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ACCELERATOR AND ION BEAM TRADEOFFS FOR STUDIES OF WARM DENSE MATTER*

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Abstract

One approach for heating a target to "Warm Dense Matter" conditions (similar, for example, to the interiors of giant planets or certain stages in inertial confinement fusion targets), is to use intense ion beams as the heating source (see refs.[6] and [7] and references therein for motivation and accelerator concepts). By consideration of ion beam phase-space constraints, both at the injector, and at the final focus, and consideration of simple equations of state and relations for ion stopping, approximate conditions at the target foil may be calculated. Thus, target temperature and pressure may be calculated as a function of ion mass, ion energy, pulse duration, velocity tilt, and other accelerator parameters. We connect some of these basic parameters to help search the extensive parameter space (including ion mass, ion energy, total charge in beam pulse, beam emittance, target thickness and density.

ION STOPPING

We first examine dE/dX, where E is the ion energy and $X \equiv \int \rho \, dz$ is the integrated range of the ion (cf, ref. [1]).

For heating solid aluminum (at room temperature) over a range of ion mass from 4 amu (helium) to 126 amu (iodine), the energy loss at the peak of the dE/dX curve (dE/dX_{max}) may be parameterized approximately as:

$$(1/Z^2) dE/dX_{max} \approx 1.09 \text{ (MeVcm}^2\text{/mg) } A^{-0.82} \qquad (1)$$

where Z and A are the ion nuclear charge and atomic mass, respectively. Expressing $dE/dX_{\rm max}$ as a function of A yields

$$dE/dX_{max} \approx 0.35 \text{ (MeVcm}^2\text{/mg) } A^{1.07}. \qquad (2)$$

Thus, the peak energy loss rate increases (nearly linearly) with ion atomic mass.

Similarly, the ion energy E at the peak increases with ion mass nearly quadratically with A according to

$$E \text{ (at } dE/dX_{max}) \approx 0.052 \text{ MeV } A^{1.803}. \qquad (3)$$

Target uniformity is another important consideration. In ref. [2] it was pointed out that target temperature uniformity can be maximized in simple planar targets if the particle energy reaches the maximum in the energy loss rate dE/dX when the particle has reached the center of the foil (see Figure 1). For any specified fractional deviation in target temperature (assuming the energy is deposited in a time short so that no hydrodynamic, radiative, or other cooling has occurred) one can determine the energy at which the ion must enter and exit the foil. From the dE/dX curves of ref. [1] we find that for the entrance energy to have less than a 5% lower energy loss rate relative to the peak in dE/dX, $\Delta E/E \approx 1.0$, where ΔE is the difference in ion energy between entering and exiting the foil, and E is the energy at which dE/dX is maximum. The spatial width of the foil Z, for a 5% temperature non-uniformity is then given by:

$$Z = \Delta E/(\rho \, dE/dX) \approx 0.77\mu \;\; A^{0.733}(\rho_{al}/\rho) \qquad (4)$$

Here we have used $\rho_{\rm al}$=2.7 g/cm^3 to convert the range into a physical distance. So by using materials of low density such as metallic foams, for example, the width of the foil can be relatively large, which allows longer heating times and accesses interesting densities.

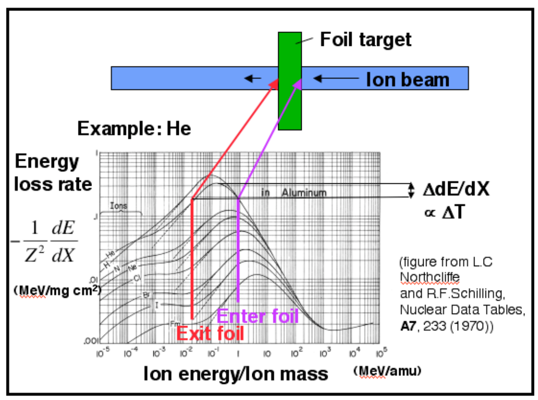

Figure 1. Temperature variations in an ion-beam heated foil can be minimized by choosing an ion and energy such that the peak in dE/dX occurs in the center of the foil (ref. [2]).

HYDRODYNAMIC DISASSEMBLY TIME AND TARGET TEMPERATURE

The sound speed c_s is given by $c_s = (\gamma P/\rho)^{1/2} = (\gamma[\gamma-1]U/\rho)^{1/2}$. For an instantaneously heated target a rarefaction wave propagates inward at about c_s while matter flows outward at about 2 c_s (for a 1D gas) (ref. [3]). Thus, for measurement of material properties, heating needs to occur on a time scale such that the rarefaction wave does not progress so far as to render the full density region of the foil samller than some minimum diagnosable spatial scale over the duration of the pulse.

In order to calculate more accurately the sound speed and the temperature achieved in the heating, one needs to understand the relation between energy density and ionization state Z*. As a first estimate, we use a model developed by Zeldovich and Raizer and summarized in ref. [4]. A second model for equation of state uses the Thomas Fermi model for calculating the distribution of electrons within an atom (see ref. [5], and reference therein for a description).

EXAMPLES OF ACCELERATOR REQUIREMENTS

Using the scaling described in the previous section for ion beam stopping, the time scale for hydrodynamic expansion, and the equation of state, we are able to make estimates of the required beam parameters for exploring the Warm Dense Matter regime. Table 1 gives examples of requirements for $Neon^{+1}$ (A=20.17) at foil entrance energy (E_{max}) of 19 MeV, The energy at the center of the foil (E_{center}) and the energy at the exit of the foil (E_{min}) are listed in the caption to the table. Three different mass densities of Aluminum target are given: Solid density (2.7 g/cm^3) and 10% and 1% of solid, which can be produced by making an aluminum "foam." In turn for each target density, three target temperatures are shown. The table is based on a minimum diagnosable length scale Z_{min} of 40 μ. It is clear from the table that solid density, although resulting in the highest energy density, requires very short pulse durations, because the foil width is smaller than Z_{min} and so only a small rarefaction wave propagation distance is allowed. But for the 1% and 10% cases, the foil is larger than Z_{min}, so that the rarefaction wave propagation distance can be 10's or 100's of microns, with concomitantly longer pulse duration. In all cases the plasma temperature is in the few to tens of eV, and the required number of particles is in the order of 10^{12} to 10^{13} particles, for equivalent focal spot radii of 1 mm.

In Table 2 the requirements for various ion species to produce a 10 eV target temperature in a 10% solid density aluminum foam are listed.

FINAL FOCUS REQUIREMENTS

In this paper we consider the case where beam plasma neutralizes both a drift compression region and the final focus. We may make simple estimates for the contribution to the spot size from chromatic effects (i.e. for the effects of a velocity spread) from particular optical systems. Here we choose a "thick" solenoidal lens in which a beam enters a solenoid with zero convergence angle and focuses to a spot within the solenoid. The focused beam can be shown to have a radius from emittance and chromatic effects r_{spot} given approximately by:

$$r^2_{spot} \approx (\pi r_0/2)^2 (\Delta v_{spread}/v)^2 + (2\varepsilon_x f/\pi r_0)^2 \qquad (5)$$

Here f is the focal length, i.e., the distance from the entrance of the solenoid to the focal spot, and ε is the beam emittance. Also, r_{spot} and r_0 are the beam radii (= $2^{1/2}\langle r^2\rangle^{1/2}$) at the focal spot and entrance to the solenoid respectively, and $\varepsilon_x \equiv 4(\langle x^2\rangle\langle x'^2\rangle-\langle xx'\rangle^2)^{1/2}$) is the unnormalized emittance. The quantity r_{spot} is minimum when $r_0^2=(2/\pi)\varepsilon f/(\Delta v_{spread}/v)$ and has the value given by

$$r_{spot}^2 = 2\varepsilon f\, \Delta v_{spread}/v. \qquad (6)$$

At minimum pulse duration a velocity "tilt" is converted to a velocity spread, so achieving high beam intensity will limit the velocity tilt.

It is apparent from equation (6) that a large velocity spread has deleterious effects in the focusing. Thus a larger velocity tilt will allow a shorter pulse but will yield a large overall spot. If the longitudinal emittance is small a larger velocity tilt is not needed to achieve the short pulse duration. Thus to obtain a small spot there are tradeoffs that can be made between longitudinal and transverse emittance; a different optimization might be made if one is easier to minimize than the other. This may be made more explicit by expressing equation (10) in terms of the transverse and longitudinal normalized emittances:

$$r_{spot}^2 = 4\varepsilon_{nxx}\varepsilon_{nz} f\ /\ (3^{1/2}\beta^3\ c\ \Delta t). \qquad (7)$$

Here ε_{nx} is the normalized x emittance (= $\beta\varepsilon_x$) and ε_{nz} is the normalized z (longitudinal) emittance (defined here as = $3\beta(\langle z^2\rangle\langle z'^2\rangle-\langle zz'\rangle^2)^{1/2}$), f is the final focal length, β is the final velocity in units of c and Δt is the final pulse duration. A prime indicates derivative with respect to

Table 1. Neon beam: Z=10, A=20.17, E_{min}=7.7 MeV, E_{center}=12.1 MeV, E_{max}=20.1 MeV, and Δz_{min}=40 μ.

ρ(g/cm³)(% solid)	0.027 (1%)			0.27 (10%)			2.7 (100%)		
Foil length (μ)	480			48			4.8		
kT (eV)	3.1	4.8	15	4.2	7.3	18	5.9	12	22
Z*	1.1	2.1	2.7	0.56	1.7	2.6	0.56	1.2	2.5
$\Gamma_{ii}=Z^{*2}e^2n_i^{1/3}/kT$	0.45	1.1	0.95	0.30	0.63	1.4	0.30	0.70	1.6
$N_{ions}/(r_{spot}/1mm)^2/10^{12}$	1	3	10	1	3	10	1	3	10
Δt (ns)	84	48	27	3.8	2.2	1.2	0.04	0.03	.014
U (J/m³)/10¹¹	.015	.045	0.15	0.15	0.45	1.5	1.5	4.5	15

Table 2. Parameters for five different ion beam species such that the central temperature of a 10% solid density Aluminum foil reaches 10 eV.

Beam Ion	Z	A (amu)	Energy at Bragg Peak (MeV)	dE/dX at Bragg Peak (MeV-cm2/mg)	Foil Entrance Energy (app) (MeV)	Delta z for 5% T variation (10% solid Al) (microns)	Beam Energy for 10 eV (J/mm2)	t_hydro= delta z/(2 cs) at 10 eV (ns)	Beam Power per sq. mm (GW/mm2)	Beam current for 1 mm diameter spot (A)
Li	3	6.94	1.6	2.68	2.4	22.1	3.3	0.5	6.1	1990.6
Na	11	22.99	15.9	11	23.9	53.5	8.0	1.3	6.1	200.3
K	19	39.10	45.6	18.6	68.4	90.8	13.6	2.2	6.1	69.8
Rb	37	85.47	158.0	39.1	237.0	149.7	22.4	3.7	6.1	20.2
Cs	55	132.91	304.0	59.2	456.0	190.2	28.5	4.7	6.1	10.5

Table 3. Comparison of requirements on a 23 MeV Na beam with final pulse duration of 1 ns, and final focal spot radius of 1 mm, assuming neutralized drift compression and solenoidal final focus (with a 15 T field corresponding to a 0.7 m focal length), satisfying equations (5) and (7). The injected beam has energy 1 MeV and pulse duration 171 ns.

Pulse duration (before drift compression) (ns)	Velocity tilt (Head to tail) dv/v_tilt	Maximum rms velocity spread dp/p_rms (befor drift comp)	Maximum emittance unnormalized 4 rms (mm-mrad)	Maximum normalized emittance 4 rms (mm-mrad)	Beam radius at solenoid entrance Ro (m)	Neutralized Drift length (m)	Maximum rms velocity spread dp/p_rms (at injector)	Normalized Long. Emittance (mm-mrad)
20	0.05	7.22E-04	49.5	2.3	0.031	5.34	1.98E-03	8.2
20	0.1	1.44E-03	24.7	1.2	0.016	2.67	3.97E-03	16.5
20	0.2	2.89E-03	12.4	0.6	0.008	1.34	7.93E-03	32.9
50	0.05	2.89E-04	49.5	2.3	0.031	13.77	1.98E-03	8.2
50	0.1	5.77E-04	24.7	1.2	0.016	6.89	3.97E-03	16.5
50	0.2	1.15E-03	12.4	0.6	0.008	3.44	7.93E-03	32.9
100	0.05	1.44E-04	49.5	2.3	0.031	27.83	1.98E-03	8.2
100	0.1	2.89E-04	24.7	1.2	0.016	13.91	3.97E-03	16.5
100	0.2	5.77E-04	12.4	0.6	0.008	6.96	7.93E-03	32.9
250	0.05	5.77E-05	49.5	2.3	0.031	69.99	1.98E-03	8.2
250	0.1	1.15E-04	24.7	1.2	0.016	35.00	3.97E-03	16.5
250	0.2	2.31E-04	12.4	0.6	0.008	17.50	7.93E-03	32.9
250	1	1.15E-03	2.5	0.1	0.002	3.50	3.97E-02	164.6

the path length s, and non-relativistic velocities are assumed.

Table 3 lists a number of parameters for possible 23 MeV Na beams, with final pulse duration Δt of 1 ns, total charge of 0.1 μC, and final spot radius of 1 mm. The table illustrates some of the tradeoffs that can be made involving pulse duration before drift compression, velocity tilt and requirements on longitudinal and transverse emittance.

We may use the ion stopping equations, together with injector and final focus equations to examine theoverall target performance as a function of ion energy, mass and other parameters. At the injector end, the normalized emittance may (ideally) be related to the temperature T_s and radius r_b of the source:

$$\varepsilon_N = 2r_b(kT_s/mc^2)^{1/2}$$
$$= 0.81 \text{ mm-mrad } (r_b/4 \text{ cm})\,(20.1/A)^{1/2}\ \ (kT_s/2 \text{ eV})^{1/2} \qquad (8)$$

Even if the injector emittance is dominated by optical aberrations an effective temperature may be used in eq. (8). To avoid voltage breakdown, the diode gap distance d must be sufficiently large:

$$d = .01 \text{ m } (V_d/100 \text{ kV})^2 \quad \text{if } d > 1 \text{ cm} \qquad (9)$$

Since we are considering large currents (d > 1 cm) is appropriate. We may combine eqs. (8) and (9) to obtain

$$\varepsilon_f = 29 \text{ mm-mrad } (4/\Delta)\ (kT_s/2 \text{ eV})(V_d/400 \text{ kV})^2 \times (20 \text{ MeV}/qV_f)^{1/2} \qquad (10)$$

The Child Langmuir current

$$I = (4\pi\varepsilon_0/9)\,(2q/m)^{1/2}\,(V_d^{3/2}/\Delta^2)$$
$$= 0.6 \text{ A } (20/A/q)^{1/2}\,(4/\Delta)^2\,(V_d/400 \text{ kV})^{3/2} \qquad (11)$$

Here $\Delta = d/r_b$ which is usually in the range 2.5 - 8 to minimize non-linearities. Here we choose 4 as a typical nominal value.

The total charge $I\Delta t$ may be written

$$I\Delta t = 0.12\ \mu C\ (20/A/q)^{1/2}\ (4/\Delta)^2\ (V_d/400 \text{ kV})^{3/2}(\Delta t_d/200ns) \qquad (12)$$

The final pulse energy E_{pulse} may be written as

$$E_{pulse} = V_f I\Delta t =$$
$$= 2.4 \text{ J } (20/A/q)^{1/2}\,(4/\Delta)^2(V_d/400kV)^{3/2}(\Delta t_d/200ns)(V_f/20 \text{ MV}) \qquad (13)$$

Equations (8) through (12) describe the phase space and total charge obtainable from an injector. The final target energy density U can be calculated from the total pulse energy, spot radius, foil thickness,

$$U = 2V_f I\Delta t/(3\pi r_{spot}^2\ \Delta z) \qquad (14)$$

and the target temperature can be expressed as:

$$kT_{targ} \approx 2UA_{targ}\text{m}_{amu}/(3(Z^*+1)\rho)$$
$$= 3 \text{ eV } (A_{targ}/27)(3/(Z^*+1))\ (2 \text{ eV}/kT_s)^{1/2}$$
$$\times (0.05/\Delta v/v_{tilt})(q/1)^{0.32}(4/\Delta)(\Delta t_d/200 \text{ ns})$$
$$\times (V_d/400 \text{ kV})^{-1/2}(V_f/20 \text{ MV})^{0.815}(0.7 \text{ m}/f) \qquad (15)$$

As discussed before, the target temperature in eq. (15) can be achieved if the pulse duration is sufficiently small compared to the hydro time. The pulse duration at the target Δt_t can be expressed as:

$$\Delta t_t = 2A \text{ m}_{amu}\ c\ \varepsilon_{nz}/\ (q\ V_f\ \Delta v/v_{tilt})$$
$$= 1 \text{ ns } (\varepsilon_{nz}/8 \text{ mm-mrad})(20 \text{ MeV}/V_f)(A/q/20)(\ 0.05/\Delta v/v_{tilt}) \qquad (16)$$

where $\Delta v/v_{tilt}$ is the head-to-tail tilt imposed on the beam during final drift compression.

These equations will be useful in evaluating concepts for accelerator-driven Warm Dense Matter studies.

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