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SHOULD CONSUMPTION SUB-AGGREGATES BE USED TO MEASURE POVERTY?

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Abstract. Frequent measurement of poverty is challenging, as measurement often relies on complex and expensive expenditure surveys which try to measure expenditures on a comprehensive consumption aggregate. We investigate the use of consumption "sub-aggregates" instead. The use of consumption sub-aggregates is theoretically justified if and only if all Engel curves are linear for any realization of prices. This is very stringent. However, one can empirically identify certain goods that happen to have linear Engel curves given prevailing prices, and hope that the effect of price changes is small, in which case a sub-aggregate might work in practice. We construct such linear sub-aggregates using data from Rwanda, Tanzania, and Uganda. We find that using sub-aggregates is ill-advised in practice as well as in theory. This raises questions about the consistency of the poverty tracking efforts currently applied across countries, since obtaining exhaustive consumption measures remains an unmet challenge.

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. JEL Classification: D11, D12, I32.

1. Introduction

Directly and comprehensively measuring non-durable consumption expenditures via large-scale household surveys such as the Living Standards Measurement Surveys (LSMS) is generally regarded as the best way in which to assess the prevalence of poverty in low income countries. But such surveys are complex and costly, and so are conducted in fewer places, with smaller samples, and with lower frequency than is desirable if our aim is to measure changes in poverty over time (Beegle et al. 2016).

In response, researchers have been looking for alternative approaches to track poverty at reduced cost and effort. One approach involves the construction of models which use observed characteristics that are easier to collect (or which have already been collected for other purposes) to predict consumption. The success of this approach hinges critically on the stability of the model parameters over time and across different survey designs. One way to mitigate this concern is to identify empirically the conditions under which this assumption is more likely to hold (Christiaensen et al. 2012; Mathiassen 2013; Kilic and Sohnesen 2019). Another is to combine the survey collection, with more complete consumption aggregates for some households, while only collecting proxies for consumption for others (Pape and Mistiaen 2018; Yoshida et al. 2015). Nonetheless, model parameter instability cannot be excluded. With more high-resolution earth observation data and more powerful machine learning expertise now readily available, another class of models predicts consumption and poverty applying artificial intelligence techniques to a combination of satellite imagery (especially daytime imagery) and existing household data. Prediction accuracy often exceeds benchmarks from more standard regression-based approaches, especially in estimating poverty levels at very disaggregated levels—so-called poverty mapping (Head et al. 2017; Jean et al. 2016; Watmough et al. 2019). Whether these models calibrated on data for a certain year are better able to make accurate predictions on data for another year, remains however largely untested.[1] The issue of model stability remains.

This paper explores the potential of a different approach, without the need for prediction and model parameters. Instead of falling back

1. An interesting exception is Bansal (2020) who demonstrates temporal transferability of a simple machine learning model applied to satellite data to estimate the evolution of a district level development index (akin to the Human Development Index) using temporal satellite data as input.

on other, more readily available data sources and consumption prediction methods, we explore whether less comprehensive consumption measures, or consumption sub-aggregates, which are also easier and less expensive to collect, might not suffice as an information base to track poverty over time. We draw on demand theory to establish the conditions under which this might be the case, and subsequently explore empirically whether these conditions hold in practice given the experience from a couple of different country settings.

The idea is inspired by Lanjouw and Lanjouw (2001) who imagine predicting poverty with food consumption only. Provided that Engel's law holds (so that the food share is larger for the poor than for the non-poor) and given a particular method of adjusting the poverty line when moving between expenditure aggregates,[2] the Lanjouws demonstrate that the Foster-Greer-Thorbecke (FGT) family of poverty measures will yield weakly smaller measured poverty when one uses only food expenditures rather than all expenditures.

This raises the question: Under what circumstances can one construct consistent poverty measures using different expenditure aggregates?[3] We extend the Lanjouws' result, and show that so long as household welfare can be summarized by total non-durable expenditures, then one can construct consistent welfare measures using smaller aggregates provided that the Engel curve (quantities vs. total expenditures) for the sub-aggregate is linear.

Accordingly, we search for sub-aggregate expenditures that are linear in total expenditure. However, if relative prices within the sub-aggregate change, then the resulting sub-aggregate expenditures are only guaranteed to *remain* linear if all of the constituent goods in the aggregate also have linear Engel curves. Thus, the key to constructing a robust proxy for total expenditures that can be used to track changes in poverty over time when prices are changing is to identify particular goods which have linear Engel curves which are fairly insensitive to changes in prices.

We use LSMS data from Rwanda, Tanzania, and Uganda and devise a simple way to identify goods with (nearly) linear Engel curves using data in an initial period. We form a linear sub-aggregate of such goods, and then ask whether this sub-aggregate can be used to accurately measure poverty in a subsequent period. Theory suggests that this will not work if there are significant changes in relative prices. Unfortunately,

2. The "upper bound" method described by Ravallion (1994).

3. In this paper, the tracking of monetary poverty is based on the commonly used "Foster-Greer-Thorbecke" (FGT) family of poverty measures described by Foster, Greer, and Thorbecke (1984).

the evidence bears out this suggestion—our constructed linear sub-aggregate does not perform well, and we conclude that one should not attempt to track poverty by trying to construct a single linear sub-aggregate. To the extent that comprehensive expenditures measures are still incomplete in practice, this also counsels caution about the validity of expenditure-based poverty tracking more broadly.

2. Theory of Demand and The Aggregation of Commodities

We are interested in identifying a particular set of expenditure goods which can be added to form a sub-aggregate, which can then be used as a proxy for total expenditures to measure poverty and changes in poverty over time. In this section we present two results that provide a set of conditions that are both necessary and sufficient for a particular sub-aggregate to serve as a proxy for total expenditures. To fix ideas, we first introduce some notation (2.1) and recall some of the standard results from demand theory (2.2). The link between consumption aggregation and sub-aggregates with different FGT poverty measures is then explored in subsections 2.3–2.5.

2.1. **Consumption Aggregates.**

Suppose that a consumer (or household) values n distinct non-durable commodities indexed by $i = 1, \dots, n$; call the bundle of these commodities consumed by the consumer $c \in X \subseteq \mathbb{R}^n$. Note our implicit assumption that consumption is continuous in all these goods. Assume also that there is a vector of prices $p \in \mathbb{R}^n_+$, so that the consumer's total expenditures on the consumption bundle c is $x = p'c$.

Because it comprises expenditures on all non-durable commodities, we call x the consumer's *total expenditures*, or *aggregate expenditures*. However, there are many possible sub-aggregates. Consider some partition or *aggregation* of the n different commodities $A = (X_1, \dots, X_m)$, with $X_j \cap X_k = \emptyset$ for $j \neq k$ and $X = \cup_{j=1}^m X_j$, so that good i is an element of the jth aggregate if $i \in X_j$.

Just as x is the consumer's total expenditures on all goods in X, given the aggregation A let x^j denote sum of expenditures on all goods in aggregate X_j. Similarly, let c^j denote the consumption bundle of goods in the jth aggregate.

2.2. **Separable Aggregates.**

Let a consumer's utility function $u : X \to \mathbb{R}$ map consumption of these n goods into utility. The consumer's demand for these different goods is said to be *separable* in the aggregation A if for any aggregate $X_i \in A$ and any two consumption goods c_j and c_k which are both in X_i the consumer's demand for c_j and c_k can be written as a function just of prices for goods in X_i and of expenditures on the aggregate x^i; demands for goods in aggregate X_i written in this form then won't depend on the prices of commodities in *other* aggregates, except to the extent that these prices affect x_i.

A sufficient condition that the aggregation A be separable is that the consumers' utility function can be written as a set of m sub-utility functions $\{u_j\}_{j=1}^m$ and an aggregating utility function $u_0 : \mathbb{R}^m \to \mathbb{R}$ such that

$$u(c_1, c_2, \ldots, c_n) \equiv u_0\left(u_1(c^1), u_2(c^2), \ldots, u_m(c^m)\right),$$

where $c^j = (c_i)_{i \in X_j}$ is a set of consumption aggregates (Gorman 1961).

2.3. **Consumption aggregation and FGT poverty measures.** Lanjouw and Lanjouw (2001) consider the case in which total non-durable expenditures x can be divided into an aggregation (X_1, X_2) with corresponding expenditures (x^1, x^2); each of these aggregates is then assumed to depend only on total non-durable expenditures x, so that $x \equiv x^1(x) + x^2(x)$.[4] Call the function relating expenditures on a particular aggregate to total expenditures the *Engel curve* for the aggregate; then the Lanjouws assume that the Engel curve $x^1(x)$ is a continuous and strictly increasing function of x, so that, observing some level of expenditures $\hat{x}^1$ one can invert $x^1(x)$ to infer what the overall level of expenditures is, obtaining, say, $\hat{x} = g_1(\hat{x}^1)$.

We extend the ideas of Lanjouw and Lanjouw (2001) in two important ways. First, they give conditions under which head-count poverty statistics for a population won't depend on whether one uses total expenditures or a expenditures on a smaller aggregate as a proxy for total expenditures. We provide conditions under which all FGT poverty measures will be similarly invariant to the use of an aggregate. Second, by assuming that expenditures on different aggregates depend only on total expenditures, the Lanjouws implicitly assume that relative prices are unchanging. In a simple application of demand theory we show how to relax this assumption as well, allowing one to apply these methods to data on expenditures over time, where we may expect changes in economic conditions to lead to changes in relative prices.

We are interested in identifying a particular aggregate that can be used to measure poverty and changes in poverty over time. Our two

4. In what follows it's useful to follow some common notational conventions in functional analysis, which blur the lines between variables and functions. For example, x^1 will always take a value, but may implicitly or explicitly be regarded as a function of other quantities, as in $x^1(x)$.

extensions leave us with a set of conditions that are both necessary and sufficient for such a particular sub-aggregate to serve as a proxy for total expenditures. We then turn our attention to a search over different possible sub-aggregates to identify sub-aggregates that may be valid proxies for total expenditures.

2.4. **Sub-aggregates and Headcount Poverty.** Consider an aggregation A with two or more sub-aggregates (i.e., with $m \geq 2$). Let P_α be a Foster-Greer-Thorbecke poverty measure, with parameter $\alpha \geq 0$; P_0 is called "headcount poverty." Let z_1 be a "poverty line" in expenditures on the first sub-aggregate, x^1 (in the Lanjouws' application x^1 is food and z_1 is a "food poverty line"). Then the z which satisfies $x^1(z) = z_1$ is what Ravallion (1994) calls the "upper bound" poverty line: we'd interpret it as the total expenditures of a household which chose food expenditures equal to the food poverty line. Let $g_1(x^1)$ be the inverse of the Engel curve $x^1(x)$, so that $z \equiv g_1(x^1(z))$; this inverse is guaranteed to exist for aggregates in A provided that x^1 is strictly increasing.

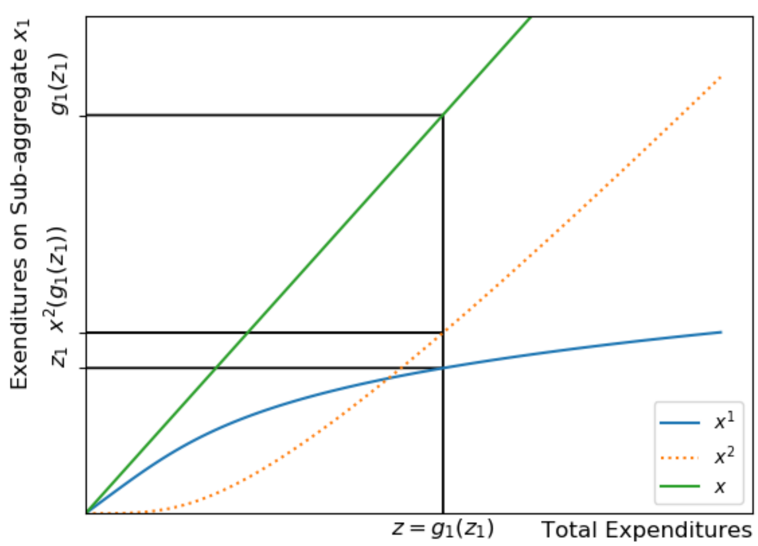

FIGURE 1. Illustration of the Construction of the 'upper bound' poverty line.

A main result from Lanjouw and Lanjouw (2001) is that the head count poverty statistic doesn't depend on whether one uses expenditures on a sub-aggregate x^j or total expenditures x; the only trick is moving from the sub-aggregate poverty line z_1 to the poverty line $z = g_1(z_1)$ relevant to total expenditures. Thus, provided one knows the mapping g_1 from the sub-aggregate poverty line to the 'full' poverty line, one ought to be able to use any monotonic sub-aggregate to measure head-count poverty.

The idea of the method is illustrated in Figure 1. Here there are two sub-aggregates with expenditures x^1 and x^2; the way in which these vary with total expenditures x is illustrated by the two lower curves in Figure 1. Then we can define a sub-aggregate poverty line $z_1 = x_1(z)$ (marked as z_1 on the vertical axis), and by construction we have $z = g_1(z_1)$, which gives the mapping between the poverty line corresponding to total expenditures z and the poverty line corresponding to expenditures on sub-aggregate one, z_1.

We first extend the Lanjouws' invariance result, allowing us to move not only from a sub-aggregate to total expenditures, but from any monotonic subaggregate to any other.

Proposition 1. *Let A be an aggregation, and let M be the subset of aggregates $X_j \in A$ such that expenditures x^j are continuous, strictly increasing functions of total expenditures x. Pick any two aggregates X_j, X_k in M. Suppose that a consumer is regarded as poor if and only if expenditures x^j on aggregate X_j are less than a poverty line z_j. Then there exists another poverty line z_k such that headcount poverty using expenditures x^j with poverty line z_j will be equal to head count poverty using expenditures x^k with poverty line $z_k = x^k(g_j(z_j))$.*

Proof. The result follows directly from the existence of the inverse functions g^j and g^k, and the existence of these is guaranteed by our assumption that X_j and X_k are in M. The poverty measure $P_0(x^j, z_j)$ is equal to the proportion of consumers with $x^j \leq z_j$, and by the definition of the inverse functions the consumer has $x^j \leq z_j$ if and only if $x^k \leq x^k(g_j(z_j))$. □

2.5. **Sub-aggregates and general FGT measures** (P_α)**.** Note that Proposition 1 does *not* require that expenditures on all aggregates are increasing; if some expenditure aggregates are decreasing we can simply set them aside.

More generally, moving from one aggregate to another *will* matter. Lanjouw and Lanjouw (2001) consider the class of Foster, Greer, and Thorbecke (1984) (FGT) poverty measures, and show that (the head-count measure aside) using two different aggregates won't generally give the same poverty statistics, even if one adjusts the poverty line as indicated above.

Define the poverty measure over a population of N households by

$$P_\alpha(\mathbf{x}, z) = \frac{1}{N}\sum_{j=1}^{N} p_\alpha(x_j, z),$$

where $\mathbf{x}$ is an N-vector of expenditures for all households and where x_j denotes total expenditures of the jth household.

The FGT class of measures can then be obtained by defining

$$p_\alpha(x, z) = \mathbb{1}(x < z)(1 - x/z)^\alpha,$$

where $\mathbb{1}$ is an indicator function which takes the value of one if $x < z$, and zero otherwise. How measured poverty changes using the FGT measure depends on how relative expenditure shares change in the different aggregates one considers. We classify some possibilities in the following proposition.

Proposition 2. *Let x^1 and x^2 be the expenditures corresponding to two different non-durable consumption aggregates X_1 and X_2 in an aggregation A, let x be total expenditures on all goods, and assume that x^1 and x^2 are both strictly increasing functions of x.*

Then for any $z > 0$ there exists a z_1^* and a z_2^* such that $P_0(x, z) = P_0(x^1, z_1^*) = P_0(x^2, z_2^*)$, and

1. *If the expenditure share of X_1 is non-increasing in total expenditures, then $P_\alpha(x^1, z_1^*) \leq P_\alpha(x, z)$. [This is the Lanjouws' result.]*
2. *If the expenditure share of X_1 is non-decreasing in total expenditures, then $P_\alpha(x^1, z_1^*) \geq P_\alpha(x, z)$. [This is the obvious converse.]*
3. *If the ratio of expenditure shares of X_1 to X_2 is non-increasing in total expenditures, then $P_\alpha(x^1, z_1^*) \leq P_\alpha(x^2, z_2^*)$. [This is an extension of the Lanjouw's second result to any two aggregates.]*
4. *If the ratio of expenditure shares of X_1 to X_2 is non-decreasing in total expenditures, then $P_\alpha(x^1, z_1^*)) \geq P_\alpha(x^2, z_2^*)$. [This is the converse of the extension.]*
5. *If the ratio of expenditure shares of X_1 to X_2 does not vary with total expenditures, then $P_\alpha(x^1, z_1^*) = P_\alpha(x^2, z_2^*)$. [This is the case of affine Engel curves.]*

Proof. We prove case (3); the remaining cases are all either immediate consequences or follow *mutatis mutandis*. The case of $\alpha = 0$ has already been addressed in Proposition 1. So fix $\alpha > 0$ and consider the difference

$$p_\alpha(x^1, z_1^*) - p_\alpha(x^2, z_2^*) = \begin{cases} (1 - x^1(x)/z_1^*(z))^\alpha - (1 - x^2(x)/z_2^*(z))^\alpha & \text{for } x < z; \\ 0 & \text{for } x \geq z. \end{cases}$$

If $x < z$ then the sign of this difference is negative if and only if $(1 - x^1(x)/z_1^*(z))^\alpha < (1 - x^2(x)/z_2^*(z))^\alpha$ which in turn is satisfied if and only if $x^1(x)/z_1^*(z) > x^2(x)/z_2^*(z)$, which is again satisfied if and only if $x^1(x)/x^2(x) > z_1^*(z)/z_2^*(z)$. At $x = z$ the two sides of this expression are equal by construction; by assumption the ratio on the left is non-increasing in x, so for any $x < z$ we have $p_\alpha(x^1, z_1^*) \leq p_\alpha(x^2, z_2^*)$, and the result follows. □

This result is, like the Lanjouws' original result, rather general. The only critical assumption is that expenditures on the aggregates we consider need to be strictly increasing in total expenditures. This in turn is promised by demand theory: Provided only that consumers are not satiated, this monotonicity property must hold for *some* aggregate; must hold for *any* separable aggregate; and more generally will hold for any aggregate of normal goods (which need not be separable). Beyond this, nothing further needs to be known or assumed regarding consumer preferences.

The result also appears to be quite useful, and perhaps an important step toward the Lanjouws' goal of devising methods for constructing possibly *small* consumption aggregates which would nevertheless allow one to construct the same poverty statistics one would obtain from measuring total expenditures. In particular, one could either use Proposition 1 to motivate using *any* small aggregate and constructing comparable headcount poverty statistics, or, using part (5) of Proposition 2, find a single small *linear* aggregate to obtain comparable statistics for any of the FGT poverty measures. This last idea is summarized in the following Corollary:

Corollary 1. *If expenditures on a separable aggregate X_1 are proportional to total expenditures x, then $P_\alpha(x^1(x), z_1^*(z)) = P_\alpha(x, z)$ for all x, for all $z > 0$ and for all $\alpha \geq 0$.*

Separability here implies an increasing relationship between x^1 and x, so that case (5) of Proposition 2 obtains, taking as x^2 total expenditures x, and we need to find just *some* aggregate which varies in a linear way with total expenditures. Note that while the property of proportionality here is presumed to always hold, the factor of proportion could be permitted to change over time, so it seems that if we could simply identify an aggregate with the necessary linearity we'd have a very useful result.

However, while the result is general, this hoped-for usefulness may at the same time be limited because the result is also both incomplete and somewhat fragile. First the incompleteness: though modest restrictions on consumer preferences can be invoked to guarantee monotonicity of x^1 and x^2, the ratio x^1/x^2 need not be monotone, and any non-monotone behavior of this ratio for $x < z$ gives us a case not covered by Proposition 2. Further, there's simply no theoretical guarantee that an sub-aggregate that varies in a linear way with x (the requirement of our Corollary) exists.

Second, the fragility: demand theory forcefully tells us that expenditure aggregates will be a function not only of total expenditures x, but also of prices. And even if the ratio x^1/x^2 is monotone for the prices that obtain in one place and one time, it needn't be monotone for other prices. This makes using this result to try to identify a linear aggregate a worrisome exercise: using data to identify an aggregate which is linear now is no guarantee that it will be linear later, when relative prices change. We revisit this point below in Proposition 3, at which point we're in a position to specify conditions under which we *can* guarantee the desired linearity.

3. Linearity of Engel Curves for Aggregates

If a particular aggregate of goods has the property that demand for the aggregate depends only on the prices of the goods that comprise it, then we say that the aggregate is *separable*. In this section we consider the circumstances under which expenditures on any such aggregate will be linear in total expenditures.

Assume that demand is separable in the aggregation A, and consider expenditures on an aggregate X_j. These expenditures will generally depend on (all) prices, and on total expenditures x. We're interested in understanding when x^j is linear in x—that is, in the case in which expenditures on the jth consumption aggregate will be a fixed share of total expenditures.

Adding up implies that there are only two cases in which some aggregate can be linear in total expenditures. The first is when expenditures for *every* aggregate $X_j \in A$ is linear in total expenditures; the second allows *some* aggregates to have non-linear Engel curves, but for non-linearities in the Engel curves for one aggregate to be offset by non-linearities in some other, so that the Engel curve for an aggregation of these two is in fact linear. We consider these two cases in turn.

3.1. **Linear Engel Curves.** The consumer will have linear Engel curves for the aggregation A if and only if the aggregating utility function u_0 is quasi-homothetic (Gorman 1959). We illustrate the case with homothetic demands in Figure 2, using expenditure shares for different aggregates in the UN "Classification of Individual Consumption According to Purpose" (COICOP) aggregation of goods for Uganda in 2005.

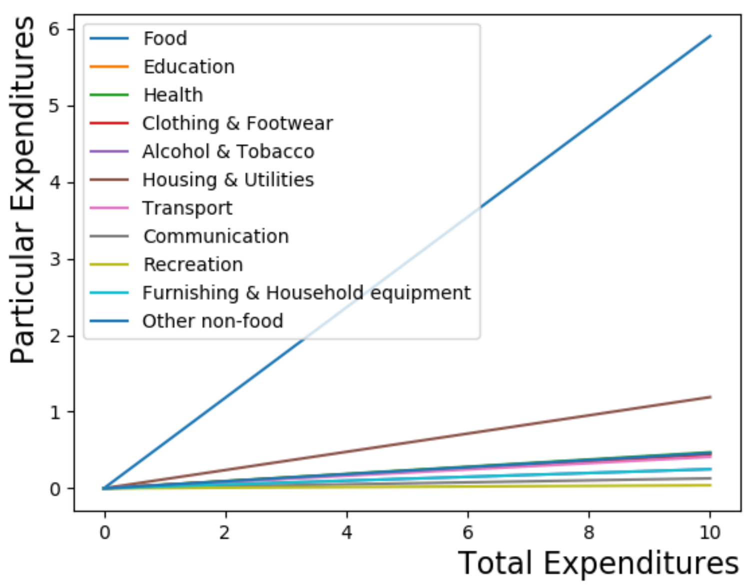

FIGURE 2. Example of Linear Engel Curves. Shares are constant.

More generally, Engel curves from quasi-homothetic (rather than homothetic) preferences need not pass through the origin, a case which is illustrated in Figure 3. Here prices of four different goods are all equal to one, and the consumer has Stone-Geary type utility given by

$$U(c_1, c_2, c_3) = \log(c_1 - 1) + \frac{1}{2}\log(c_2 + 1) + 2\log(c_3).$$

A consequence of having linear Engel curves which don't pass through the origin is that budget balance requires that for any good for which consumption is positive when total expenditures are zero, others must be negative, as illustrated in the figure, where a subsistence demand of 1 for good 1 is financed by consuming a quantity of good two equal to minus one when total expenditures are equal to zero.

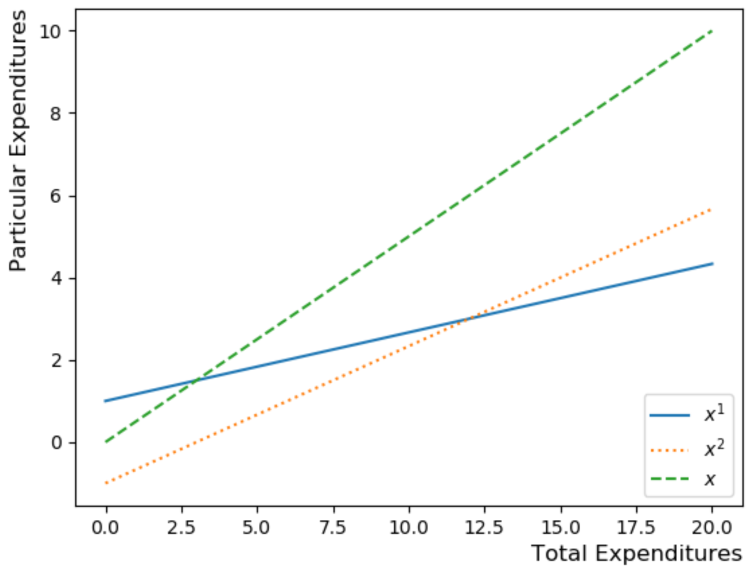

FIGURE 3. Example of Linear Engel Curves; Shares are *not* constant.

Requiring a consumer to have positive levels of consumption of e.g., food seems perfectly sensible, but allowing a consumer with no resources to finance her food consumption through negative consumption of some other good may be problematical. In such cases the consumer's Engel curves will simply not be defined for levels of total expenditure below that necessary to finance subsistence consumption. This circumstance is illustrated in Figure 4. Here a minimum expenditure of one is required for good 1, so that Engel curves aren't defined for levels of expenditure below this. Related, consumption of good two is zero at the lowest levels of income because of the non-negativity constraint on consumption. Thus, when the corner constraint is no longer binding this induces a "kink" in the Engel curves for other goods.

These 'kinks' in the Engel curves created by corner solutions for demand for some goods violate the claim made above that a quasi-homothetic aggregating utility u_0 is sufficient for linear Engel curves. To extend the claim to cover the case in which consumption is required to be non-negative, we restrict the domain of total expenditures so that over this domain the consumer never finds herself at a corner.

The requirement that the consumer not find herself at a corner is less restrictive than one might suppose, because the separability of the aggregation A implies that demand for every $X_j \in A$ is normal.

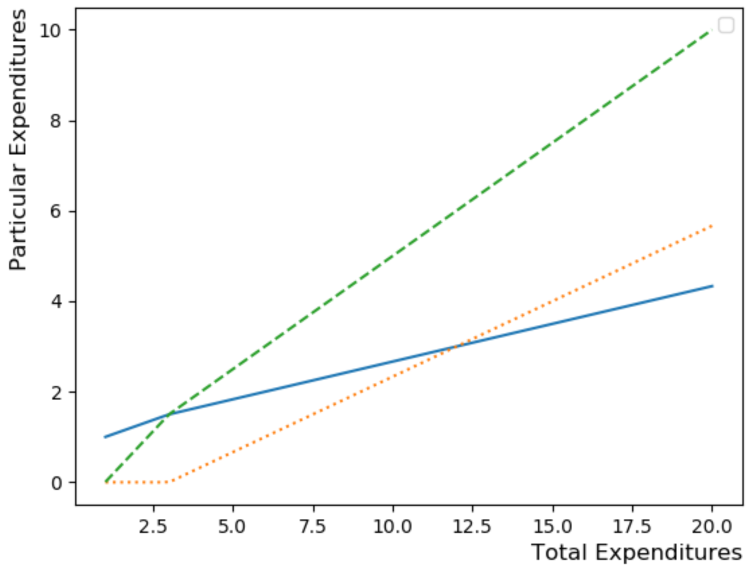

FIGURE 4. Example of Linear Engel Curves, but with corner solutions.

Accordingly, there exists a lower bound on total expenditures $\underline{x}$ such that Engel curves for all $X_j \in A$ are linear for all $x \geq \underline{x}$.

3.2. **Non-linear Engel Curves.** The second case allows for some Engel curves to be non-linear, but for the non-linearity of an aggregate X_j to be offset by non-linearities in the Engel curves of the remaining aggregates, as illustrated in Figure 5. However, in cases where two or more non-linear curves sum (or average) to a straight line it's a bit of a balancing act. The weights of the curves in the sum depend on prices, and if relative prices change *at all* then the curves will no longer deliver a linear sum.

This point is made formally in the following proposition.

Proposition 3. *If the sum of the Engel curves of some set of goods is a linear function of total expenditures x for all prices p, then either (i) the Engel curves of each good in the sum are linear, or (ii) all goods are included in the sum.*

Proof. If the sum is over a single good, then case (i) obtains trivially. Consider the case of summing the Engel curves of two goods 1 and 2. Assume that the sum of Engel curves is linear; i.e., that $p_1c_1 + p_2c_2 = a(p) + b(p)x$ for all p for some functions a and b that may depend on

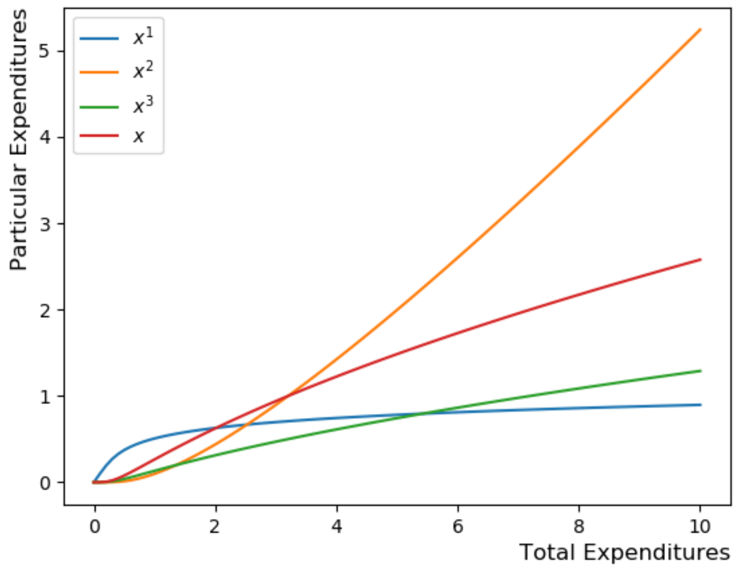

FIGURE 5. Example of Non-linear Engel Curves, with approximately linear aggregates.

prices. Taking derivatives with respect to x, we have $b(p) = p_1\partial c_1/\partial x + p_2\partial c_2/\partial x$. Taking derivatives with respect to prices, we have

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} c_{11} & c_{21} \\ c_{12} & c_{22} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} a_1(p) & b_1(p) \\ a_2(p) & b_2(p) \end{bmatrix} \begin{bmatrix} 1 \\ x \end{bmatrix},$$

where $a_i(p)$ and $b_i(p)$ indicate the partial derivatives of these functions with respect to the price of the ith good, and where c_{ij} indicates the partial derivative of demand for good i with respect to price p_j.

Now, using the Slutsky decomposition to substitute for these last partial derivatives, we obtain

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} a_1(p) & b_1(p) \\ a_2(p) & b_2(p) \end{bmatrix} \begin{bmatrix} 1 \\ x \end{bmatrix} - \Sigma \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \begin{bmatrix} \partial c_1/\partial x & \partial c_2/\partial x \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix},$$

where Σ is the Slutsky substitution matrix. We've already demonstrated that the last inner product in this expression is equal to $b(p)$, so re-arranging we obtain

$$(1 - b(p)) \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} a_1(p) & b_1(p) \\ a_2(p) & b_2(p) \end{bmatrix} \begin{bmatrix} 1 \\ x \end{bmatrix} - \Sigma \begin{bmatrix} p_1 \\ p_2 \end{bmatrix},$$

from which it can be immediately seen that provided that $b(p) \neq 1$ that both c_1 and c_2 are linear functions of x. This leaves only the second case; here suppose that there are only the two goods c_1 and c_2. Then from the consumer's budget constraint it follows that $b(p) = 1$. □

This proposition establishes that a linear expenditure aggregate must be exclusively comprised of linear goods. The following corollary establishes that if *any* good is linear in total expenditures, then *all* goods must also be.

Corollary 2. *If the Engel curve of a good is a linear function of total expenditures x for all prices p, then the Engel curves of all goods are also linear functions of total expenditure.*

Proof. Suppose that good 1 has linear Engel curves, with $p_1c_1(p, x) = a^1(p) + b^1(p)x$. Then from the budget constraint it follows that an aggregate of the remaining goods also have linear Engel curves, since $\sum_{i=2}^{n} p_ic_i = x - p_1c_1 = -a^1(p) + (1 - b^1(p))x$. But then by Proposition 3 all goods $i = 2, \ldots, n$ also have linear Engel curves. □

4. Sub-aggregates and Poverty in Practice

Corollary 1 tells us that we can use any separable sub-aggregate provided expenditures on the sub-aggregate vary in proportion to total expenditures. But Proposition 3 and its corollary tell us that such a sub-aggregate will exist only under very special (and highly implausible) conditions. In particular, even if one can identify goods with linear Engel curves given prevailing prices, under different prices this linearity may fail.

However, from a practical point of view it may be possible to identify goods with linear Engel curves in one period, and then we may get lucky: in a subsequent period perhaps relative prices won't have changed in such a way that linearity will be compromised, and our sub-aggregate will work for measuring poverty.

4.1. Data.

In the remainder of this paper we test our luck using data from expenditure surveys in Rwanda, Uganda, and Tanzania.[5] All surveys are large multipurpose household consumption surveys, representative at both national and urban/rural levels with large sample sizes. In Uganda about 3000 households were interviewed each year, in Tanzania between 3000 and 4000 each year, and in Rwanda the number of households increased substantially from 6900 to 14300 household observations per year.

5. Rwanda: Enquete Intégrale sur les Conditions de Vie des ménages de Rwanda (EICV1) 2001 and (EICV2) 2006 . Uganda: Uganda National Household Survey (UNHS) 2005/06 and 2009/10. Tanzania: Tanzania National Panel Survey (NPS) 2008/09 and 2010/11.

The goods included in the consumption aggregates follow the guidelines for consumption aggregates found in Deaton and Zaidi (2002).[6] In all three countries, FGT measures rely on consumption data collected by recall questions with probing for each consumption good. The number of consumption goods probed for in the questionnaire vary across countries, with 112 in Tanzania, 126 in Uganda, and 284 in Rwanda.[7] Beegle et al. (2012) review recall-based consumption surveys and report ranges from 37 to 305 goods, with a mean of 137 goods in total. Thus, the sample of countries seems to represent typical recall consumption surveys, even if the total number of goods doesn't actually encompass all non-durable consumption. In each country, consumption goods are valued in year one's prices corrected by the same price adjustment as used to evaluate the trend in poverty in the surveys. In each year values are spatially and temporally deflated.

The three countries are chosen to display variation in level and trend in poverty headcount. Headcount poverty stood at 15 percent in Tanzania in year one, compared to 57 percent in Rwanda, and poverty fell substantially in Uganda and Rwanda, while it increased in Tanzania (Development Research Group of the World Bank, Downloaded 2017). The five year time span between surveys in Rwanda can also be seen as an upper bound for the approach, as most countries implement full consumption surveys every five years or less.

4.2. **Looking for Linearity.** Moving from theory to practice, we would like to gauge the consumption patterns of households, as some goods might be more suitable than others. Figure 2 illustrates how a sum of linear goods will always lead to a linear aggregate, but Figure 5 also shows that a sum of expenditures on non-linear goods can be linear for a particular set of prices. To gauge properties of goods in relation to the total aggregate, we run an OLS regression of household-level log consumption expenditures for each good on a constant and the log total aggregate:

$$\log(x_i) = \alpha_i + \beta_i \log(x) + \epsilon_i. \tag{1}$$

6. The included goods in the aggregates are very similar to those defined by the national statstical agencies, but are not neccesarily identical.

7. In Tanzania and Uganda, the questionnaire is designed so that each row captures consumption of own production, gifts and purchased, summed into total consumption of items. In Rwanda, own consumption, and gifts and purchased items are split into two separate questionnaires. Hence, in the data it appears as if Rwanda has twice as many items ($2\times$ 284). For the analysis we have combined the values of each item from both questionnaires into one value, following the same principle as in Tanzania and Uganda.

Note that this equation can be re-written to give an expression for expenditure shares:

$$\frac{x_i}{x} = e^{\alpha_i+\epsilon_i} x^{\beta_i-1}.$$

In this regression we would expect that the linear goods we are looking for will have $\beta_i = 1$, and a constant share of consumption expenditures. Any goods with expenditures decreasing in total expenditures will have a negative β_i. Goods with $\beta_i > 1$ will have an increasing share of total consumption expenditures, while other goods will have $0 < \beta_i < 1$ (and a declining share of total consumption). Only for goods with income elasticity of $\beta_i = 1$ will the consumption share remain constant. We emphasize that these regressions should be regarded as *temporary*, since theory tells us to expect the estimated coefficients to be stable across periods only if prices don't change (much) or if all Engel curves are linear. The empirical question is whether in real-world data actual price changes and preferences are such that (1) provides a reasonable approximation or not. If so, then we may use goods with estimated β_i close to one to form a sub-aggregate. If not, then the construction of sub-aggregates for the purpose of tracking poverty may simply not be wise.

Thus, we estimate the regression equation (1) for the many different consumption goods across three countries described above. We then use the estimated values of β_i to classify goods:

$\beta_i < 1$: Shares decreasing;
$\beta_i > 1$: Shares increasing;
$\beta_i = 1$: Shares constant.

Our idea is then to use goods for which expenditures shares are constant to construct a sub-aggregate. If we can construct such a linear sub-aggregate and if it *remains* linear when relative prices change then we can confidently use the sub-aggregate and result of Proposition 2.5 to construct any of the FGT poverty measures.

To implement this classification, we first choose two different rounds for each of the three countries we consider. We first exclude any observations of zero expenditures, then exclude any consumption goods that have fewer than five observations. We regard some estimated point values of β_i as insignificant; these are those having a t-statistic (associated with the test that the point estimate is zero) less than 2.326 (the critical value associated with having the probability of a type I error of one percent). We classify the remaining point estimates according to two simple criteria: first, are they greater or less than one; and second, are they "approximately linear", in the sense that the point estimates lie within the interval $(0.9, 1.1)$?

Table 1 gives results of this exercise. We first use just data from the first round of data for each country to estimate and classify the β_i. For Tanzania 83% of all 112 goods can be classified (i.e., have enough observations and are significant); of these 80% are less than one, so that their expenditure shares fall on average with increasing total expenditures, while the remainder have β_i greater than one, and so increasing shares. Of these, 18% indicate an approximately linear good (i.e., β_i is within 0.1 of unity). Similarly, for Rwanda 88% of 284 goods can be classified, of which 92% have decreasing shares, while 11% are approximately linear. And for Uganda 95% of 126 goods can be classified, of which 81% have decreasing shares, while 15% are approximately linear.

This exercise allows us to construct linear sub-aggregates for each country consisting respectively of 17, 28, and 18 goods, and an appeal to Proposition 2.5 assures us that these sub-aggregates can be used to compute the same FGT poverty measures we could have computed using all observed expenditures. Further, if we were to assume that the estimated parameters (α_i, β_i) obtained at a *different* time (for the same population) remained constant, we could use the quite small amount of data required on expenditures on the sub-aggregate to compute FGT poverty measures.

Table 1. Consumption goods and linearity. Counts of goods for each country by classification; numbers in parentheses are counts of goods with the same classification in the second year as in the first.

Classification	Tanzania	Rwanda	Uganda
Fewer than 5 observations	1 (1)	3 (1)	0 (0)
Insignificant relationship	18 (7)	31 (12)	6 (5)
Decreasing shares ($0 < \beta < 1$)	74 (69)	229 (209)	97 (89)
Increasing goods ($\beta > 1$)	18 (14)	21 (9)	23 (13)
Approximately linear ($0.9 < \beta < 1.1$)	17 (10)	28 (12)	18 (7)
Total number of expenditure goods	112	284	126

However, as discussed in previous sections we have no good theoretical justification to suppose that these parameters will be stable over time; and indeed, if relative prices change we should expect the estimated β_i to also change. So what do the data tell us?

We re-estimate (1) using data from a *second* round for each country, and Table 1 reports (in parentheses) the number of goods where our estimate of β_i is unchanged. Of chief interest is the number of goods which remain approximately linear. In Tanzania this is 10 of 17; in Rwanda 12 of 28; and in Uganda 7 of 18. Overall less than half of the goods initially classified as "approximately linear" received the same classification in a second round of data. It's impossible to comfortably conclude that the linear classification is stable over time.

Why are these parameters not stable? Corollary 2 tells us that any given good can have a linear Engel curve (for all prices) if and only if *all* goods have linear Engel curves. In this situation (and only in this situation) changes in relative prices will not affect relative expenditures. The classification exercise we report in Table 1 provides evidence that some goods are not linear given prevailing prices; it follows that *no* good is linear in expenditures for all prices. Or, put differently, the parameters β_i must be functions of prices.

4.3. **Performance of sub-aggregates in measuring poverty.** Perhaps deviations from linearity are quantitatively unimportant for constructing FGT poverty measures? Guided by the theory above, we consider identifying expenditure items which are linear in a base period and using these to construct a sub-aggregate used for measuring poverty in a subsequent period. In our exercise we *also* have data on all other expenditures, so we can evaluate the performance of the sub-aggregate in measuring poverty. To make this work, we need to identify the g_i functions of Proposition 1. Here this amounts to a simple rescaling. For example, if a poverty headcount of 20 percent was observed for the full aggregate in year one, the poverty line for the reduced aggregate is set to the value at the 20th percentile of the reduced aggregate in the same year. Then valuing year two quantities using year one's prices, we can use exactly this same poverty line in the second year.

We construct our sub-aggregate by beginning with the "best" good; i.e., the good i with β_i closest to one in absolute value. Then the second best good, and so on, until all goods have been added to the aggregate. For each such sub-aggregate we then evaluate its ability to replicate measured poverty levels against poverty levels measured using the complete set of expenditures.

This procedure does not provide good approximations of poverty headcounts in the second year (Figure 6). In all three countries the estimates are systematically off in one direction, until almost all goods are added to the aggregate. We see this by the approximated poverty headcounts being clearly outside the 95 percent confidence interval of the poverty estimates for the full consumption aggregate, illustrated by the dotted lines. The reason for this pattern is the exclusion of goods with large expenditure shares, usually staple foods, that have values of β_i far from one. Examples are cassava and sweet potatoes in Uganda, and maize in Tanzania. The latter changes from 12 percent of total consumption to nine percent. These are examples of goods that have large changes in their relative share of consumption over time—these changes could be due to changes in relative prices, or changes in the distribution of wealth with non-linear Engel curves. Either way, these large changes in expenditure shares illustrate the fragility of the approach.

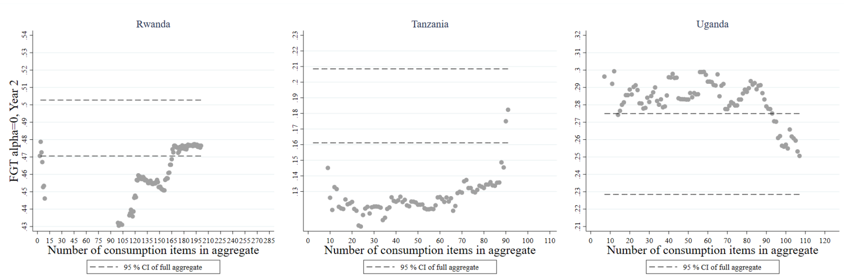

Figure 6. Selecting linear goods, FGT P_0.

5. Conclusion

A standard approach to measuring consumer or household welfare involves first constructing a measure of total aggregate consumption expenditures. These totals are usually elicited by asking about many detailed individual expenditures, and then summing over these to obtain the desired aggregate. The data collection involved is both expensive and time-consuming.

This paper explores the idea of choosing, *ex ante*, a subset of all consumption goods, and forming a 'sub-aggregate' consisting only of expenditures on this subset. If such a sub-aggregate could be used instead of total expenditures this could lead to less expensive data collection. A result due to Lanjouw and Lanjouw (2001) is encouraging in this regard, but requires the Engel curve for the sub-aggregate to be stable over time. We extend their result, and show that the critical feature is that the Engel curve for the sub-aggregate be linear regardless of prevailing prices.

We use some basic consumer theory to establish necessary and sufficient conditions for the linearity of the sub-aggregate Engel curve. Unfortunately these conditions are quite stringent if relative prices vary over time—effectively *all* Engel curves must be linear.

It remains possible that Engel curves in fact possess the desired linearity, or that changes in Engel curves due to changes in relative prices are negligible in real-world data. Accordingly, we use data from Rwanda, Uganda, and Tanzania and search across different goods to identify those which feature (nearly) linear Engel curves in a base period. We then use these goods to construct a sub-aggregate in a subsequent period. Consistent with theory, the performance of this method turns out to be extremely bad at measuring head-count poverty rates. One should *not* construct sub-aggregates for this purpose by relying on the fact that Engel curves may be linear at some time and place, because they are unlikely to be linear in a different time or place since prices are likely to be different.

Perhaps there are better ways to construct a sub-aggregate? Our conditions are both necessary and sufficient, so theory tells us that the answer is "no". Another idea often used in practice involves constructing a sub-aggregate by choosing goods with large expenditure shares. If one can do this in such a way that a very large percentage of *all* households expenditures were in the sub-aggregate this would hold promise. But if Engel curves aren't linear then there will be variation in expenditure shares across households, and identifying goods that have large shares on average will tend to select goods that have a low income elasticity. This is more or less the opposite of what one would wish, since expenditures on these goods will convey little information about underlying household resources.

Finally, as a practical matter, any measured consumption aggregate is likely to be an incomplete or a reduced aggregate approximation to the full aggregate. For instance, the number of consumption items (or categories) for which data are collected from households in LSMS surveys ranges from 37 to 305, with the mean being 137 and the median 130 (Beegle et al. 2010). Arguably the value of public goods and time and leisure should also be included in a comprehensive or "full" consumption expenditure aggregate for measurement of poverty, but in most cases this consumption is excluded. The consumption of publicly provided health and education, as well as the use value of durable goods are currently included in some countries, but excluded in other countries. These matters have been largely ignored in poverty tracking, but our results strongly suggest that they may be of first order importance.

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