LBL--13880

DE82 007750

THE SOGGY SADDLE THEORY OF FISSION

L.G. Moretto and G. Guarino

January 1982

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics and by the Nuclear Sciences of the Basic Energy Science Program of the U.S. Department of Energy under Contract W-7405-ENG-48.

The soggy saddle theory of fission

L.G. Moretto and G. Guarino

Abstract: The transition state theory of fission is generalized to allow for trajectories that return from saddle to compound nucleus.

The standard Bohr Wheeler (BW) theory of fission decay[1], identical with the transition state theory for chemical reactions, is subject to serious limitations of both quantal and classical nature. We want to consider here the most crucial approximation of the theory, its possible failure, and a generalization designed to overcome part of the difficulty. The basis of the BW theory is to calculate the flux of the density distribution in phase space across a suitably chosen hypersurface normal to the reaction coordinate. This flux is then identified with the reaction rate; this is both the beauty and the trap of the theory. The flux and the reaction rate can be identified if and only if no phase-space trajectory, after crossing the hypersurface, comes back and crosses it again returning to the reactant's region. In order to eliminate, or at least to alleviate, the problem, the "transition state", or the position of the hypersurface, is chosen to cut across the saddle point in coordinate space, on the hope that, once the saddle point is negotiated, the system irreversibly rolls down towards the product region. This is certainly an extreme approximation, requiring a substantial decoupling (low viscosity) between collective and internal degrees of freedom.

An alternative approximation is to assume high viscosity in the general saddle point neighborhood. As a result, the flux from the compound nucleus is trapped in the saddle region and the associated randomization leads to a backflow towards the compound nucleus. For this case the natural way to handle the problem is the use of the Master Equation.

Let us consider a compound nucleus A, a saddle point region B, a region C far down the scission valley, and a nucleus D after one neutron emission. The transition probabilities are λ_1 (from A to B), λ_2 (from B to A), λ_3 (from B to C), λ_n (from A to D), $\lambda_{n'}$ (from B to D) and are illustrated in fig. 1a.

The master equations are

$$\dot{\varphi}_A = \varphi_B\lambda_2 - \varphi_A(\lambda_1 + \lambda_n); \qquad \dot{\varphi}_B = \varphi_A\lambda_1 - \varphi_B(\lambda_1 + \lambda_3 + \lambda_{n'});$$

$$\dot{\varphi}_C = \varphi_B\lambda_3 \qquad ; \qquad \dot{\varphi}_D = \varphi_A\lambda_n + \varphi_B\lambda_{n'}$$

where the φs are the time dependent populations. Two main differences with respect to the standard theory are visible: a) there is a backflow from B to A that makes the decay of A nonexponential (notice that by setting $\lambda_2 = 0$ we recover the BW expression); b) neutrons are allowed to be emitted from the saddle region.

The system of differential equations can be solved in a straightforward way. From the populations at time infinity one can obtain the following expressions for Γ_N/Γ_F

$$\Gamma_N/\Gamma_F = \frac{\varphi_D(\infty)}{\varphi_C(\infty)} = \frac{\lambda_n}{\lambda_1} + \frac{\lambda_1\lambda_{n'}+\lambda_2\lambda_n+\lambda_n\lambda_{n'}}{\lambda_1\lambda_3}$$

The first term to the right is the standard result. The above expressions can be obtained without solving the differential equation by summing over the probability tree (fig. 1b).

One can try to assign values to the λs by using detailed balance. One obtains

$$\lambda_1 = \frac{T}{h}\,\frac{\rho_B(E-B_F)}{\rho_A(E)} \cong \frac{T}{h}\,e^{-B_F/T} \quad ;\lambda_n \cong \frac{T}{h}\,e^{-B_N/T}$$

$$\lambda_{n'} \cong \frac{T}{h}\,e^{-(B_N+B_F)/T} \quad ;\lambda_2 \cong \frac{T}{h} \cong \lambda_3$$

Thus

$$\Gamma_N/\Gamma_F = 2e^{(B_F-B_N)/T} + e^{-(B_F+B_N)/T} + e^{-2B_N/T}$$

The assumption $\lambda_2=\lambda_3$ has been used although it is doubtful. Experimental data should verify it.

The new expressions favor neutron decay in two ways, a) by allowing neutron decay from the saddle; b) more important, by feeding back the flux from the saddle region to the compound nucleus.

An intermediate situation can be envisaged as follows. For a given viscosity at the saddle, there will be a critical velocity along the fission coordinate, above which the system escapes altogether towards fission and below which the system gets trapped in the saddle region. The treatment can be modified by splitting λ_1 as follows:

$$\lambda_1 = \frac{1}{2\pi\rho(E)}\left[\int_0^{\varepsilon_0}\rho(E-B_F-\varepsilon)\,d\varepsilon + \int_{\varepsilon_0}^{\infty}\rho(E-B_F-\varepsilon)\,d\varepsilon\right]$$

$$= \frac{T}{2\pi\rho(E)}\left\{\rho(E-B_F)\left[1-e^{\varepsilon_0/T}\right] + \rho(E-B_F)e^{-\varepsilon_0/T}\right\} = \lambda_{1S}+\lambda_{1F}$$

The first term corresponds to saddle trapping and the second to complete saddle negotiation (figs. 1c, 1d). The general result is

$$\Gamma_F/\Gamma_N = \frac{\lambda_{1F}(\lambda_2+\lambda_3+\lambda_{n'}) + \lambda_{1S}\lambda_3}{\lambda_n(\lambda_2+\lambda_3+\lambda_{n'}) + \lambda_{1S}\lambda_{n'}}$$

Again it is reasonable, although not necessary, that, for the systems trapped in the saddle region $\lambda_2 = \lambda_3$. If one disregards the contribution of the neutron decay from the saddle region and obtains the simple form:

$$\Gamma_F/\Gamma_N = \frac{\lambda_{1F}}{\lambda_n} + \frac{1}{2}\frac{\lambda_{1S}}{\lambda_n} = \frac{1}{2}\left(e^{-(B_F-B_N)/T} + e^{-(B_F+\varepsilon_0-B_N)/T}\right)$$

Interesting effects can be observed by introducing a temperature dependence of ε_0, which is equivalent to assuming a temperature dependent friction coefficient. Results for the BW case and for various temperature dependences of ε_0 are shown in fig. 2.

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics and by Nuclear Sciences of the Basic Energy Sciences Program of the U.S. Department of Energy under Contract W-7405-ENG-48.

Reference: 1. N. Bohr and J.A. Wheeler, Phys. Rev. 56, 426 (1939)

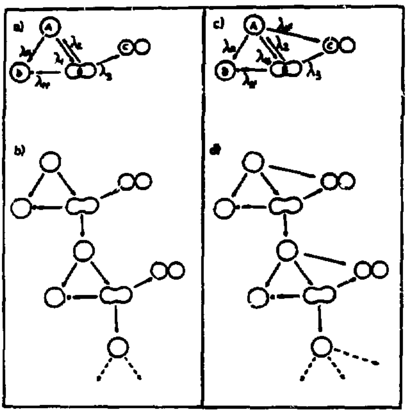

Fig. 1

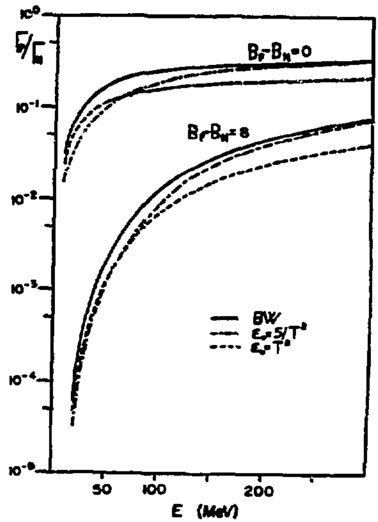

Fig. 2