K. Frisk *et al.* (1981) have explored the sensitivity of their fit to variations in assumed log g, abundances of individual elements, mixing length parameter, and opacities. Only the last matters much ($\sim$ 50 K). This exploration and the way the Frisk *et al.* (1981) data were obtained probably make their value the best we have for the effective temperature of Arcturus. One of the implications is that other K giants are likely also to be a bit hotter than has been thought.

Temperatures derived from continuum fluxes will best represent the region where the continuum photons are produced – deeper and hotter in the atmosphere than the region making the line photons that must be analysed to get log g. This problem is further discussed in Section 5. One way around it is to use the lines themselves to get a temperature. Line profiles have the advantage that temperatures found from them are not very sensitive to gravity (± 10 K for 0·4 dex in g, according to Gustafsson). Unfortunately, they are also not very sensitive to the absolute value of T, responding mostly to difference in temperature between line- and continuum-forming regions of the atmosphere, according to H.Holweger (Kiel). As a result, where profiles have been used (e.g. Martin 1977), they don't really contribute information about T_e that isn't already in the continuum and line-strength-ratio analysis. Line profile data can, on the other hand, usefully constrain the temperature gradient (Ayres & Linsky 1975; Mäckle *et al.* 1975).

Finally, an excitation temperature can be found by matching curves of growth for lines of different excitation potential; or, in more elegant treatments, requiring spectrum-synthesis techniques to give the same abundance of an ion for lines of different EP yields a model atmosphere temperature reflecting excitation conditions. Normally, the result varies from ion to ion (see, e.g. Griffin & Griffin 1967). In addition, an excitation temperature applies to the line formation region, higher and cooler than the continuum photosphere, and must be transformed into an effective temperature via a model atmosphere, as is done implicitly in the synthesis methods. Typical determinations of T_e by these methods yield numbers in the same range as the continuum analyses (e.g. 4350 K, van Paradijs & Meurs 1974), though Lambert & Ries (1981) found a record high of 4490 K. The errors are at least as large, coming from uncertainties in f values, etc. (see Section 2) as well as in model atmosphere structure. On the other hand, an excitation temperature can be used directly in further analysis of lines formed in the same atmospheric layer to derive spectroscopic abundances and gravities with relatively little sensitivity to atmospheric model. Problems will arise if the lines involved are not all in fact formed in the same region or if non-LTE effects (especially ionization by hot photons coming from below) are important. You can hardly ever win on both parameters at once. For instance, MgH molecular lines are largely free of non-LTE effects, according to Gustafsson, but the excitation temperature obtained from them is very low, according to R.E.M.Griffin, because they are formed higher in the atmosphere than are the atomic lines one wants to analyse for log g.

No allowance has been made for interstellar absorption. One analysis (Scargle & Strecker 1979) found appreciable reddening, apparently as a result of inappropriate weighting of the various wavelength points and difficulty in interpolating model atmospheres across large temperature differences (Manduca *et al.* 1981). In any case, their value of $E(B-V) = 0{\cdot}1$ would imply an average hydrogen density along the line of sight of some 10 atoms per cc; and we don't believe it.

4 DETERMINATIONS OF SURFACE GRAVITY

No spectroscopic parameter measures surface gravity (still less stellar mass) directly. But electron and gas pressure can be turned into surface gravity in a relatively straightforward way (see, e.g. Gray 1976). The transformations are sensitive to composition (via both mean molecular weight and number of electron donors) and $T(\tau)$, but less so than the pressure indicators themselves. These include molecular and ionization equilibria, the shapes of damping wings of strong lines, and changes in continuous opacity source from H$^-$ (proportional to P^2) to H I Rayleigh scattering (proportional to P) at short wavelengths. Though these all depend on pressure, they all, unfortunately, depend more sensitively on something else, particularly temperature, but also abundances, atmospheric structure, and accuracy of laboratory data. This section reviews the history of surface gravity determinations (summarized in Fig. 2), focusing on the major sources of uncertainty in each.

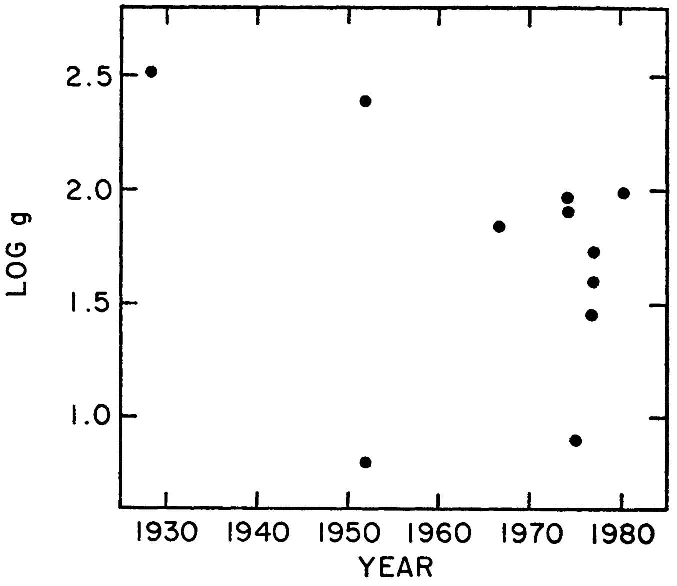

FIG. 2. Published surface gravities (log g in cgs units) for Arcturus as a function of publication date. There are no conspicuous secular trends, even toward convergence.

The continuous opacity method has not been applied to Arcturus, although several colour indices (including DDO C (45–48) and the Danish narrow-band λλ 4000–4900 colours gnkmf) are known to be sensitive to surface gravity. The problem is that the dependence has to be calibrated on stars of known g and the same composition, because our poor theoretical understanding of ultraviolet opacity sources does not permit a useful calculation of the indices from model atmospheres. The main culprits are line crowding and the fudge factor discussed in Section 5 (a). Gustafsson suggests that a study of these colours in Arcturus and other metal-deficient red giants would be interesting.

Molecular equilibrium has also not been much applied. Mäckle *et al.* (1975) used MgH to check the abundance of Mg rather than log g; but the consistency of their atomic and molecular abundances suggests that they would have found the same low gravity from MgH as from ionization equilibrium if they had gone the other way. Adopting the revised MgH dissociation energy from Balfour & Cartwright (1976) would probably destroy this consistency.

Unpublished work by Bell used an atomic, Mg I, abundance ([Mg/H] = −0·2) and the atmospheres discussed by Frisk *et al.* (1981) to synthesize MgH band profiles. The best-fit log g is 2·0, 1·5, or 1·0 for an effective temperature of 4350, 4250, or 4150 K. Adopting a much less likely low abundance of Mg ([Mg/H] = [Fe/H] = −0·7) and/or another equally likely set of *f* values would raise the best-fit log *g* at each temperature by about 0·3. These numbers are distressing only in connection with an analysis, discussed below, of iron ionization equilibrium using the same model atmospheres. The chief difficulty with the molecular dissociation method is extreme sensitivity to adopted temperature, as shown in Bell's results for MgH.

Other molecules, like CN, CH and OH, whose bands are strong in Arcturus could be analysed for surface gravity if independent abundances can be found from C I and [O I] lines; but the necessary molecular constants (*f* values; dissociation energies) are, in general, even less well known than the corresponding atomic parameters.

Several groups have derived log *g* from the wings of strong lines, via a combination of ionization equilibrium and spectrum synthesis techniques. Blackwell & Willis (1977), Ayres & Johnson (1977) and Martin (1977) found log *g* = 1·48, 1·6 and 1·74 respectively, with statistical uncertainties of about 0·2 dex. The methods have been critically discussed in the papers cited and by Ruland *et al.* (1980). All three used *Atlas* data, but the model atmospheres, *f* values, and damping constants came from different compilations. Adopted microturbulence values varied by 50 per cent; and the temperatures used were 4400, 4250 and 4300 K. Thus the good agreement among the three studies must be at least partially fortuitous. The Blackwell and Willis approach has the lowest temperature sensitivity (d log g/dT = 0·07/100 K), but it is also not very sensitive to surface gravity at small values of *g*, because the ratio of collisional to radiative damping drops with *g*. No allowance was made in these studies for the effects of line blending. This is likely to be important (particularly contamination by CO), but is fairly incalculable.

Surface gravities based primarily on weak line ionization equilibria have been reported by Griffin & Griffin (1967), van Paradijs & Meurs (1974), Mäckle *et al.* (1975) and Lambert & Ries (1981). The values – 1·74, 1·95, 0·9 and 2·0 – show a remarkable lack of bandwagon effect, both among themselves and relative to the results mentioned above. The wide range is largely a result of different choices of effective temperature and model atmosphere. Gustafsson calculates that d log g/dT = 0·3–0·5/100 K for these methods. Thus, if Mäckle *et al.* and Lambert & Ries had used each other's *T*s (4250 and 4490 K) they would roughly have derived each other's surface gravities. The dependence on choice of model atmosphere is also large and calculable. M.Spite (Meudon) has performed the interesting experiment of putting exactly the same Fe line data through three standard model atmospheres, assuming the same microturbulence, *f* values, damping, and so forth, and requiring Fe I and Fe II to yield the same total iron abundance. The abundance at which matching occurred varied only slightly among the atmospheres ([Fe/H] = −0·7±0·05), but the log *g*s were 1·2 (model from Mäckle *et al.* 1975), 1·4 (model from Johnson *et al.* 1977), and 1·6 (model from Bell *et al.* 1976). Allowing temperature, model, and microturbulence parameter to vary independently would permit log *g* to range from 0·5 to 2·5, according to Gustafsson. This is overstating the horror of the situation somewhat (the models don't all fit at all possible temperatures!), but not by much, as no allowance has been made for uncertainties in either observational or laboratory data.

We seem to have identified the problem. Solving it is another matter. Section 5 addresses some of the difficulties in constructing and using model atmospheres. Temperature determinations were discussed in Section 3. In addition, efforts have been made to work around the problem. Bell has looked at Fe ionization in Arcturus, using only those Fe I lines that arise from high excitation levels. The atoms making these should be close to the Fe II atoms in the star's atmosphere, reducing sensitivity to atmospheric model and (especially) temperature. This works, in the sense that both the iron abundance and the surface gravity at which matching occurs vary by only $\pm 0{\cdot}1$ dex over $T = 4150$–4350 K. The centroid values are [Fe/H] $= -0{\cdot}8$ and $\log g = 1{\cdot}0$. Notice that this matches the MgH $\log g$, derived from exactly the same atmosphere grid, only at $T = 4150$ K, which (still using the same atmospheres) is a lousy fit to continuum fluxes. The model atmosphere dependence of this method has not been explored.

Studies of three other stars shed additional darkness on the situation. Holweger has analysed the *Procyon Atlas* (Griffin and Griffin 1979) following the techniques of Mäckle *et al.* (1975). Procyon is in a binary system, and its known mass implies $\log g = 4{\cdot}04$. Using this, the total iron abundance comes out 0·23 dex higher for Fe II than for Fe I, implying a 'spectroscopic g' that is smaller than the real one by about 0·4. This does not, however, mean that we should automatically kick the Mäckle *et al.* Arcturus gravity up by 0·4, unless the main problem is errors in the Fe II gf values. In fact, Procyon is a good deal hotter, and its Fe I/Fe II discrepancy is probably due to nonlocal control of ionization.

FTS spectra of Pollux (having rather higher resolution than the *Atlas* data) analysed in the same way by Ruland *et al.* (1980) yield a 'reasonable' $\log g$ of 2·24 (or 2·54 using Bell *et al.* 1976 model atmospheres). This cannot be checked against the real $\log g$, as Pollux is a single star (practically the only one in Gemini!). But the patterns of abundance as determined from lines of different strength and excitation potential indicate that non-LTE effects are of considerable importance in this 4850 K star. Spite finds very similar patterns in Arcturus. Looking at these three studies together produces in our minds the impression that, even neglecting uncertainties in temperature, the Mäckle *et al.* $\log g$ for Arcturus is probably too low by 0·3–0·4 dex. Ruland *et al.* (1980) do not agree.

Finally, Ruland *et al.* have calculated Fe ionization equilibrium surface gravities for the Sun, using a single set of stellar and atomic data, but five different solar model atmospheres (three the work of people who have also published Arcturus models). Answers range from 4·14 to 5·05, corresponding to masses of $0{\cdot}51$–$4{\cdot}09\ M_\odot$. The Bell *et al.* (1976) model yields too low a surface gravity (4·30) and the Holweger & Müller (1974) model too high a one (4·63) in the opposite sense to the Arcturus discrepancy. Ruland *et al.* conclude only that (at least until absolute f values are better known) this is not a good way to choose among model atmosphere types. We heartily agree!

5 MODEL ATMOSPHERES

The phrase 'model dependence' is widely and somewhat loosely used in astrophysics, often meaning not much more than 'I don't like your answer'. We shall use it here to refer to circumstances where the result of some calculation depends on the difference between published grids of model atmospheres or on differences between models in general and real stars. The atmospheric input parameters that result in model dependence in this sense include (but undoubtedly are not limited to) opacities, deviations from plane parallel structure, local thermodynamic equilibrium or its absence, and convection, taken here to include microturbulence, which is really a spectroscopic fitting parameter, expressing our ignorance of line broadening mechanisms. The most important output parameter for model dependence is the temperature gradient in the atmosphere, $T(\tau)$ for short.

(a) *Opacities*. The dominant source of continuous opacity in cool stars is H^-. There is about a 25 per cent difference between the two sets of H^- opacities (Doughty & Fraser 1966; Bell *et al.* 1975) so far incorporated in models. This translates into a 50 K difference in effective temperature for Arcturus. For instance, the 4375 K found by Frisk *et al.* (1981) slides up to 4425 K using the more recent set. The discrepancy in calculated opacities is largest at 1–2 μm, where convective flux contributes appreciably to the continuum radiation, thus preventing observation resolution of the problem. More recent calculations (Wishart 1979a, b; John 1979) have not yet been incorporated into stellar models, but John's absorption coefficients fall between those of Doughty & Fraser (1976) and Bell *et al.* (1975), though closer to the former, suggesting, e.g. $T \sim 4390$ K.

In addition, Frisk *et al.* find that fitting the ultraviolet continuum data requires a 'fudge factor' increasing κ by about 10 per cent over that due to H^- and standard line-blocking. The most successful fudge factor varies with wavelength in proportion to the number of weak lines. Models that do not include something of the sort yield lower effective temperatures when matched to continuum fluxes. This is at least part of the reason for the low temperature found by Mäckle *et al.* (1975) according to Gustafsson.

Kurucz believes that the need for extra UV opacity merely reflects inadequate treatment of weak lines of Fe II and similar ions. Thus when *f* values and such have been calculated for all relevant lines and the lines included individually in model atmosphere programs, there should be no need for this fudge factor (or even line-blocking coefficients). We hope to live long enough to see this come to pass!

(b) *Sphericity*. No star is really stratified in plane parallel layers. This matters only when we see photons coming from a range of depth not negligible compared to the radius of the star, i.e. for stars of small surface gravity. Far infrared fluxes are particularly affected because total opacity is low there. As a result, the $V-K$ colour of cool giants is changed by some 0·4 mag, and even $R-I$ shifted by 0·2 mag, in the direction of making the stars, if analysed with a plane parallel model, look cooler than they really are, because you see less surface area at the longer wavelength (Watanabe & Kodaira 1979). R.Wehrse (Heidelberg) estimates that this effect makes the effective temperature of Arcturus uncertain, because undefined, by about 90 K. This may be at least part of the answer to Rabin's worry about the difference between infrared colour temperatures and other measures of T_e for Arcturus.

(c) *Non-LTE.* When local thermodynamic equilibrium breaks down, the Saha equation usually goes first (and the Maxwell velocity distribution last). This occurs in the cool upper atmospheres of intermediate temperature giants where appreciable ionization is contributed by UV photons leaking out from hot layers below (see Ruland *et al.* 1980 on Pollux). Arcturus is cool enough that this should be unimportant. But in fact, Spite finds that abundances of Fe, etc. found from lines of different excitation potential show just the systematic discrepancies that would be caused by this deviation from LTE in Arcturus. Other spectral features, like the oxygen triplet and lithium lines, do show non-LTE effects in cool giants, according to Gustafsson. M.Edmunds (Cardiff) suggests that Arcturus may act like a somewhat hotter star because the low metal abundance allows UV photons to stray farther from where they belong than would otherwise be the case. A proper calculation of these effects requires collisional ionization cross-sections which are not now adequately known. Neglect of non-LTE effects is, according to Gustafsson, the probable cause of the high temperatures found by Lambert & Ries (1981) for Arcturus and about 30 other stars. Non-LTE effects in the Sun are one of the sources of uncertainty in solar *f* values.

(d) *Convection and microturbulence.* In one sense, convection is not a major problem for stars like Arcturus: both temperature and gravity found from model atmospheres are quite insensitive to choice of mixing length. But this is apparently not the whole story. Many models agree that as you go from dwarfs to giants, and from low to high layers in the atmosphere of a cool star, the fraction of flux carried by convection drops. Nevertheless, giants show higher microturbulence parameters than dwarfs (2–3 km s^{-1} *v.* 0·5–1 km s^{-1}); and Arcturus, according to Sikorski (1976) has microturbulence inversely proportional to optical depth. These patterns presumably reflect the fact that it takes a large velocity to move even a modest amount of flux at the low gas densities found in giant atmospheres.

For the Sun, at least, measured convective cell speeds are only slightly smaller than the spectroscopically determined microturbulence. And calculations by Dravins, Lindegren & Nordlund (1981) suggest that, when convective overshoot is included, large-scale velocity fields may directly provide the line broadening we see. Another possibility is line broadening by acoustic waves generated in convection zones (Edmunds 1978). For either mechanism, even small amounts of convective flux can imply large broadening velocities in giant atmospheres.

In addition, convection itself may be important if overshoot takes place and materially flattens the real atmospheric temperature gradient from values calculated ignoring the effect. This can happen over the whole surface of the star or in patches. The situation will be still more complex if convective velocities are anisotropic as, apparently, in the Sun.

Microturbulence, viewed as an adjustable parameter, ξ, is by no means unimportant for Arcturus or stellar spectroscopy in general. Solar microturbulence is, according to Blackwell, the dominant source of error in calibrating laboratory *f* values against the Sun (or conversely). For Arcturus itself, the published range of ξ (1·5–2·2 km s^{-1} since 1970) introduces a 0·4 dex uncertainty into surface gravities found from ionization equilibrium, according to Gustafsson. The situation is worst when there is a microturbulence gradient in the atmosphere and lines of different ions are formed at different levels. Finally, improper choice of ξ can make abundances seem to be functions of line strength and excitation potential. Improper choices of temperature or temperature gradient, *f*-values, and damping constants have similar effects, and it is not always obvious, according to N.Kovács (Berlin) which parameter needs changing.

Some observational check of the importance of convection in Arcturus is possible. A few large convective cells rising and falling at ~ 2 km s^{-1} should introduce jitter into the measured radial velocity at a detectable 0·1–0·2 km s^{-1} level. Arcturus shows no such jitter (Ruland *et al.* 1980), though some other stars with still more extended atmospheres may, according to R.F.Griffin (Cambridge). This means either that convection carries very little flux or that there are many cells on the surface at a given instant. The latter is perhaps more likely. Even with many cells, convective velocity should show up via a relationship between radial velocities of individual lines and their ionization and excitation potentials, since rising cells will be hotter than falling ones (Trimble 1974; Dravins *et al.* 1981). Again the effect is not seen at the 0·1 km s^{-1} level, according to R.F.Griffin. The catch is that, since most lines are really blends, and we can measure stellar wavelengths to higher precision than that of most published laboratory values, solar wavelengths have to be used in measuring stellar radial velocities. Thus, the existence of convective effects in the Sun will conceal them in other stars. A spectrum synthesis approach, given sufficiently accurate laboratory wavelengths, could circumvent this problem.

Clearly convection and turbulence do matter in the analysis of stars like Arcturus, at the same level as several other uncertainties ($\pm$50 K in T_e, 0·4 dex in *g*, etc.), but given the present state of theory, it is not at all obvious what to do about them.

(e) $T(\tau)$. The single most important difference among published model atmospheres and between the models and real stars nominally of the same *T*, *g*, and composition is in their temperature distributions as a function of continuum optical depth, $T(\tau)$. Causes, in addition to those discussed above, include neglect or improper treatment of molecular cooling, line-blocking, deviations from hydrostatic equilibrium and spherical symmetry (star spots), and effects of rotation and magnetic fields. Among the results are that the effective temperature of the best purely theoretical (flux constant) models can be wrong by 100–200 K even for the Sun (according to Holweger), and that the empirical ones are not necessarily any better, except for the Sun (according to Gustafsson). It is particularly important not to mix the two kinds in a single analysis (e.g. use one sort for the Sun and the other for the star of interest in a differential abundance determination), as the inherent errors are different.

In the case of Arcturus, we have already seen that spectroscopic values of T, g and ξ are quite sensitive to choice of model atmosphere. In addition, several points suggest that none of the current models is entirely adequate. These include (a) The best fit to continuum fluxes is achieved at a higher temperature for visible and near infrared data than for far infrared data (R.E.M.Griffin), (b) Atomic ionization and molecular dissociation equilibrium calculations do not readily yield the same surface gravity at a reasonable temperature, although the large error bars overlap slightly (Bell), and (c) Abundances derived from individual line intensities vary with line strength and excitation potential in ways that cannot entirely be blamed on poor laboratory data or wrong choices of T, ξ, etc. (Kovács, Holweger).

What should be done about this? The most far-sighted view was expressed by Kurucz, who believes that, once all the right physics is included in atmosphere programs, the models will automatically come out right, and we will know it, because they will provide perfect matches to high-precision data. In the meantime, one can play relatively safe by using some more approximate model that matches fluxes over a limited wavelength range for analysis of lines only in that range and by exploring as carefully as possible the model dependence of derived parameters.

6 ABUNDANCES – EFFECTS AND DETERMINATIONS

Chaotic as the model atmosphere situation may seem, the calculations are nevertheless good enough that assuming all heavy elements to be deficient by the same amount is not appropriate. Inevitably, the abundances chosen (e.g. for CNO, Ne-Si, iron group, etc.) modify the atmospheric structure (largely via opacities) enough to affect abundance determinations themselves. In principle, therefore, the calculations should be done self-consistently. This is too complex for current programs. The alternative is iteration. Frisk *et al.* (1981) used slight modifications of the abundances found by Mäckle *et al.* (1975) and Lambert & Ries (1977) in constructing their detailed Arcturus models. They checked that the new model yielded the assumed abundance only for calcium. Further work on this might be interesting, as the models differ noticeably from the uniform [M/H] set, especially in $P_e(\tau)$.

The most unconstrained abundance of all is helium. O'Brien & Lambert (1979) have reported detection of the λ 10830 line, but it is variable on a one-week time scale and not useful for measuring He/H. Not knowing the helium abundance introduces a modest additional uncertainty into model atmospheres via mean molecular weight and opacities. The effect, as summarized by Y.Chmielewski (Geneva) is that an increase in He abundance mimics a decrease in g. In principle, Arcturus might have He/H either lower than solar (because it is old and metal-poor) or higher than solar (because it is evolved and partially mixed). The resulting change in log g determination has not been explored quantitatively.

Recent measurements of average [M/H] in Arcturus are remarkably free of the scatter seen in T and g. [Fe/H] $= -0{\cdot}7 \pm 0{\cdot}2$ covers every post-*Atlas* value of which we are aware. Gustafsson suggest that the dominant effect here may be that of the bandwagon, given the relatively high sensitivity of abundances to choice of spectroscopic parameters; ∂ [Fe/H]/∂ log g = 0·4, holding everything else constant, for example, according to Bell. There is also general agreement that Mg and Si are rather less, and the heaviest elements rather more, deficient than the iron group. A similar trend is found, at a lower level, in Pollux (Ruland *et al.* 1980) and R.E.M.Griffin suggests that it may be characteristic of giants. Alternatively, such a trend has also been identified in some moderately metal deficient field dwarfs (Peterson 1976, 1981) and may be telling us about when different elements were synthesized in the galaxy. Or, if you don't want to believe any of the trends, Holweger has pointed out that they can be generated artificially by errors in f, ξ, damping constants, etc. that vary systematically with ionization and excitation potential. We prefer not to vote on this one.

Isotope ratios, apart from CNO, have been published only for Mg. Tomkin & Lambert (1976) reported Mg^{26}/M^{24} as solar to within the errors and Mg^{25}/Mg^{24} as approximately half the solar value.

Elemental and isotopic abundances in the CNO group are particularly interesting probes of nuclear reactions, mixing, and mass loss in evolved stars, because they are strongly modified by CNO cycle hydrogen shell burning during post-main-sequence phases. Arcturus is a bit difficult to understand, even with this assortment of processes. The C^{13} abundance is about as high as has been found for any giant, $C^{12}/C^{13} = 7$ (Lambert & Ries 1977). If this is due to CNO cycle processing, one would expect an associated depletion of C. But B.Barbuy (Meudon) finds that, by the time allowance is made for uncertainties T and ξ, even the sign of [C/Fe] cannot be confidently determined. She used lines that have the advantage of being insensitive to surface gravity, but the disadvantage of being rather strong. The abundances published by Mäckle *et al.* (1975) and Lambert and Ries concur that carbon is deficient by about the same amount as iron ([C/H] = −0·70 and −0·62 respectively). But they disagree on the sign of [N/C], −0·20 *v.* +0·34, and on the oxygen abundances as well ([O/H] = −0·60 *v.* −0·27), at least partially because of their different choices of surface gravities.

The combination of high C^{13} and low-to-normal N^{14} is puzzling if the C^{13} is to be attributed to CNO cycle processing. One way out, suggested by Edmunds, may be to assume that initially the star had very little nitrogen (a secondary nucleus), so that it has, in fact, been enhanced along with C^{13}.

7 THE SPECTROSCOPIC MASS OF ARCTURUS

Table I gives the mass implied by assorted values of log g (vertical axis) and angular diameter (horizontal axis) over the ranges permitted by the uncertainties discussed in previous sections. A distance of 11·1 pc was assumed. The probable error of the parallax introduces only another 5 per cent uncertainty in mass. Pick your favourite value before continuing to the next section!

8 EVOLUTIONARY CONSIDERATIONS

There is no inherent impossibility in the combination of K2III atmospheric structure and any of the masses that have been suggested for Arcturus. On the one hand, Popper's (1980) list of well-determined masses in semidetached, spectroscopic, eclipsing binary systems shows seven late type giants with masses between 0·2 and 0·4 $M_\odot$; and the average for all 17 evolved components of such systems is 0·7 $M_\odot$. On the other hand, R.F. Griffin has called attention to three non-eclipsing, double-lined, detached systems, each consisting of two giants with essentially identical spectral type, for which the minimum values of $(M_1+M_2) \sin^3 i$ are $4{\cdot}9 \pm 0{\cdot}2$ $M_\odot$ (HD 44780, K2III), $4{\cdot}7 \pm 0{\cdot}2$ $M_\odot$ (6 Gem, K2III), and $6{\cdot}38 \pm 0{\cdot}13$ $M_\odot$ (HR 2081, G5III). Neither set of examples is directly relevant to Arcturus, the former because extensive mass transfer between the binary components has undoubtedly occurred; the latter because the stars belong (on evidence of kinematics and metallicity) to a young disc population. Arcturus, with [Fe/H] $\sim -0{\cdot}7$ and $U, V, W = -25, -115, -3$ km s^{-1} (Woolley *et al.* 1970), clearly does not.

TABLE I

Possible masses for Arcturus implied by admissible values of surface gravity (log g, *vertical axis) and angular diameter (θ arcsec, horizontal axis) in solar masses*

log g (cm/sec^2)	θ (arcsec) 0·018	0·021	0·024	0·027	0·030
0·5	0·053	0·073	0·095	0·12	0·15
1·0	0·17	0·23	0·30	0·38	0·47
1·5	0·53	0·72	0·95	1·2	1·5
2·0	1·7	2·3	3·0	3·8	4·7
2·5	5·3	7·2	9·5	12·0	14·8

Gamma Leo could be a closer analogue. This mildly-metal deficient, high velocity visual binary yields masses of $\sim 0{\cdot}3$ $M_\odot$ for both G7III and K0III components (Wilson 1967). Unfortunately, although the wide separation ($\sim$ 60 AU) makes mass transfer between the components unlikely, it also makes the orbit rather uncertain. There are no K giants in detached systems that have first-class orbit determinations by any technique (Popper 1980).

Evolutionary tracks *per se* (the modern equivalent of the mass-luminosity relation!) provide very little information for several reasons, discussed at some length by Mäckle *et al.* (1975). First, the post-main-sequence tracks for a wide range of initial masses tend to converge in the upper right hand corner of the HR diagram. Second, there are no published tracks for 0·5–2·0 $M_\odot$ stars with significant mass loss, except approximate ones intended for analysing mass-exchange binaries, and none for such stars with low [M/H]. Third, the tracks usually used will be somewhat modified (especially in colour) by inclusion of molecular opacities and realistic boundary conditions (Cheung 1980; Whitaker 1981). And fourth, we do not know whether we should be trying to match Arcturus to the giant branch, the clump (field-star analogue of the horizontal branch in globular clusters), or the asymptotic giant branch phase of an evolutionary track.

Arcturus' high C^{13} abundance slightly favours the asymptotic giant phase. On the giant branch, mixing, as indicated by carbon depletion, may set in as faint as $M_v = +3$ (Carbon *et al.* 1981; Kraft & Oke 1981, on M92, a metal-poor globular cluster), which would be relevant to Arcturus, or perhaps not until about $M_v = -0{\cdot}7$ (Bell, Dickens & Gustafsson 1979, on the same cluster), which would not be relevant to Arcturus. In either case, mixing results in C^{12}/C^{13} as extreme as seven during hydrogen shell burning only in the lowest-Z clusters. Later phases are likely to bring more C^{13} to the surface. This is normally accompanied by large amounts of N^{14}, not seen in our star. Since $C^{12}\,(p, \gamma)\,N^{13}\,(\beta^-)\,C^{13}$ is the first step in the CNO cycle and goes at the lowest temperature, incomplete processing at the top of the H-burning shell, accompanied by very carefully placed mixing, could perhaps bring up only C^{13}. No models that actually accomplish this have been published, and Lambert & Ries (1977) regard the CNO chemistry of Arcturus as a mystery.

Lambert *et al.* (1980) reported a very low lithium abundance ($[\mathrm{Li/H}] \lesssim -1{\cdot}5$) and regard that, together with the high C^{13}/C^{12} ratio as a signature of a star with initial mass $\lesssim 1{\cdot}3\ M_\odot$, neither very surprising nor very informative.

Evolutionary considerations in a broader sense can constrain the mass slightly. In metallicity, Arcturus is on the ragged edge between the highest-Z (youngest?) globular clusters and the lowest-Z, oldest open clusters, assuming the traditional abundance scales (see, e.g. Demarque 1980; but Cohen 1980 and Pilachowski *et al.* 1980 report much lower metallicities for some of the highest-Z globular clusters). Other things being equal, this would imply that Arcturus must have started with whatever mass takes about the age of the galactic disc (8–15×10^9 yr?) to leave the main sequence. Thus the present mass should be $0{\cdot}85$–$1{\cdot}1\ M_\odot$ minus whatever has been lost along the way.

We should not be too firm about this, however, as the large, negative V velocity of Arcturus means that it could have been formed quite far out in the galactic disc, where the metal abundance has probably always been lower than that in the solar neighbourhood at the same time. This permits a larger initial mass ($\sim 1{\cdot}5\ M_\odot$?), again modified by mass loss over the life of the star.

How much ought to be allowed for mass loss? Linsky *et al.* (1974) reported a probable detection of $5 \times 10^{-9}\ M_\odot$/yr from Arcturus, later modified by Chiu *et al.* (1977) to an upper limit of $10^{-8}\ M_\odot$/yr. Integrated mass loss at this rate will be at most $0{\cdot}2\ M_\odot$ prior to helium ignition for a $1\ M_\odot$ star (Ayres & Johnson 1977), but could be important if Arcturus is an asymptotic giant and mass loss has continued over its entire post-MS life. Large total mass loss would make the C^{13} abundance easier to understand, in the sense that it takes less mixing to pollute a thin atmosphere outside the hydrogen-burning shell than a thick one. Notice that if only incompletely-processed CNO cycle material has reached the surface, helium should have been enhanced by less than 1 per cent over its initial abundance.

Still less firm constraints come from looking at the last phases of stellar evolution. On the one hand, if Arcturus is an asymptotic giant, whose history is similar to that of the stars that make the $0{\cdot}5\ M_\odot$ RR Lyrae variables in globular clusters, then its present mass might well be $0{\cdot}4\ M_\odot$ or less. If, on the other hand, it is more like the average sort of field star that, after losing a planetary nebula, gives birth to a $0{\cdot}7\ M_\odot$ degenerate dwarf, then a present mass of $0{\cdot}8\ M_\odot$ or more is likely. Our best guess is $0{\cdot}9\ M_\odot$ on the main sequence, 0·75 now, and eventually a $0{\cdot}6\ M_\odot$ degenerate dwarf. A great many other scenarios present difficulties not significantly worse, or better, than this one.

9 RECOMMENDATIONS

Given the web of interconnected uncertainties and errors presented here, one is initially tempted to give up on stellar atmospheres completely and turn to something simple like cosmology. But this is a snare and a delusion, as the more esoteric branches of astrophysics have been built up on the assumption that we understand things like stellar masses and evolution. Thus, if the foundation is shaky, so is the superstructure. The workshop participants, therefore, gave some thought to what should be done next to shore up the underpinnings. Our recommendations divide naturally into three classes: good advice, things that someone (else) should observe, and things that someone (else) ought to calculate.

(a) *Good advice*. Authors ought to make it as easy as possible for readers to evaluate and use their results. Thus publishers of spectra should include the instrumental profile; and publishers of spectral analyses should include discussions of errors and uncertainties that are as careful and honest as possible. In particular, where many lines are used and yield a scatter in the derived parameter (T, abundance, or whatever), the possibility should be addressed that the scatter is systematic rather than statistical and that the answer may be no better known than the full spread of points.

(b) *Useful observations*. The effective temperature problem could quite possibly be resolved by absolute continuum flux measurements, accurate to 1 per cent, over the 0·4–3·0 μm range. This may be impossible, owing to the uncertain fluxes of laboratory standards at the shorter wavelengths and assorted noise problems at the longer ones. Could synchrotron UV eventually be used as the laboratory comparison source? A step in the right direction might be one or more standard stars, observed as carefully as Vega, but cooler. Pollux and Aldebaran were suggested.

Additional light could perhaps be shed on non-LTE effects by extending to cooler stars the work of Holweger and Kovács, which examines abundances implied by many individual lines as a function of line strength, ionization, and excitation potential (i.e. depth of formation in atmosphere).

The nature and effects of convection and turbulence might be better defined by measuring high-dispersion profiles of moderately strong lines. These should be repeated at least once a month (to probe changes correlated with stellar rotation and growth and decay of individual cells). At the moment all that can be said is that the profiles do change (for a variety of reasons) and may be asymmetric in the opposite sense from what one would have expected (Gray 1980).

(c) *Useful calculations*. Current knowledge of physics would permit better calculations of H^- opacities and of the effects of deviations from plane parallel atmospheric structure than have thus far been published.

Several new methods of determining log g deserve further investigation, particularly of their sensitivity to differences among model atmospheres. These include molecular dissociation equilibrium, blue and UV colours as an index of the switch from H^- to HI opacity, the profiles of strong and weak lines arising from the same lower level, and atomic ionization equilibrium determined from high excitation lines of the neutral species.

Finally, if several K giants with known binary masses (or lower limits) could be analysed spectroscopically with the techniques used on Arcturus, the results would be exceedingly interesting. The chief difficulty is de-blending the lines from the two stars, which characteristically have very similar spectral types but rather different luminosities.

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