Title
Ultra Sensitive Laser Induced Fluorescence Sensor (SEN 4)

Authors
Leyla Sabet
Arnaud Benahmed
Chih-Ming Ho

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Optimal Power Allocation in Distributed Sensing

Gautam Thatte and Urbashi Mitra
Communication Sciences Institute, USC

Problem Description: Minimize BLUE MSE subject to a total network power constraint

Introduction to Parameter Estimation

In this work, we consider the problem of optimal power allocation for parameter estimation and detection in a distributed sensor network setting. For the simple star topology, an analysis of the effect of the measurement noise variance on the optimal power allocation policy is presented. Relaying nodes are introduced to form more complicated branch, tree and linear topologies (depicted in Figure 1). Analytical solutions for these cases for both amplify-and-forward (AF) and estimate-and-forward (EF) transmission protocols are intractable, and thus asymptotically optimal (for increasing measurement noise variance) solutions are derived.

- **Simple signal model for the star topology**
  The received signal at the fusion center from the i^{th} sensing node is:
  $$y_i = \sqrt{P_i} h_i (\theta + z_i) + n_i$$
  - θ is the deterministic ***scalar*** parameter to be estimated
  - z_i and n_i are zero-mean and unknown PDF noise terms, independent
  - h_i is non-random channel attenuation factor known at fusion center (FC)
  - P_i is power gain factor, $0 \leq P_i \leq 0, \quad \forall i$
- **The best linear unbiased estimator (BLUE) is optimal**
  Since the measurement and channel noise terms are only defined using second-order statistics, the best linear unbiased estimate is the optimal ***linear*** estimator.
- **Constrained optimization problem is considered**
  The generic optimization problem, for any topology, is
  $$\text{minimize MSE subject to} \quad \sum_{i=1}^{N} P_i = P_T$$

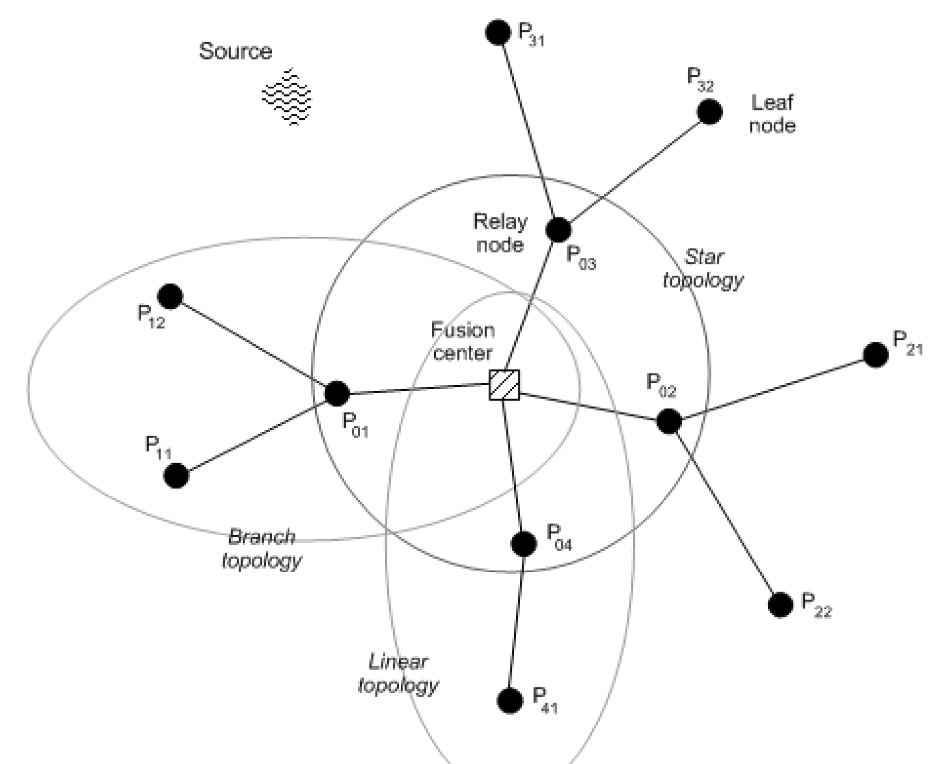

Figure 1: Different generic topologies considered for distributed parameter estimation: linear, star and tree topologies.

Extending optimizations to complex topologies

- **Introduce relay transmission protocols**
  Consider the simple two-hop linear network shown in Figure 1.
  - Using amplify-and-forward (AF), the FC receives:
    $$y_{FC,AF} = \sqrt{P_0} h_0 \left[\sqrt{P_1} h_1 (\theta + z) + n_1\right] + n_0$$
  - For the estimate-and-forward (EF) protocol, the signal model is:
    $$y_{FC,EF} = \sqrt{P_0} h_0 \hat{\theta}_R + n_0$$
    where $\hat{\theta}_R = \theta + w$ is the BLU estimate formed at the relay.

Proposed Solution: Optimal solution evolves from sensor selection to power equalization

Analysis of the Star topology

- **The MSE for the star topology, and optimal solution**
  $$\text{MSE} = \left(\sum_{i=1}^{N} \frac{1}{\sigma_{z_i}^2 + \frac{1}{r_i P_i}} \right)^{-1}, \quad P_i^\star = \frac{1}{r_i \sigma_{z_i}^2} \left(\sqrt{\frac{r_i}{\nu^\star}} - 1 \right)^+$$
  where $r_i = |h_i|^2 / \sigma_{n_i}^2$ is the channel SNR for the i^{th} node.

  The solution is obtained using Lagrangian optimization and KKT conditions, and $\nu^\star$ is the optimal Lagrange multiplier for the equality constraint
- **Optimal power allocation strategy evolves from a waterfilling solution to power equalization as measurement noise increases**
  The evolution of the optimal solution is shown as a function of the measurement noise variance in Figure 2. The no measurement noise case is an example of extreme waterfilling; only the sensor with the strongest SNR is active, and ***sensor selection*** is optimal for $\sigma_z^2 = 0$.

  As the measurement noise variance increases, the sensors with weaker channel SNRs become active, and a ***waterfilling*** solution is optimal.

  The asymptotic solution, for high measurement noise, is ***power equalization***. All sensors are active, and the sensor with the weakest channel SNR is allocated the greatest fraction of the total power.

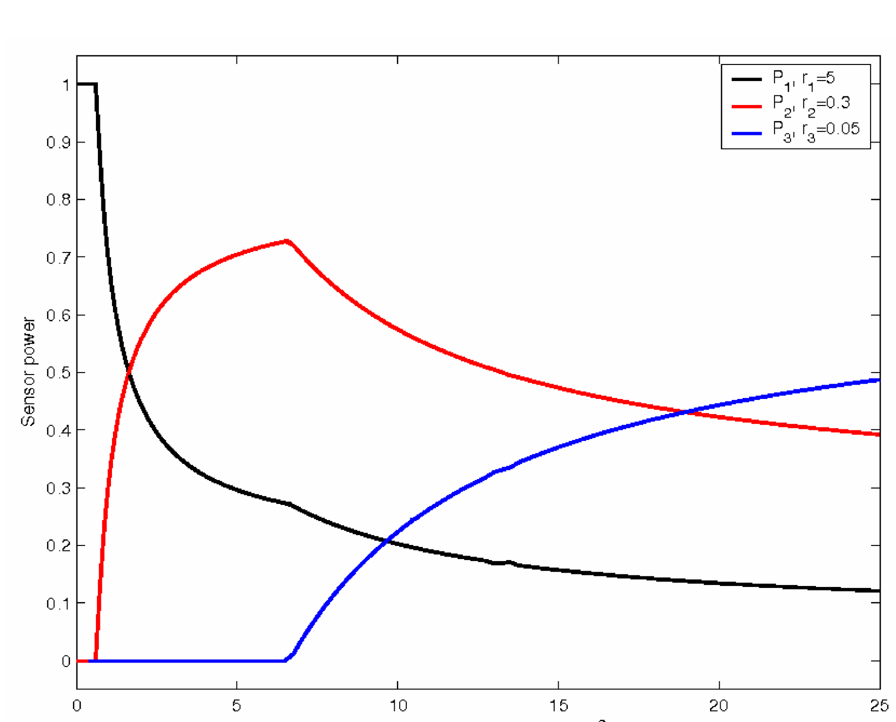

Figure 2: Evolution of optimal power allocation scheme, for N=3 sensors in the star topology, for increasing measurement noise.

Asymptotically optimal solutions

- **Solutions for branch, tree and linear networks are intractable**
  - Use solution techniques and results from star topology to develop the asymptotically optimal (for increasing measurement noise variance) solutions to these more complex topologies.
  - For low measurement noise in tree topologies, branch selection is optimal; and in the case of linear network, sensors further away from the fusion center remain inactive.
  - As the measurement noise increases, all sensors become active. Power equalization is optimal for the leaves of a branch topology; for a linear network, weighted power equalization is optimal.
- **Topology comparisons**

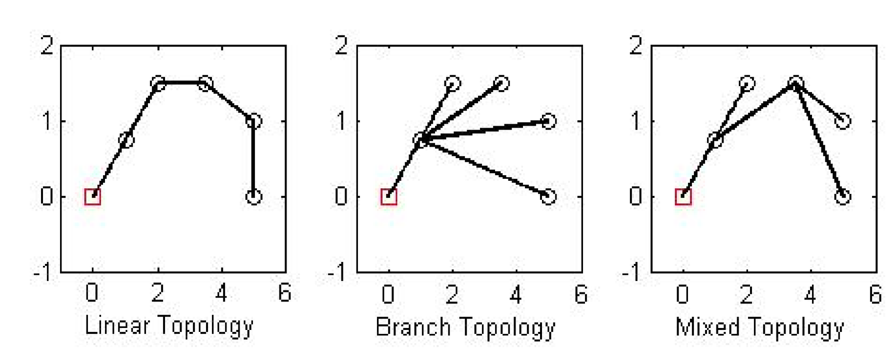

Figure 3: Three topologies for a fixed location of nodes for optimal power allocation and sensor selection.

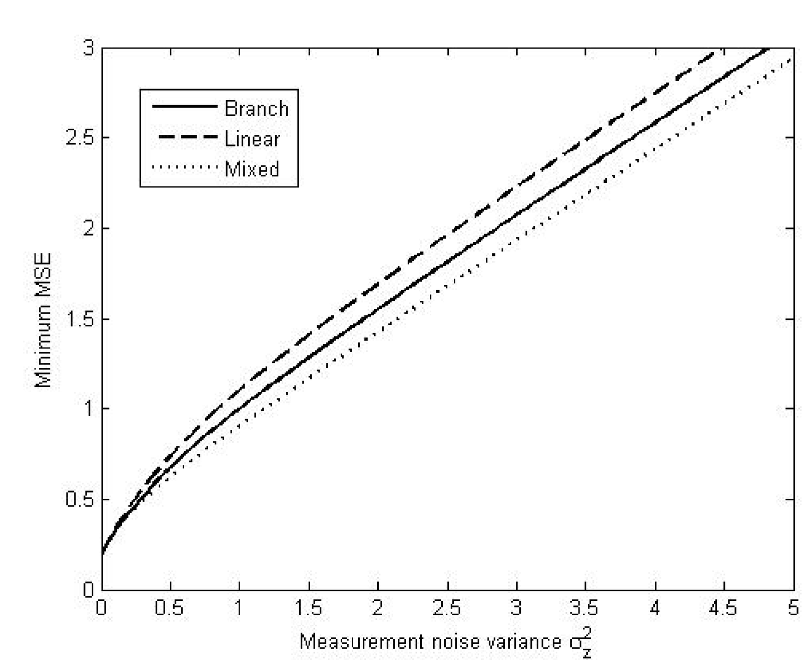

Figure 4: MMSE achieved from optimal power allocation and sensor selection for the three topologies of Figure 3 as a function of the measurement noise.

– Figure 4 illustrates that more branching with shorter hops is preferred to linear topologies which use longer direct hops.